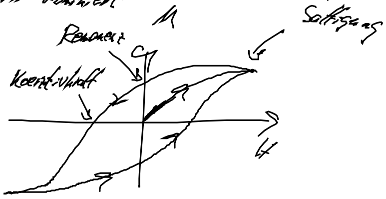
WdH

Suszeptibilität   magnet. Feldstärke

$$\underbrace{M}_{\text{Magnetisierung}} = \chi_m H \qquad\qquad B = \underbrace{(1+4\pi\chi_m)}_{\mu\ \hat{=}\ \text{Permeabilität}}\cdot H$$

- Diamagnetismus: überall, Dipole werden induziert $\quad \chi_m < 0$
- Paramagnetismus: perm. Dipole richten sich aus $\quad \chi_m > 0$
- Kollektiver Magnetismus: Dipol-Dipol-WW dominiert



- $$\oint_{\partial F} \underline{E}\cdot d\underline{r} = -\frac{1}{c}\frac{d}{dt}\iint_F \underline{B}\cdot d\underline{F} \quad \Leftrightarrow \quad \nabla\times E = -\frac{1}{c}\dot{B}$$
  "Lenz'sche Regel"   Faraday-Gesetz
- MS: $\nabla\cdot j = 0 \quad \rightarrow$ jetzt $\frac{\partial\rho}{\partial t} + \nabla\cdot j = 0$
- $$\boxed{\begin{aligned} \nabla\cdot D &= 4\pi\rho \\ \nabla\cdot B &= 0 \\ \nabla\times E + \frac{1}{c}\partial_t B &= 0 \\ \nabla\times H - \frac{1}{c}\partial_t D &= \frac{4\pi}{c} j \end{aligned}}$$
  $$\left.\begin{aligned} D &= \varepsilon\cdot E \\ B &= \mu\cdot H \end{aligned}\right\} \text{für lin. Medien}$$
  $$j = \sigma_c \cdot E$$
  σ_c: Leitfähigkeit
- Potentiale
  $$B = \nabla\times A \qquad E = -\nabla\Phi - \frac{1}{c}\partial_t A$$
  A: Vektorpotential $\qquad A \rightarrow A + \nabla\cdot\Lambda(r,t)$
  $$\left.\begin{aligned} \Box\,\Phi &= \left\{\Delta - \frac{1}{c^2}\partial_t^2\right\}\Phi = -4\pi\rho \\ \Box\,\underline{A} &= \left\{\Delta - \frac{1}{c^2}\partial_t^2\right\}\underline{A} = -\frac{4\pi}{c}\cdot j \end{aligned}\right\} \text{ in Lorenz-Eichung } \nabla A + \frac{1}{c}\partial_t\Phi = 0$$

Lorenz-Eichung ist nicht eindeutig

§.3.2. Coulomb-Eichung

$\nabla\cdot A = 0$

$$-4\pi\rho = \Delta\Phi + \cancel{\frac{1}{c}\partial_t(\nabla\cdot A)}$$
$$-\frac{4\pi}{c}j = \cancel{-\nabla(\nabla\cdot A)} + \Delta A - \frac{1}{c^2}\partial_t^2 A - \frac{1}{c}\nabla\cdot\partial_t\Phi$$

$$\Delta\Phi = -4\pi\rho \quad \longrightarrow \quad \Phi(r,t) = \int \frac{\rho(r',t)}{|r-r'|}\,d^3r' \qquad \text{"instantane Eichung"}$$

$$\Box A = -\frac{4\pi}{c}\cdot j + \frac{1}{c}\nabla\partial_t\Phi$$
$$= -\frac{4\pi}{c}\cdot j - \underbrace{\frac{1}{c}\nabla\int\frac{[\nabla'\cdot j(r',t)]}{|r-r'|}d^3r'}$$ aus Kontin.-gl.

