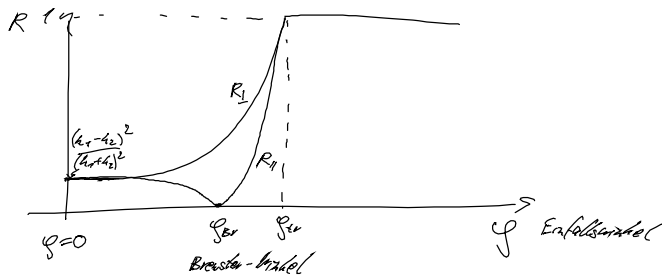


Wdlt

- Fresnel'sche Formeln



• inhomogene Medien $n(z) = \sqrt{\epsilon(z) \mu(z)}$

$\rightarrow \left[\epsilon + \frac{\omega^2}{c^2} \chi(z, \omega) \right] \psi(z)$

$\psi(z) = \Phi(z) e^{-k \cdot S(z)}$
Erfahrung

\rightarrow Eikonalgleichung $(\nabla \cdot S)^2 = k^2(z) = \epsilon(z) \mu(z)$

$\frac{\nabla \cdot S}{k} = c_R \stackrel{!}{=} \text{Tangenten-Einheitsvektor}$

Lichtstrahlen breiten sich nicht mehr geradlinig aus

• Wellenausbreitung in Hohlleitern



$\underline{E} \perp \underline{z}$
 $\underline{B} \parallel \underline{z}$

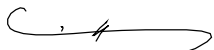
"TE"-Mode transv. elektrisch

Dämpfung möglich

$\frac{\omega^2}{c^2} = k^2 = \left(\frac{\omega \cdot k}{a}\right)^2 + \left(\frac{\omega \cdot k}{b}\right)^2$

$k_1, k_2 \in \{0, 1, 2, \dots\}$

k kann negativ sein



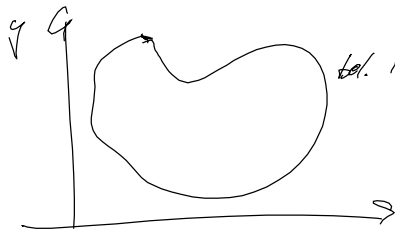
andere $\underline{B} \perp \underline{z}$ "TM"-Moden

$\underline{B} \perp \underline{z} \perp \underline{E}$ "TE_z"-Moden
 unged. Ausbreitung

Koaxialkabel

hier $\langle S \rangle = \frac{c}{4\pi} \langle \underline{E}(r, t) \times \underline{B}(r, t) \rangle = \frac{c}{4\pi} \frac{1}{2} \langle (\underline{E}(r) \cdot e^{-i\omega t} + \underline{E}^*(r) \cdot e^{i\omega t}) \times (\underline{B}(r) \cdot e^{-i\omega t} + \underline{B}^*(r) \cdot e^{i\omega t}) \rangle$
 $= \frac{c}{8\pi} \text{Re} \langle \underline{E}(r) \times \underline{B}^*(r) \rangle$

8.1.2. Wellenfunk. mit konst. Querschnitt



all. Form

Ansatz: $\underline{E} = \underline{E}_0(x, y) e^{-i(k_z z - \omega t)}$

$\underline{B} = \underline{B}_0(x, y) e^{-i(k_z z - \omega t)}$

$\{k_z - \frac{\omega}{c} k_z\} E/B = 0$

Idee

$\underline{E} = \underline{E}_L + E_z \cdot \underline{e}_z$

analog \underline{B}

\underline{E}_L	\underline{B}_L	lösen von	\underline{E}_z, B_z
			ab

$\underline{V} = \underline{V}_L + e_z \cdot \partial_z$

vert. $\underline{V}_L = \underline{e}_x \partial_x + \underline{e}_y \partial_y$

z. kom. $\underline{V}_L = \underline{e}_x \partial_y - \underline{e}_y \partial_x$

