Wdh.

- holomorphe Fkt $f(z) = u(x,y) + i\,v(x,y)$ $\qquad z = x + iy$

$$\frac{\partial u}{\partial x} = + \frac{\partial v}{\partial y} \qquad \frac{\partial u}{\partial y} = - \frac{\partial v}{\partial x}$$

$$\rightsquigarrow \left\{\partial_x^2 + \partial_y^2\right\} u(x,y) = 0$$

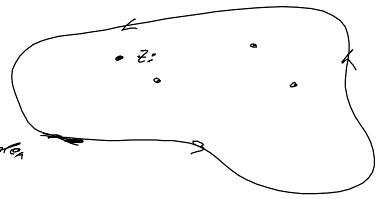
- Laurent-Reihe

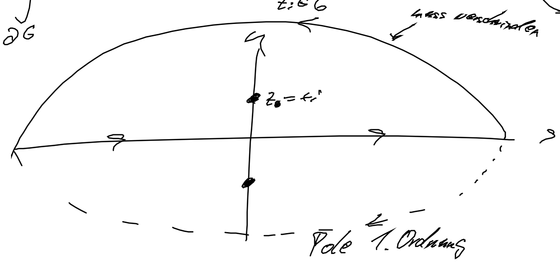
$$f(z) = \sum_{n=-\infty}^{\infty} a_n \cdot (z-z_0)^n$$

+ hebbare Sing. $a_{n<0} = 0 \longrightarrow \frac{\sin(z)}{z}$ bei $z=0$
+ Pol der Ordnung ($n>0$): $a_{n<-n} = 0 \qquad a_{-n} \neq 0 \to \frac{1}{(z-i)^3}$
+ essentielle Sing. $e^{1/z}$
+ $a_{-1} \longrightarrow$ Residuum von $f(z)$ an z_0

- Residuensatz

$$\oint_{\partial G} f(z)\, dz = 2\pi i \sum_{z_i \in G} \underset{z=z_i}{\mathrm{Res}}\, f(z)$$





Pole 1. Ordnung

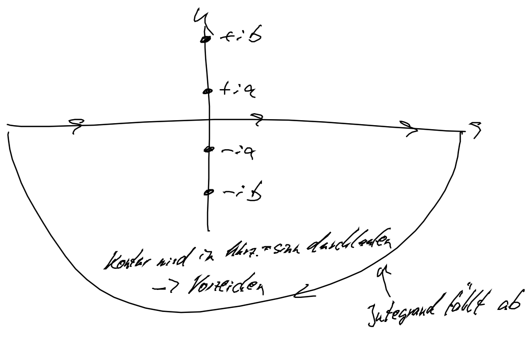
$$a_{-1} = \lim_{z\to z_0} (z-z_0)\cdot f(z)$$

$$\int_{-\infty}^{\infty} \frac{dx}{x^2+1} = \oint \frac{dz}{(z-i)(z+i)} = 2\pi i \left.\frac{1}{z+i}\right|_{z=i} = \pi$$

$$F(k) = \int \frac{dx\; e^{-ikx}}{(x^2+a^2)(x^2+b^2)} = \int \frac{dz \quad e^{-ikz}}{\underline{(z+ia)}(z-ia)(z+ib)(z-ib)}$$

$$e^{-ik(x+iy)} = e^{-ikx}\, e^{+ky}$$

$a>0 \qquad b>0 \qquad F(-k) = F(+k) \quad \rightsquigarrow \quad a, b, k > 0$



$$\underset{z=-ia}{\mathrm{Res}}\, f(z) = \frac{e^{-ik(-ia)}}{(-2ia)(-ia+ib)(-ia-ib)}$$

$$F(k) = \frac{\pi}{b^2-a^2}\left[\frac{e^{-k|a|}}{|a|} - \frac{e^{-k|b|}}{|b|}\right]$$

- Hauptwert-Integrale $(a>0)$

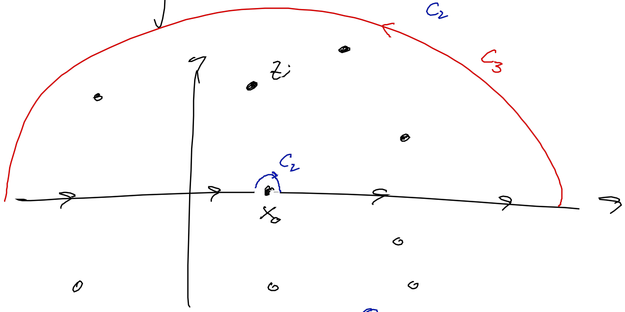
$$\mathcal{P}\int_{-a}^{+a} \frac{dx}{x} = 0 \qquad\qquad \mathcal{P}\int_a^b g(x)\,dx := \lim_{\varepsilon\to 0}\left[\int_a^{c-\varepsilon} g(x)\,dx + \int_{c+\varepsilon}^{b} g(x)\,dx\right], \quad c \in (a,b)$$