Stromdichte kontrahiert A $+\frac{4\pi}{c}\cdot j_\parallel$

$$j_\parallel = -\frac{1}{4\pi}\nabla\cdot\int\frac{\nabla'\cdot j(r',t)}{|r-r'|}d^3r'$$

$$j_\perp = \frac{1}{4\pi}\nabla\times\left(\nabla\times\int\frac{j(r',t)}{|r-r'|}d^3r'\right) \qquad \nabla\times(\nabla\times b) = \nabla(\nabla\cdot b) - \Delta b$$

$$j_\parallel + j_\perp = +\frac{1}{4\pi}\nabla\left(\nabla\cdot\int\frac{j(r',t)}{|r-r'|}d^3r'\right) + j(r,t) - \underline{\frac{1}{4\pi}\nabla\cdot\int\frac{\nabla' j(r',t)}{|r-r'|}d^3r'}$$
$$= \cancel{-\frac{1}{4\pi}\nabla\int j(r',t)\left(\nabla'\frac{1}{|r-r'|}\right)d^3r'} + j(r,t)$$
$$\underline{-\frac{1}{4\pi}\nabla\int\nabla'\left(\frac{j(r',t)}{|r-r'|}\right)d^3r' + \cancel{\frac{1}{4\pi}\nabla\int j(r',t)\left(\nabla'\frac{1}{|r-r'|}\right)d^3r'}}$$
$$= j(r,t)$$ Gauss

$$\Rightarrow \Box A = -\frac{4\pi}{c}j_\perp \qquad \text{"transversale Eichung"}$$

z.B.: $\rho = 0$ $\quad j = 0 \quad \rightarrow \Phi = 0 \qquad \Box A = 0$

dann: $B = \nabla\times A \qquad E = -\frac{1}{c}\partial_t A$

jetzt $A' = A + \nabla\Lambda \qquad \Phi' = \Phi - \frac{1}{c}\frac{\partial\Lambda}{\partial t}$

$$\nabla\cdot A' = 0 = \nabla\cdot A + \nabla(\nabla\Lambda) = \nabla\cdot A + \Delta\Lambda = 0$$
$$\Delta\Lambda = -\nabla\cdot A$$

5.4. Erhaltungssätze und Poynting-Vektor

5.4.1. Energie-Erhaltung

$$\underline{F} = q\left(\underline{E} + \frac{1}{c}\underline{v}\times\underline{B}\right) \qquad v = \frac{dr}{dt}$$

$$dW = \underline{F}\cdot d\underline{r} = q\cdot\underline{E}\cdot d\underline{r} \qquad \text{für PL}$$

$$q \to \rho(r,t)\cdot d^3r$$

Kraftdichte $f(r,t) = \rho(r,t)\left(\underline{E} + \frac{1}{c}\cdot\underline{v}\times\underline{B}\right)$

Leistungsdichte $f(r,t)\cdot v(r,t) = \rho(r,t)\,(\underline{v}\cdot\underline{E}) = (j\cdot E)$

Leistung in Volumen V: $\frac{dW_V^{(mech)}}{dt} = \int_V (j\cdot E)\,d^3r$

nutze $\nabla\times H - \frac{1}{c}\partial_t D = \frac{4\pi}{c}\cdot j$

$$\to (E\cdot j) = \frac{c}{4\pi}\underline{E\cdot(\nabla\times H)} - \frac{1}{c}E\cdot\frac{\partial D}{\partial t} \qquad \varepsilon_{ijk} = \varepsilon_{kij} = -\varepsilon_{jik}$$

$$\nabla\cdot(E\times H) = \underline{H\cdot(\nabla\times E)} - E(\nabla\times H) = \sum_{ijk} \varepsilon_{ijk}\, \partial_i (E_j \cdot H_k)$$

$$(E\cdot j) = \nabla\cdot(E\times H) - \frac{1}{4\pi} E\cdot\frac{\partial D}{\partial t} - \frac{1}{4\pi} H\cdot\frac{\partial B}{\partial t}$$

$$\frac{dW_V}{dt} = -\frac{1}{4\pi}\int d^3r \left[c\,\nabla\cdot\left(E\times H + \underline{E\cdot\partial_t D + H\cdot\partial_t B}\right)\right]$$

Poynting-Vektor $S = \frac{c}{4\pi}(E\times H)$

$w = \frac{1}{8\pi}(E\cdot D + B\cdot H)$

Def. der Energiedichte des EM-Feldes

für lineare Medien

$H\cdot\partial_t B = \frac{1}{2}\partial_t (H\cdot B)$

$E\cdot\partial_t D = \frac{1}{2}\partial_t (E\cdot D)$

$$\frac{dW_V^{\mathrm{mech}}}{dt} = \int_V j\cdot E\, d^3r = -\int_V \left(\frac{\partial w}{\partial t} + \underline{\nabla\cdot S}\right) d^3r$$

$$\boxed{\frac{\partial w}{\partial t} + \nabla\cdot S = -j\cdot E}$$

Poynting-Theorem

mech Leistungsdichte

EM Leistungsdichte

Energiefluss durch ∂V

$$\frac{d}{dt}\left(W_V^{\mathrm{mech}} + W_V^{\mathrm{Feld}}\right) = -\oint_{\partial V} \underline{S}\cdot d\underline{F}$$

