

WdL

• Konsistenz als RB ande GF

$$\left\{ \Delta - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right\} G(\underline{r}-\underline{r}', t-t') = -4\pi \delta(\underline{r}-\underline{r}') \delta(t-t')$$

$$FT \left[-\partial^2 + \frac{\omega^2}{c^2} \right] g(\underline{k}, \omega) = -4\pi$$

$\Rightarrow G(\underline{r}-\underline{r}', t-t') = \frac{\delta\left(t-t' - \frac{|\underline{r}-\underline{r}'|}{c}\right)}{|\underline{r}-\underline{r}'|}$ "retardierte" GF

$t' = t - \frac{|\underline{r}-\underline{r}'|}{c} < t$

retard. Potentiale

$$\underline{\Phi}(\underline{r}, t) = \int \frac{\rho(\underline{r}', t - \frac{|\underline{r}-\underline{r}'|}{c})}{|\underline{r}-\underline{r}'|} d^3 r'$$

$$\underline{A}(\underline{r}, t) = \frac{1}{c} \int \frac{j(\underline{r}', t - \frac{|\underline{r}-\underline{r}'|}{c})}{|\underline{r}-\underline{r}'|} d^3 r'$$

C. f.

6.3. Pot. einer bewegten PL

$$\rho(\underline{r}, t) = e \cdot \delta(\underline{r} - \underline{r}_0(t)) \quad j(\underline{r}, t) = e \cdot \underline{v}_0(t) \delta(\underline{r} - \underline{r}_0(t))$$

↑
vorgegeben

$$\underline{v}_0(t) = \frac{d\underline{r}_0(t)}{dt}$$

wichtig $\underline{v}_0(t)$ kann sich ändern

$$\underline{\Phi}(\underline{r}, t) = e \int d^3 r' \frac{\delta(\underline{r}' - \underline{r}_0(t - \frac{|\underline{r}-\underline{r}'|}{c}))}{|\underline{r}-\underline{r}'|} \quad \text{schwierig}$$

$$= e \int d^3 r' dt' \frac{\delta(\underline{r}' - \underline{r}_0(t')) \cdot \delta(t' - t + \frac{|\underline{r}-\underline{r}'|}{c})}{|\underline{r}-\underline{r}'|}$$

$$= e \int dt' \frac{\delta(t' - t + \frac{|\underline{r}-\underline{r}_0(t')|}{c})}{|\underline{r}-\underline{r}_0(t')|} \frac{d\underline{h}}{d\underline{h}} \quad \text{hier noch schwierig}$$

$$h = t' - t + \frac{1}{c} \sqrt{(x-x_0(t'))^2 + (y-y_0(t'))^2 + (z-z_0(t'))^2} \quad \left(\begin{matrix} \cdot \\ \cdot \\ \cdot \end{matrix} \right) \neq 0 \mid \text{Subst.}$$

$$\frac{dh}{dt'} = 1 - \frac{(x-x_0(t')) \frac{dx_0}{dt'} + (y-y_0(t')) \frac{dy_0}{dt'} + (z-z_0(t')) \frac{dz_0}{dt'}}{c \sqrt{\dots}} = 1 - \frac{1}{c} \cdot \underline{h}(t') \cdot \underline{v}_0(t')$$

$$h(t') = \frac{r - r_0(t')}{|r - r_0(t')|}$$

$$\Phi(r, t) = e \int dt' \frac{dt'}{dt} \frac{1}{|r - r_0(t')|} \delta(u)$$

$$\Phi(r, t) = \frac{e}{|r - r_0(t')| \left[1 - \frac{1}{c} \dot{r}_0(t') \cdot \underline{u}(t') \right]} \quad \left| \quad t' = t - \frac{|r - r_0(t')|}{c} \right.$$

$$A(r, t) = \frac{\frac{e}{c} \dot{u}(t')}{|r - r_0(t')| \left[1 - \frac{1}{c} \dot{r}_0(t') \cdot \underline{u}(t') \right]} \quad \left| \quad t' = t - \frac{|r - r_0(t')|}{c} \right.$$