$\{k_z^2 - k_z^2 + \frac{\omega^2}{c^2}\} E_{0,z}(x, y) = 0$

$B_{0,z}(x, y)$ analog

Faraday $-\frac{1}{c} \partial_t B = \nabla \times E$

$i \frac{\omega}{c} (B_L + B_z \cdot \underline{e}_z) = (\underline{V}_L + \underline{e}_z \cdot \partial_z) \times (E_L + E_z \cdot \underline{e}_z)$

$= \underline{V}_L \times E_L + \underline{V}_L \times (E_z \underline{e}_z) + i \partial_z (E_z \times \underline{e}_L)$

$\underline{e}_z \times [\dots]$

$\underline{e}_z \times (\underline{V}_L \times \underline{E}_L) = \underline{V}_L (\underline{e}_z \cdot \underline{E}_L) - (\underline{e}_z \cdot \underline{V}_L) \underline{E}_L = 0$

$\underline{V}_L E_z - i k_z \cdot E_L = i \frac{\omega}{c} \underline{e}_z \times B_L \leftarrow \nabla \times E = -\frac{1}{c} \partial_z B$

$\underline{e}_z \times [\dots]$

$\underline{V}_L B_z - i k_z \cdot B_L = -i \frac{\omega}{c} \underline{e}_z \times E_L \leftarrow \nabla \times B = +\frac{1}{c} \partial_z E$

$B_L = \frac{i c^2}{\omega^2 - k_z^2 c^2} \left[+ \frac{\omega}{c} \underline{e}_z \times (\underline{V}_L E_z) + k_z \underline{V}_L B_z \right]$
$E_L = \frac{i c^2}{\omega^2 - k_z^2 c^2} \left[- \frac{\omega}{c} \underline{e}_z \times (\underline{V}_L B_z) + k_z \underline{V}_L E_z \right]$

\perp -Anteile sind durch z. kom. bestrahlt falls $\omega^2 - k_z^2 c^2 \neq 0$

a) TE-Moden ($E_z = 0$)

$B_L = \frac{i c^2}{\omega^2 - k_z^2 c^2} k_z \underline{V}_L B_z$

$E_L = \frac{-i c^2 \omega/c}{\omega^2 - k_z^2 c^2} \underline{e}_z \times (\underline{V}_L B_z) = -\frac{\omega}{k_z c} \underline{e}_z \times B_L$

$B_L \perp E$

überprüfen für Beispiel

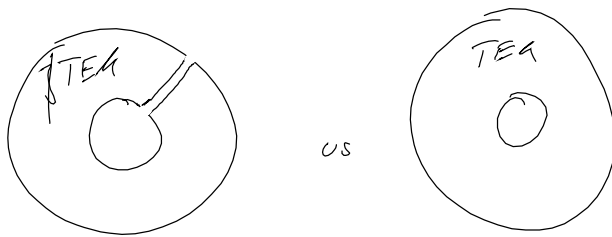
b) analog TM-Moden ($B_z = 0$)

$B_L \perp E$

\Rightarrow für TE- & TM-Moden muss man nur eine WG lösen

$$\left\{ \epsilon_1 - \epsilon_2 \frac{v^2}{c^2} \right\} B_{0z}(x, y) = 0$$

c) TE₁ - Moden ($E_z = 0 = B_z$) haben $\omega^2 = k^2 c^2$ keine Dispersion



8.2. Lagrange-Hamilton-Formalismus

- Noether Theorem \rightarrow EG
- Quantisierung \rightarrow QM II

8.2.1. Punktteilchen - relativistisch (frei)

$$L(q_i, \dot{q}_i, t) \quad \text{gen. Koordinaten} \quad \text{gen. Geschwindigkeiten}$$

$$H(q_i, p_i) = \sum_i \dot{q}_i p_i - L(q_i, \dot{q}_i)$$

$$p_i = \frac{\partial L}{\partial \dot{q}_i} \quad \text{gen. Impulse}$$

Eisenstein-Papals-Bes: $E^2 = p^2 c^2 + m_0^2 c^4$

$$H(q, p) = \sqrt{p^2 c^2 + m_0^2 c^4}$$

$$\dot{q}_i = + \frac{\partial H}{\partial p_i} = \frac{p_i c^2}{\sqrt{p^2 c^2 + m_0^2 c^4}}$$

