

Wdh

$$S = \frac{c}{4\pi} E \times H \quad \hat{=} \text{Energie-Flaschdichte}$$

Poynting-Vektor  
o.k. Energiedichte

$$\frac{\partial w}{\partial t} + \underbrace{j \cdot E}_{\text{lokal. Energiedichte}} = -\nabla \cdot S = -\text{div } S \quad \text{Poynting-Theorem}$$

$$W = \frac{1}{8\pi} [E \cdot D + B \cdot H]$$

$S \neq 0$  heißt nicht nur Energiefluss,  
man braucht  $\nabla \cdot S \neq 0$

$$\frac{d}{dt} [W_V^{\text{Feld}} + W_V^{\text{med}}] = - \oint_{\partial V} \underline{S} \cdot d\underline{F}$$

Inputs linear in Vakuum:  $E=D$   $B=H$   $\rightarrow S = \frac{c}{4\pi} E \times B$

$$\underline{P}_V^{\text{Feld}} = \frac{1}{4\pi c} \int_V (\underline{E} \times \underline{B}) d^3x = \frac{1}{c^2} \int_V \underline{S} d^3x$$

$$\frac{d}{dt} (P_V^{\text{Feld}} + P_V^{\text{med}}) = \oint_{\partial V} \left( \sum_j T_{ij} h_j \right) \cdot d\underline{F}$$

Maxwell'scher Spannungstensor

$$T_{ij} = \frac{1}{4\pi} [E_i E_j + B_i B_j - \frac{1}{2} \delta_{ij} (E \cdot E + B \cdot B)]$$

$$\text{Tr}\{T\} = \text{Sp}\{T\} = \sum_i T_{ii} = -w$$

$$P = - \sum_{ij} T_{ij} h_j$$

### 5.5. Lösungen in Vakuum

einfach  $B=H$   $D=E$   $\rho=0$   $j=0$

$$\nabla \cdot E = 0$$

$$\nabla \cdot B = 0$$

$$\nabla \times E = -\frac{1}{c} \partial_t B$$

$$\nabla \times B = +\frac{1}{c} \partial_t E$$

$$\nabla \times (\nabla \times E) = -\frac{1}{c} \partial_t \nabla \times B = -\frac{1}{c^2} \partial_t^2 E = \nabla_0 (\nabla \cdot E) - \Delta E$$

$$\Rightarrow -\Delta E = -\frac{1}{c^2} \partial_t^2 E \quad \rightarrow \quad \boxed{\nabla^2 E_i = 0}$$

analog  $\boxed{\nabla^2 B_i = 0}$

z.B. 1D:  $\left\{ \partial_x^2 - \frac{1}{c^2} \partial_t^2 \right\} E_i(x, t) = 0$

Ansatz:  $f_{\pm}(\underline{k} \cdot \underline{r} \pm \omega t)$  (skalare Fkt)

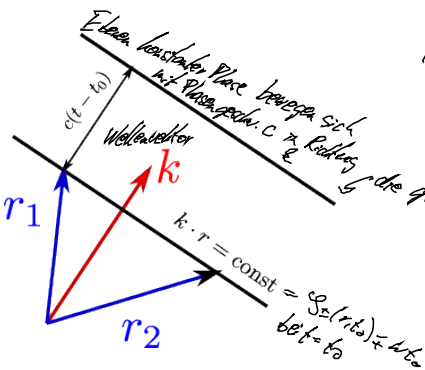
z.B.  $\Delta f_{\pm}(x) = f''(x) \cdot \underline{k} \cdot \underline{k}$   
 $\partial_t^2 f_{\pm}(x) = f''(x) \cdot \omega^2$   $\left[ \underline{k} \cdot \underline{k} - \frac{1}{c^2} \cdot \omega^2 \right] = 0$

$\omega = \pm c |\underline{k}|$  "Dispersionsrelation"

5.5.1. Ebene Wellen

$f_{\pm}(\underline{k} \cdot \underline{r} \pm \omega t)$  skalare Lsg der Wellengl.

betr. Argument  $\varphi_{\pm}(r, t_0) = \text{const} = \underline{k} \cdot \underline{r} \pm \omega \cdot t_0$



für feste Zeiten ist dies eine Ebenen-Gleichung

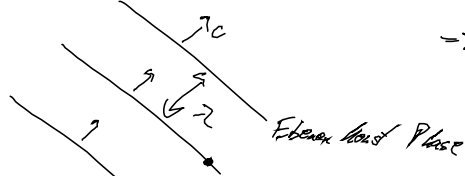
später  $\varphi_{\pm}(r, t) = \underline{k} \cdot \underline{r} \pm \omega \cdot t = \varphi_{\pm}(r, t_0)$

$$\frac{\varphi_{\pm}(r, t_0)}{|\underline{k}|} = \frac{\underline{k} \cdot \underline{r}}{|\underline{k}|} \pm \frac{\omega}{|\underline{k}|} \cdot t$$

falls  $f_{\pm}(\underline{k} \cdot \underline{r} \pm \omega t)$  periodische Fkt. sind, spricht man von ebenen Wellen

z.B.  $f_{\pm}(\underline{k} \cdot \underline{r} \pm \omega t) = A e^{i(\underline{k} \cdot \underline{r} \pm \omega t)}$

• Abstand zw. benachbarten Wellenfronten  $\underline{k} \cdot \underline{r} = 2\pi \cdot 1$



$\Rightarrow \Delta r_{\perp} = \lambda = \frac{2\pi}{|\underline{k}|}$   
Wellenlänge

