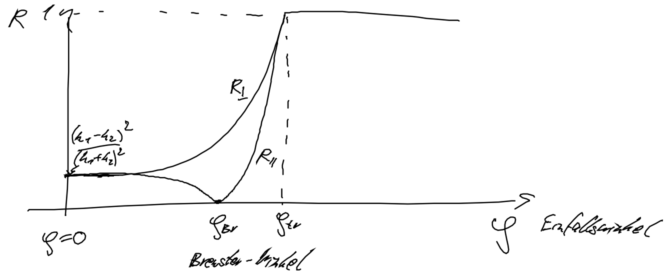
Wdh

· Fresnelsche Formeln



· inhomogene Medien $n(r) = \sqrt{\varepsilon(r)\mu(r)}$

Eikonal

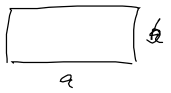
$\to \left[\Delta + \frac{\omega^2}{c^2} n^2(r,\omega)\right] \Psi(r)$ $\Psi(r) = \Phi(r)\, e^{-i k S(r)}$

$\to$ Eikonalgleichung $(\nabla S)^2 = n^2(r) = \varepsilon(r)\mu(r)$

$\frac{\nabla S}{n} = e_k$ ≙ Tangential-Einheitsvektor

Lichtstrahlen breiten sich nicht mehr geradlinig aus

· Wellenausbreitung in Hohlleitern



$E \perp k$ "TE"-Mode transv. elektrisch

$B \not\perp k$ Dämpfung möglich

$$\frac{\omega^2}{c^2} = k^2 + \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 \qquad m,n \in \{0,1,2,\dots\}$$

k kann imaginär werden

anders · $B \perp k$ "TM"-Moden

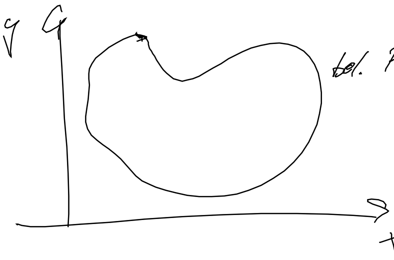
· $B \perp k \perp E$ "TEM"-Moden koaxial Kabel

unged. Ausbreitung

hier $\langle S\rangle = \frac{c}{4\pi}\langle E(r,t)\times B(r,t)\rangle = \frac{c}{4\pi}\frac{1}{2}\frac{1}{2}\left\langle \left(E(r)e^{-i\omega t} + E^*(r)e^{+i\omega t}\right) \times \left(B(r)e^{-i\omega t} + B^*(r)e^{+i\omega t}\right)\right\rangle$

$= \frac{c}{8\pi}\, \mathrm{Re}\{E(r) \times B^*(r)\}$

8.1.2 Hohlleiter mit konst. Querschnitt



Ansatz: $\underline{E} = \underline{E}_0(x,y)\, e^{i(k_z z - \omega t)}$

$\underline{B} = \underline{B}_0(x,y)\, e^{i(k_z z - \omega t)}$

$$\left\{\Delta - \frac{1}{c^2}\partial_t^2\right\} \underline{E}/\underline{B} = 0$$

<u>Idee</u> $\underline{E} = \underline{E}_\perp + E_z \cdot \underline{e}_z$ analog $\underline{B}$

$\nabla = \nabla_\perp + e_z \cdot \partial_z$

kart. $\nabla_\perp = \underline{e}_x \partial_x + \underline{e}_y \partial_y$

Zyl.-koord. $\nabla_\perp = \underline{e}_\rho \partial_\rho + \frac{1}{\rho} \underline{e}_\varphi \partial_\varphi$

$\boxed{\underline{E}_\perp \quad \underline{B}_\perp \text{ hängen von } E_z, B_z \text{ ab}}$

$$\left\{\Delta_\perp - k_z^2 + \frac{\omega^2}{c^2}\right\} E_{0,z}(x,y) = 0$$

$B_{0,z}(x,y)$ analog

<u>Faraday</u> $-\frac{1}{c}\partial_t B = \nabla \times E$

$$i\frac{\omega}{c}\left(B_\perp + B_z \cdot e_z\right) = \left(\nabla_\perp + e_z \cdot \partial_z\right) \times \left(E_\perp + E_z \cdot e_z\right)$$

$$= \nabla_\perp \times E_\perp + \nabla_\perp \times (E_z e_z) + i k (e_z \times E_\perp) \qquad \Big| \; e_z \times [\ldots]$$

$$e_z \times (\nabla_\perp \times E_\perp) = \nabla_\perp (\underbrace{e_z \cdot E_\perp}_{0}) - (\underbrace{e_z \cdot \nabla_\perp}_{0}) E_\perp = 0$$

$$\nabla_\perp E_z - i k_z \cdot E_\perp = i \frac{\omega}{c} e_z \times B_\perp \longleftarrow \nabla \times E = -\frac{1}{c}\partial_t B \qquad \Big| \; e_z \times [\ldots]$$

$$\nabla_\perp B_z - i k_z \cdot B_\perp = -i \frac{\omega}{c} e_z \times E_\perp \longleftarrow \nabla \times B = +\frac{1}{c}\partial_t E$$

$$\boxed{\begin{aligned} B_\perp &= \frac{i c^2}{\omega^2 - k_z^2 c^2}\left[+\frac{\omega}{c}\, e_z \times (\nabla_\perp E_z) + k_z \nabla_\perp B_z\right] \\ E_\perp &= \frac{i c^2}{\omega^2 - k_z^2 c^2}\left[-\frac{\omega}{c}\, e_z \times (\nabla_\perp B_z) + k_z \nabla_\perp E_z\right] \end{aligned}}$$