$$p_i = - \frac{\partial H}{\partial q_i} = 0$$

$$\frac{\dot{q}^2}{c^2} = \frac{p^2 c^4}{p^2 c^2 + m_0^2 c^4}$$

$$[p^2 c^2 + m_0^2 c^4] \frac{\dot{q}^2}{c^2} = p^2 c^4$$

$$m_0^2 c^4 \frac{\dot{q}^2}{c^2} = p^2 c^4 \left[1 - \frac{\dot{q}^2}{c^2} \right]$$

$$\rightarrow \left[p = \gamma m_0 \frac{\dot{q}}{c} \right] \quad \gamma = \frac{1}{\sqrt{1 - \frac{\dot{q}^2}{c^2}}}$$

$$L(q, \dot{q}) = p \cdot \dot{q} - H(q, p)$$

nicht-rel. GF \rightarrow konsistent mit Newton

$$= \gamma m_0 \dot{q}^2 - \sqrt{\gamma^2 m_0^2 c^2 \dot{q}^2 + m_0^2 c^4}$$

$$= -m_0 c^2 \sqrt{1 - \frac{\dot{q}^2}{c^2}} = -\frac{m_0 c^2}{\gamma}$$

8.2.2. Lorentz-Kraft - nicht-rel

Masse von PT \downarrow

hochbel. \uparrow

$$m_0 \ddot{q} = e E_s + \frac{e}{c} (\underline{v} \times \underline{B});$$

e Ladung von PT

$$rel. \frac{d}{dt} \left(\gamma m_0 \dot{q} \right)$$

$$L = \frac{1}{2} m_0 \dot{\underline{q}}^2 - e \overline{\Phi}(\underline{q}, t) + \frac{e}{c} \dot{\underline{q}} \cdot \underline{A}(\underline{q}, t)$$

$$\frac{d}{dt} A(\underline{q}, t) = \frac{\partial A}{\partial t} + \sum_j \frac{\partial A}{\partial q_j} \dot{q}_j$$

$$0 = \frac{d}{dt} \frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i}$$

$$= \frac{d}{dt} \left[m_0 \dot{q}_i + \frac{e}{c} A_i \right] - \left[-e \frac{\partial \overline{\Phi}}{\partial q_i} + \frac{e}{c} \dot{q}_j \frac{\partial A_j}{\partial q_i} \right]$$

$$= m_0 \ddot{q}_i - e \underbrace{\left(-\frac{\partial \overline{\Phi}}{\partial q_i} - \frac{1}{c} \frac{\partial A_i}{\partial t} \right)}_{E_i(\underline{q})} - \frac{e}{c} \sum_j \dot{q}_j \underbrace{\left(\frac{\partial A_j}{\partial q_i} - \frac{\partial A_i}{\partial q_j} \right)}_{(\underline{v} \times \underline{B})_i}$$

$$\left[\dot{\underline{q}} \times (\underline{v} \times \underline{A}) \right]_i = \left[\underline{v} (\dot{\underline{q}} \cdot \underline{A}) - (\dot{\underline{q}} \cdot \underline{v}) \underline{A} \right]_i = \dots$$

$$P_i = \frac{\partial L}{\partial \dot{q}_i} = \underbrace{m_0 \dot{q}_i}_{P_{kin}} + \frac{e}{c} A_i$$

kan. kin. Impuls

$$\underline{P} = \underline{P}_{kin} + \frac{e}{c} \underline{A}$$

$$P_{kin} \rightarrow \underline{P} - \frac{e}{c} \underline{A}$$

"virtuelle" Kopplung

$$H = \underline{P} \cdot \dot{\underline{q}} - L$$

$$\boxed{H(\underline{q}, \underline{P}) = \frac{1}{2} \frac{(\underline{P} - \frac{e}{c} \underline{A})^2}{m_0} + e \overline{\Phi}} = \frac{P_{kin}^2}{2m_0} + e \overline{\Phi}$$