Def. Hauptwert

Wenn $g(x) = \frac{f(x)}{x - x_0}$ $\quad f(x)$ holomorph $\underline{\forall x \in \mathbb{R}}$

$$\mathcal{P}\int \frac{f(x)}{x-x_0}\,dx + \int_{C_2} \frac{f(z)}{z-x_0}\,dz + \underbrace{\int_{C_3} \frac{f(z)}{z-x_0}}_{\to 0} = 2\pi i \sum_{\substack{z=z_i \\ z_i:\, \mathrm{Im}(z_i)>0}} \mathrm{Res}\, \frac{f(z)}{z-z_0}$$

falls $f(z) \propto \frac{1}{R^{1+\epsilon}}$



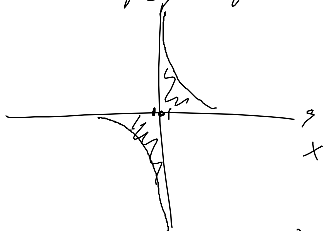
$$\int_{C_2} \frac{f(z)}{z-x_0}\,dz = \lim_{R\to 0} \int_{\pi}^{0} \frac{f(x_0+R\cdot e^{i\varphi})}{\cancel{R\cdot e^{i\varphi}}}\, i \cancel{R\cdot e^{i\varphi}}\, d\varphi = f(x_0)\cdot(-\pi i)$$

$$\boxed{\mathcal{P}\int \frac{f(x)}{x-x_0}\,dx = \pi i \cdot f(x_0) + 2\pi i \sum_{\substack{z=z_k \\ z_k:\, \mathrm{Im}(z_k)>0}} \mathrm{Res}\, \frac{f(z)}{z-z_0}} \in \mathbb{R}$$

$$I = \int \frac{\sin(a\cdot x)}{x}\,dx = (-i)\,\mathcal{P}\int \frac{e^{+i a\cdot x}}{x}\,dx = (-i)\,\mathcal{P}\int \frac{\cos(ax) + i\sin(ax)}{x}\,dx$$

$$= \pi\cdot i\,(-i) = \pi$$

$$\mathcal{P}\int \underbrace{\frac{\cos(ax)}{x}}_{\tilde{g}(x)}\,dx = 0 \qquad \tilde{g}(x) \to \tilde{g}(-x) = -\tilde{g}(x)$$



6.2 Retardierte Potentiale

$$\left.\begin{aligned} \Box\, \Phi &= -4\pi \overset{\text{Ladungsdichte}}{\rho} \\ \Box\, \underline{A} &= -\frac{4\pi}{c}\, \underset{\text{Stromdichte}}{\underline{j}} \end{aligned}\right\}$$ WG für Pot. in Lorenz-Eichung

einfacher

$$\Box\, \psi = \left\{\Delta - \frac{1}{c^2}\partial_t^2\right\}\psi(r,t) = -4\pi\cdot f(\underline{r},t)$$

$$\boxed{\left\{\Delta - \frac{1}{c^2}\partial_t^2\right\} G(\underline{r},t,\underline{r}',t') = -4\pi\,\delta(\underline{r}-\underline{r}')\,\delta(t-t')}$$

Poisson-Gl.: $\Phi(\underline{r}) = \int \frac{\overset{\text{Quelle}}{\rho(\underline{r}')}}{\underset{\text{GF}}{|\underline{r}-\underline{r}'|}}\, d^3 r'$

Wenn G gefunden ist $\quad$ Translationsinvarianz: $G(\underline{r},t,\underline{r}',t') = G(\underline{r}-\underline{r}', t-t')$

$$\psi(\underline{r},t) = \int d^3 r'\, dt'\, G(\underline{r}-\underline{r}', t-t')\, f(\underline{r}',t')$$

betrachte FT $\quad g(\underline{k},\omega) = \int G(\underline{r},t)\, e^{-i(\underline{k}\cdot\underline{r} - \omega t)}\, d^3 r\, dt$

$$G(\underline{r},t) = \frac{1}{(2\pi)^4} \int g(\underline{k},\omega)\, e^{+i(\underline{k}\cdot\underline{r} - \omega t)}\, d^3k\, d\omega$$

$$\circ \quad \delta^{(3)}(\underline{r}-\underline{r}')\,\delta(t-t') = \frac{1}{(2\pi)^4} \int e^{+i[\underline{k}(\underline{r}-\underline{r}') - \omega(t-t')]}\, d^3k\, d\omega$$

$$\int g(\underline{k},\omega)\, \Box\, e^{+i\overbrace{[\underline{k}(\underline{r}-\underline{r}') - \omega(t-t')]}^{}_{[\dots]}}\, d^3k\, d\omega = \int g(\underline{k},\omega) \left[\frac{1}{c^2}\cdot\omega^2 - \underline{k}\cdot\underline{k}\right] e^{+i[\dots]}$$