$\perp$-Anteile sind durch z-komp. bestimmt falls $\omega^2 - k_z^2 c^2 \neq 0$

<u>a) TE-Moden ($E_z = 0$)</u>

$$B_\perp = \frac{i c^2}{\omega^2 - k_z^2 c^2} k_z \nabla_\perp B_z$$

$$E_\perp = \frac{-i c^2 \, \omega/c}{\omega^2 - k_z^2 c^2}\, e_z \times (\nabla_\perp B_z) = -\frac{\omega}{k_z c}\, e_z \times B_\perp$$

$B \perp E$

überprüfen für Beispiel

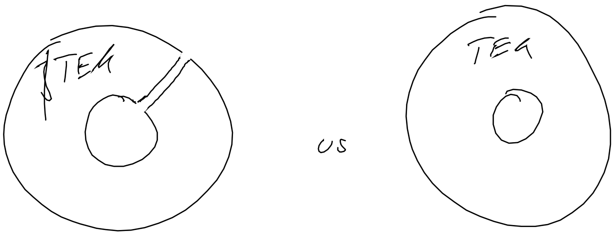
<u>b) analog TM-Moden ($B_z = 0$)</u>

$B \perp E$

$\Rightarrow$ für TE- & TM-Moden muss man nur eine WG lösen

$$\left\{\Delta_\perp - k_z^2 + \frac{\omega^2}{c^2}\right\} B_{0z}(x,y) = 0$$

c) TEM-Moden $(E_z = 0 = B_z)$ haben $\omega^2 = k^2 c^2$ keine Dämpfung



8.2. Lagrange-Hamilton-Formalismus

- Moderne Theorien → EG
- Quantisierung → QM II

8.2.1. Punktteilchen – relativistisch (frei)

$L(q_i, \dot q_i, t)$ gen. Geschwindigkeiten

gen. Koordinaten

$$H(q_i,p_i) = \sum_i \dot q_i p_i - L(q_i,\dot q_i)$$

$p_i = \frac{\partial L}{\partial \dot q_i}$ gen. Impulse

Energie-Impuls-Bez.: $E^2 = p^2c^2 + m_0^2 c^4$

$$H(q,p) = \sqrt{p^2c^2+m_0^2c^4}$$

$$\dot q_i = +\frac{\partial H}{\partial p_i} = \frac{p_i \cdot c^2}{\sqrt{p^2c^2+m_0^2c^4}}$$

$$\dot p_i = -\frac{\partial H}{\partial q_i} = 0$$

$$\dot q^2 = \frac{p^2c^4}{p^2c^2+m_0^2c^4}$$

$$[p^2c^2+m_0^2c^4]\,\dot q^2 = p^2c^4$$

$$m_0^2 \cancel{c^4} \dot q^2 = \cancel{c^4} p^2 \left[1 - \frac{\dot q^2}{c^2}\right]$$

$$\to \boxed{p = \underbrace{\gamma m_0}_{m_{eff}} \cdot \dot q} \qquad \gamma = \frac{1}{\sqrt{1-\frac{\dot q^2}{c^2}}}$$

$$L(q,\dot q) = p\cdot\dot q - H(q,p)$$
$$= \gamma m_0 \dot q^2 - \sqrt{\gamma^2 m_0^2 c^2 \dot q^2 + m_0^2c^4}$$
$$= -m_0c^2\sqrt{1-\frac{\dot q^2}{c^2}} = -\frac{m_0c^2}{\gamma}$$

nicht-rel. GF → konsistent mit Mechanik

8.2.2. Lorentz-Kraft – nichtrel

Masse von PT

$$m_0 \cdot \ddot q_i = e\, E_i + \frac{e}{c}(v\times B)_i \qquad \text{rel. } \frac{d}{dt}(\gamma m_0 \dot q_i)$$

e ≙ Ladung von PT

nichtrel.

→

$$L = \frac{1}{2} m_0 \dot{q}^2 - e\,\Phi(q,t) + \frac{e}{c}\,\dot{q}(t)\cdot A(q(t),t)$$

$$\frac{d}{dt} A(q(t),t) = \frac{\partial A}{\partial t} + \sum_j \frac{\partial A}{\partial q_j}\,\dot{q}_j$$

$$0 = \frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i}$$

$$= \frac{d}{dt}\left[m_0 \cdot \dot{q}_i + \frac{e}{c} A_i\right] - \left[-e\frac{\partial \Phi}{\partial q_i} + \frac{e}{c}\,\dot{q}\cdot\frac{\partial A}{\partial q_i}\right]$$

$$= m_0\ddot{q}_i - e\underbrace{\left(-\frac{\partial \Phi}{\partial q_i} - \frac{1}{c}\frac{\partial A_i}{\partial t}\right)}_{E_i(q)} - \frac{e}{c}\underbrace{\sum_j \dot{q}_j\left(\frac{\partial A_j}{\partial q_i} - \frac{\partial A_i}{\partial q_j}\right)}_{(v\times B)_i}$$

$$\left[\dot{q}\times(\nabla\times A)\right]_i = \left[\nabla(\dot{q}\cdot A) - (\dot{q}\cdot\nabla)A\right]_i = \ldots$$

$$p_i = \frac{\partial L}{\partial \dot{q}_i} = \underbrace{m_0\cdot\dot{q}_i}_{p_{kin}} + \frac{e}{c}A_i$$

kan. konj. Impuls

$$p = p_{kin} + \frac{e}{c}A$$

$$p_{kin} \to p - \frac{e}{c}A$$

„minimale Kopplung"

$$H = p\cdot\dot{q} - L$$

$$\boxed{H(p,q) = \frac{1}{2}\frac{\left(p - \frac{e}{c}A\right)^2}{m} + e\,\Phi} = \frac{p_{kin}^2}{2m} + e\,\Phi$$