Lienard-Wiechert Potentiale

• $\dot{v}_0 = 0 \rightarrow$ konstant $\Phi(r, t) \rightarrow \frac{e}{|r - r_0|}$ $A(r, t) = 0$
 $r_0(t) = r_0 = \text{const}$

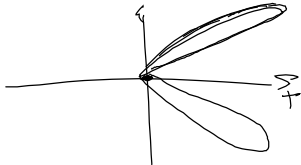
• $v_0 = \text{const}$ $\Phi(r, t) = e \cdot \delta(r - v_0 \underline{e}_x \cdot t)$

löse $t' = t - \frac{1}{c} \sqrt{(x - v_0 t')^2 + y^2 + z^2}$
 $t' = \frac{t - \frac{v_0 x}{c^2} \pm \frac{1}{c} \sqrt{(x - v_0 t')^2 + y^2 + z^2}}{1 - \frac{v_0^2}{c^2}}$

$$\Phi(r, t) = \frac{r \cdot e}{\sqrt{(\gamma x - \beta \gamma ct)^2 + y^2 + z^2}} \quad A_x(r, t) = \beta \cdot \Phi(r, t)$$

Ausdrück $\underline{A}, \underline{\Phi} \rightarrow \underline{E} \ \& \ \underline{B} \rightarrow \underline{S} = \frac{c}{4\pi} \underline{E} \times \underline{B}$ (Volumen)

$$\frac{dP}{d\Omega} = R^2 \cdot (\underline{S} \cdot \underline{e}_R) \quad \text{beschreibt Energiefluss in Raumwinkel } d\Omega$$



6.4. Strahlung zeitlich oszillierender Dipole

wellen mikrosk. MWG

$$\nabla \cdot \underline{E} = 4\pi \rho$$

$$\nabla \cdot \underline{B} = 0$$

$$\nabla \times \underline{E} + \frac{1}{c} \partial_t \underline{B} = 0$$

$$\nabla \times \underline{B} - \frac{1}{c} \partial_t \underline{E} = \frac{4\pi}{c} \underline{j}$$

sind linear in \underline{E} & \underline{B} und ρ & \underline{j}

$$\rho, \underline{j} \in \mathbb{C} \rightarrow \underline{E}, \underline{B} \in \mathbb{C}$$

\rightarrow wir können die Felder & Quellen komplex ansetzen

$$E(r, t) = \text{Re} \{ \underline{E} \} \in \mathbb{R} \in \mathbb{C} \quad \underline{E}(r), \underline{E} \in \mathbb{C}$$

Quellen

$$\varphi(r, t) = \operatorname{Re} \int \rho(r') \cdot e^{-i\omega t} d^3r'$$

$$j(r, t) = \operatorname{Re} \int j(r') \cdot e^{-i\omega t} d^3r'$$

$$A(r, t) = \frac{1}{c} \int d^3r' \int dt' \frac{j(r', t') \cdot \delta(t - t' + \frac{|r-r'|}{c})}{|r-r'|}$$

$$= \frac{1}{c} \operatorname{Re} \int d^3r' \frac{j(r') e^{i\frac{\omega}{c}|r-r'|}}{|r-r'|} e^{-i\omega t}$$

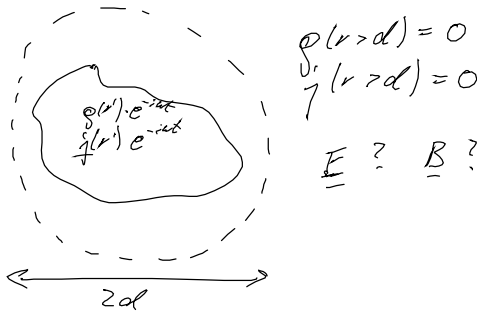
$$= \operatorname{Re} \int d^3r' \frac{j(r') e^{-i\omega t}}{|r-r'|}$$

\uparrow
 $\in \mathbb{C}$

$$\underline{A}(r) = \frac{1}{c} \int d^3r' \frac{j(r') e^{-i\omega |r-r'|}}{|r-r'|}$$

raum. Komp. des Vektorpot.