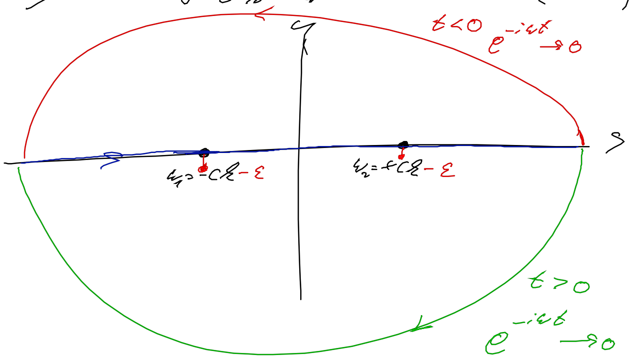
$$= -4\pi \int [1]\, e^{+i[\dots]}$$

$$\Rightarrow g(k,\omega) = \frac{4\pi}{k^2 - \frac{\omega^2}{c^2}}$$

$$\Rightarrow G(\underline{r},t) = \frac{1}{4\pi^3} \int \frac{c^2}{c^2k^2 - \omega^2}\, e^{+i[\underline{k}\cdot\underline{r} - \omega t]}\, d^3k\, d\omega$$

$$= \frac{1}{4\pi^3} \int d^3k\, e^{i\underline{k}\cdot\underline{r}} \cdot \underline{I}(k,t)$$

$$\underline{I}(k,t) = \int \frac{e^{-i\omega t}\, c^2}{c^2k^2 - \omega^2}\, d\omega = -\int \frac{c^2 e^{-i\omega t}}{(\omega - ck)(\omega + ck)}\, d\omega$$



per Hand: Randbedingung

Folge muss nach der Ursache erfolgen

"Kausalität"

$$G(\underline{r}, t<0) \overset{!}{=} 0$$

$$\underline{I}(k,t) = \lim_{\varepsilon\to 0} \int \frac{-c^2\, e^{-i\omega t}}{(\omega - ck + i\varepsilon)(\omega + ck + i\varepsilon)} \to \underline{I}(k, t<0) = 0 \Rightarrow$$

$$= \Theta(t)\cdot 2\pi i \left[\operatorname{Res}_{\omega = ck - i\varepsilon} \frac{c^2 e^{-i\omega t}}{(\omega - ck + i\varepsilon)(\omega + ck + i\varepsilon)} + \operatorname{Res}_{\omega = -ck - i\varepsilon} \frac{c^2 e^{-i\omega t}}{(\omega - ck + i\varepsilon)(\omega + ck + i\varepsilon)}\right]$$

$$= \Theta(t)\cdot 2\pi i \left[\frac{c^2 e^{-ick\cdot t}}{2ck} + \frac{c^2 e^{+ickt}}{-2ck}\right]$$

$$= 2\pi\cdot\Theta(t)\cdot\frac{c}{k}\cdot\sin(c\cdot k\cdot t)$$

$$G(\underline{r},t) = \frac{1}{2\pi^2}\Theta(t) \int_0^\infty k^2\, dk \int_{-1}^{+1} dx \int_0^{2\pi} d\varphi\, \frac{c}{k}\, \sin(c\cdot k\cdot t)\, e^{+i k\cdot r\cdot x}$$

$$= \frac{2}{\pi}\,\Theta\,\frac{c}{r}\frac{1}{2} \int_{-\infty}^{\infty} \underbrace{\sin(c\cdot k\cdot t)}_{\text{analog}}\ \underbrace{\sin(k\cdot r)}_{\frac{1}{2i}\left(e^{+ikr} - e^{-ikr}\right)}\, dk$$

$$= \frac{-1}{4\pi}\,\Theta(t)\cdot\frac{c}{r}\cdot(2\pi)\cdot(2)\left[\delta(r+ct) - \underbrace{\delta(r-ct)}_{\text{trägt bei}}\right]$$

$$= \frac{c}{r}\,\delta(r-c\cdot t) = \boxed{\frac{1}{r}\cdot\delta\left(\frac{|r|}{c}-t\right) = G(\underline{r},t)}$$

$$\boxed{G(r-r',t-t') = \frac{\delta\left(t-t'-\frac{|\underline{r}-\underline{r}'|}{c}\right)}{|r-r'|}}$$

GF d. WG in $(3+1)$d

$$\Psi(r,t) = \int d^3r'\,dt'\,\frac{f(r',t')}{|r-r'|}\,\delta\left(t-t'-\frac{|r-r'|}{c}\right)$$

$$\boxed{\begin{aligned} \underline{\Phi}(r,t) &= \int d^3r'\,\frac{\varrho\left(r',t-\frac{|r-r'|}{c}\right)}{|r-r'|} \\ \underline{A}(\underline{r},t) &= \frac{1}{c}\int d^3r'\,\frac{\underline{j}\left(r',t-\frac{|r-r'|}{c}\right)}{|r-r'|} \end{aligned}}$$

„retardierte Potentiale"

$$t' = t - \frac{|r-r'|}{c}$$