$S \neq 0$ muss nicht immer mit einem Energiestrom korrespondieren

denn $S \to S + \nabla\times b$ lässt die Bilanz invariant

Beispiel: homogene Felder

$$S = \frac{c}{4\pi}\begin{pmatrix} E \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ H \end{pmatrix} = \frac{c}{4\pi}\begin{pmatrix} 0 \\ -E\cdot H \\ 0 \end{pmatrix} \neq 0$$

aber: $\underline{\nabla\cdot S} = 0 \longrightarrow$ kein Energiestrom

5.4.2. Impulsbilanz

$$F = \frac{d}{dt} p^{mech} = q\left(E + \frac{1}{c} v \times B\right) \quad \text{für 1 PL}$$

$$\frac{d}{dt} P_V^{mech} = \int_V \left(\rho E + \frac{1}{c} j \times B\right) d^3r$$

$$\nabla \cdot D = 4\pi \rho$$

$$\nabla \times H - \frac{1}{c}\partial_t D = \frac{4\pi}{c} j$$

Gesamtimpuls in V

$$= \frac{1}{4\pi} \int_V \left(E \cdot (\nabla \cdot D) - B \times (\nabla \times H) + \frac{1}{c} B \times \frac{\partial D}{\partial t}\right) d^3r$$

$$= \frac{1}{4\pi} \int_V \left[E(\nabla D) + H(\nabla B) - B \times (\nabla \times H) - \underline{\frac{1}{c}\partial_t (B \times D)} - D \times (\nabla \times E)\right] d^3r$$

im Vakuum $\quad D = E \quad B = H$

$$\underline{P}_V^{Feld} = \frac{1}{4\pi c} \int_V (E \times B)\, d^3r = \frac{1}{c^2} \int \underline{S}\, d^3r$$

$$\frac{d}{dt}\left(P_V^{mech} + P_V^{Feld}\right) = \frac{1}{4\pi} \int_V \left[\underline{E \cdot (\nabla \cdot E)} + B \cdot (\nabla B) - B \times (\nabla \times B) \underline{- E \times (\nabla \times E)}\right] d^3r$$

$$\left[E(\nabla \cdot E) - E \times (\nabla \times E)\right]_i = \sum_j \partial_j \left[E_i E_j - \frac{\delta_{ij}}{2} \sum_k E_k E_k\right]$$

$$\sum_j \partial_j F_{ij} \qquad \sum_k \varepsilon_{ijk} \varepsilon_{lmk} = \delta_{il}\delta_{jm} - \delta_{im}\delta_{jl}$$

auch für $E \to B$

Maxwell'scher Spannungstensor

$$\boxed{T_{ij} = \frac{1}{4\pi}\left[E_i E_j + B_i B_j - \frac{1}{2}\delta_{ij} \sum_k \left(E_k E_k + B_k B_k\right)\right]}$$

$$\sum_i T_{ii} = -w$$

$$T_{ij} = T_{ji}$$

$$\frac{d}{dt}\left(P_V^{mech} + P_V^{Feld}\right)_i = \int_V \left(\sum_j \partial_j T_{ij}\right) d^3r$$

$$\underline{t}_i = \begin{pmatrix} T_{i1} \\ T_{i2} \\ T_{i3} \end{pmatrix}$$

$$\Rightarrow \sum_j \partial_j T_{ij} = \nabla \cdot t_i$$

$$= \oint_{\partial V} \underline{t}_i \cdot \underline{n}\, dF = \oint_{\partial V} \sum_j T_{ij} \cdot n_j\, dF$$

Kraft auf eine Randfläche der Größe ΔF

$$F_i = -\sum_j T_{ij} \cdot n_j \cdot \Delta F$$

$$\underline{F} \cdot \underline{n} = \sum_i F_i \cdot n_i = -\sum_{ij} T_{ij} n_i n_j \Delta F$$

$$\boxed{P_{Strahlung} = \frac{F \cdot n}{\Delta F} = -\sum_{ij} T_{ij} n_i n_j}$$

Strahlungsdruck