$$B(r, t) = \operatorname{Re} (\nabla \times A(r)) \cdot e^{-i\omega t}$$



$$\nabla \times B - \frac{1}{c} \partial_t E = \frac{\mu_0}{c} j \quad \xrightarrow{\text{in Fernfeld}} 0$$

$$\partial_t E = c \cdot \nabla \times (\nabla \times A(r)) \cdot e^{-i\omega t} \quad ; \quad r > d$$

$\uparrow \in \mathbb{C} \quad \quad \quad \uparrow \in \mathbb{C}$

$$\underline{E} = \frac{i}{\omega} \nabla \times (\nabla \times A(r)) \cdot e^{-i\omega t} \quad ; \quad r > d$$

$$\underline{E}(r, t) = \operatorname{Re} \{ \underline{E} \} \quad \rightarrow \text{Vorgabe von } j(r', t') \text{ ist ausreichend}$$

a.) Strahlungs-Näherung

$$\lambda = \frac{2\pi c}{\omega} = \frac{2\pi}{k}$$

$$r, r' \ll \lambda$$

$$e^{i\omega |r-r'|/c} = e^{i \cdot 2\pi \frac{|r-r'|}{\lambda}} \approx 1$$

$$A(r) = \frac{1}{c} \int d^3r' \frac{j(r')}{|r-r'|}$$

Retardierung hat keinen Effekt

b) Fernzone $d \ll r$ $d \ll \lambda$

"Langwellen-Näherung"

$$|r-r'| = \sqrt{r^2 + (r')^2 - 2r \cdot \frac{r'}{r}} \approx r \left(1 - \frac{1}{r} \underline{h} \cdot \underline{r}' + \dots \right)$$

$$h = \frac{r}{|r|}$$

$$\rightarrow e^{i k |r-r'|} \approx e^{i k r} e^{-i k \cdot \underline{h} \cdot \underline{r}'} = e^{i k r} \left[1 - i k \cdot (\underline{h} \cdot \underline{r}') + \dots \right]$$

$$\frac{1}{|r-r'|} \approx \frac{1}{r} \left(1 + \frac{1}{r} \cdot \underline{h} \cdot \underline{r}' + \dots \right)$$

$$\frac{e^{i k |r-r'|}}{|r-r'|} = \frac{e^{i k r}}{r} \left[1 + (\underline{h} \cdot \underline{r}') \left(\frac{1}{r} - i k \right) + \dots \right]$$

Kugelwelle (laufende)

$$A(\underline{r}) = \frac{1}{c} \frac{e^{i k r}}{r} \int d^3 r' j(\underline{r}') + \frac{1}{c} \left(\frac{1}{r} - i k \right) \frac{e^{i k r}}{r} \int j(\underline{r}') (\underline{h} \cdot \underline{r}') d^3 r' + \dots$$

elekt. Dipolterm

elekt. Quadrupol & magnet. Dipol

hier

G.S. Dipolstrahlung

kont.-Gleichung $\partial \rho + \nabla \cdot \underline{j} = 0 \rightarrow -i \omega \cdot \overset{EC}{\rho}(\underline{r}) + \nabla \cdot \overset{EC}{\underline{j}}(\underline{r}) = 0$

$$\int d^3 r' j(\underline{r}') = \int d^3 r' (j \cdot \underline{r}) \overset{EC}{\perp} \frac{1}{r} - \int d^3 r' \underline{r} (\nabla \cdot \underline{j})$$

$$= -i \omega \int d^3 r' \underline{r} \cdot \overset{EC}{\rho}(\underline{r}) = -i \omega \underline{P}$$

EC

$$\underline{P} = \int d^3 r' \underline{r} \cdot \rho(\underline{r}) \quad \text{stat. Dipolmoment}$$

$$P(t) = \int d^3 r' \underline{r} \cdot \rho(\underline{r}, t) = \text{Re} \left\{ \underline{P} \cdot e^{-i \omega t} \right\}$$

$$A(\underline{r}) = -i \cdot k \cdot \underline{P} \cdot \frac{e^{i k r}}{r} + \dots \quad A(\underline{r}, t) = \text{Re} A(\underline{r}) \cdot e^{-i \omega t}$$

nur Dipolterm

$$\partial_r \frac{e^{i k r}}{r} = i k \frac{e^{i k r}}{r} - \frac{e^{i k r}}{r^2} \approx i k \frac{e^{i k r}}{r}$$

$$k = \frac{2\pi}{\lambda}$$

$$\lambda \ll r$$

vernachl.

$$\boxed{d \ll \lambda \ll r}$$

Ersetzung in Fernfeld: $\nabla \rightarrow i \underline{e}_r \cdot \underline{e}_r$

$$B(r) = \frac{1}{r^2} (\underline{e}_r \times \underline{p}) \frac{e^{i \omega r}}{r} \quad B = \nabla \times A(r) \approx i \omega \underline{e}_r \times A(r)$$

$$E(r) = \frac{1}{2} \nabla \times B(r) \approx -\frac{1}{2} \omega^2 \underline{e}_r \times (\underline{e}_r \times \underline{p}) \frac{e^{i \omega r}}{r} \quad E(r) \perp B(r) \perp \underline{e}_r$$

Fernfeld eines oszillierenden Dipols (radiating dip.)

Poynting-Vektor

$$S(r) = \frac{c}{4\pi} \langle E(r,t) \times B(r,t) \rangle = \frac{c}{4\pi} \langle [\text{Re } \underline{E}(r) \cdot e^{-i\omega t}] \times [\text{Re } \underline{B}(r) \cdot e^{-i\omega t}] \rangle$$

betriebe $a(t) = \text{Re } a_0 e^{-i\omega t}$ $b(t) = \text{Re } b_0 e^{-i\omega t}$

$$= \frac{1}{2} a_0 e^{-i\omega t} + \frac{1}{2} a_0^* e^{+i\omega t}$$

$$\langle a(t) \cdot b(t) \rangle = \frac{1}{4} \langle (a_0 e^{-i\omega t} + a_0^* e^{+i\omega t}) (b_0 e^{-i\omega t} + b_0^* e^{+i\omega t}) \rangle$$

$$= \frac{1}{4} (a_0 b_0^* + a_0^* b_0) = \frac{1}{2} \cdot \text{Re} \{ a_0 \cdot b_0^* \}$$

$\Rightarrow S(r)$

Leistungs in Raumwinkel

$$d\Omega = \sin \vartheta d\vartheta d\varphi$$

$$dP = \langle S \rangle \cdot \underline{e}_r \cdot r^2 \cdot d\Omega$$

$$\frac{dP}{d\Omega} = r^2 \langle \underline{e}_r \cdot \underline{S} \rangle$$

$$\frac{dP}{d\Omega} = \frac{c r^2}{4\pi} \underline{e}_r \cdot \langle E(r,t) \times B(r,t) \rangle = \frac{c r^2}{4\pi} \frac{1}{2} \underline{e}_r \cdot [\text{Re} \{ \underline{E}(r) \times \underline{B}^*(r) \}]$$

$$= \frac{c \cdot \omega^4}{8\pi} \underline{e}_r \cdot [(\underline{e}_r \times \underline{p}) \times \underline{e}_r] \times (\underline{e}_r \times \underline{p}^*)$$

$$= \frac{\omega^4}{8\pi c^3} [(\underline{e}_r \times \underline{p}) \times \underline{e}_r] \cdot [(\underline{e}_r \times \underline{p}^*) \times \underline{e}_r]$$

$$= \frac{\omega^4}{8\pi c^3} (\underline{e}_r \times \underline{p}) \cdot (\underline{e}_r \times \underline{p}^*) = \frac{\omega^4}{8\pi c^3} |\underline{e}_r \times \underline{p}|^2$$

Strahlungsleistung pro Raumwinkel

falls $\underline{p} = \begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix} \cdot e^{i\delta}$

$$\frac{dP}{d\Omega} = \frac{\omega^4}{8\pi c^3} |\underline{p}_r|^2 \cdot \sin^2 \vartheta$$

Strahlungschar. eines Dipols

$$\vartheta = \angle(\underline{p}_r, \underline{e}_r)$$