• Periode  $\tilde{c} = \frac{2\pi}{\omega}$  ;  $\omega \cdot (t_0 + \tilde{c}) = \omega \cdot t_0 + 2\pi$  ;  $\omega \cdot t_0 = 2\pi$

Frequenz  $\nu = \frac{\omega}{2\pi} = \frac{1}{\tilde{c}}$  Kreisfrequenz  $\omega = 2\pi \cdot \nu$

$\Rightarrow \nu \cdot \lambda = \frac{\omega}{2\pi} \cdot \frac{2\pi}{\omega} = c$

Wellenlänge  $\cdot$  Frequenz = Phasengeschw.

haben  $\nabla \cdot \underline{E} = 0$   $\nabla \cdot \underline{B} = 0$

$\underline{E}(r, t) = \text{Re } \underline{E}_0 e^{i(\underline{k} \cdot \underline{r} - \omega t)}$

$\underline{B}(r, t) = \text{Re } \underline{B}_0 e^{i(\underline{k} \cdot \underline{r} - \omega t)}$

in Vakuum  $\nabla \cdot \underline{E} = 0$   $\nabla \cdot \underline{B} = 0$

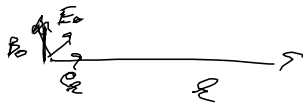
$\rightarrow \underline{\epsilon} \cdot \underline{E}_0 = 0$   $\underline{\epsilon}' \cdot \underline{B}_0 = 0$   $\underline{E}_0 \perp \underline{\epsilon}$   $\underline{B}_0 \perp \underline{\epsilon}'$   
 $\nabla \times \underline{E} = -\frac{1}{c} \frac{\partial \underline{B}}{\partial t}$  z.B.  $\nabla \times \underline{E} = \sum_{j,k} (\partial_j E_k) \cdot \underline{e}_j \rightarrow \underline{\epsilon} \times \underline{E}$

$\rightarrow \partial_x E_0^x e^{i(kr - \omega t)} + \partial_y E_0^y e^{i(kr - \omega t)} + \partial_z E_0^z e^{i(kr - \omega t)}$   
 $= i k_x E_0^x e^{i(\dots)} + i k_y E_0^y e^{i(\dots)} + i k_z E_0^z e^{i(\dots)}$   
 $= i (\underline{k} \cdot \underline{E}_0) \cdot e^{i(kr - \omega t)} = 0$

$i (\underline{k} \times \underline{E}_0) \cdot e^{i(kr - \omega t)} = i \frac{\omega'}{c} \underline{B}_0 e^{i(kr - \omega t)}$   $\forall r, \forall t$

$\rightarrow \omega' = \omega$   $\epsilon' = \epsilon$

$\rightarrow \underline{\epsilon} \times \underline{E}_0 = |\epsilon| \cdot \underline{B}_0 \rightarrow \underline{B}_0 = c \underline{\epsilon} \times \underline{E}_0$



Beispiel  $\underline{\epsilon} = \epsilon \cdot \underline{e}_z$

$$E = \text{Re} \left[ \left( E_x^0 \cdot \underline{e}_x + E_y^0 \cdot \underline{e}_y \right) \cdot e^{i(kz - \omega t)} \right]$$

$$B = \text{Re} \left[ \left( -E_y^0 \cdot \underline{e}_x + E_x^0 \cdot \underline{e}_y \right) \cdot e^{i(kz - \omega t)} \right]$$

$E_x = E_0, E_y = 0$

$$\frac{1}{\sqrt{\epsilon}} = \frac{1}{\sqrt{\epsilon_0}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -E_0^2 \cdot \cos^2(kz - \omega t) \end{pmatrix}$$

5.5.2. Polarisation ebener Wellen

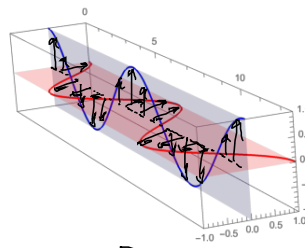
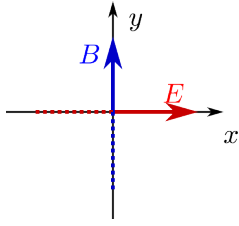
rel. Phase  $\delta$

$$E_x^0 = |E_x^0| \cdot e^{i\varphi} \quad E_y^0 = |E_y^0| \cdot e^{i(\varphi + \delta)}$$

$$E = \begin{pmatrix} |E_x^0| \cdot \cos(kz - \omega t + \varphi) \\ |E_y^0| \cdot \cos(kz - \omega t + \varphi + \delta) \\ 0 \end{pmatrix} \quad B = \underline{e}_z \times E$$

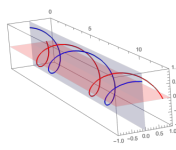
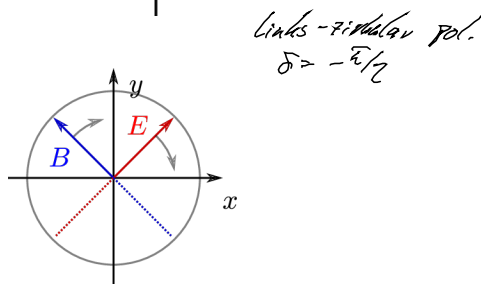
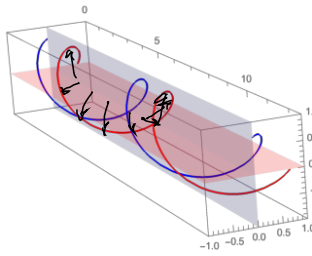
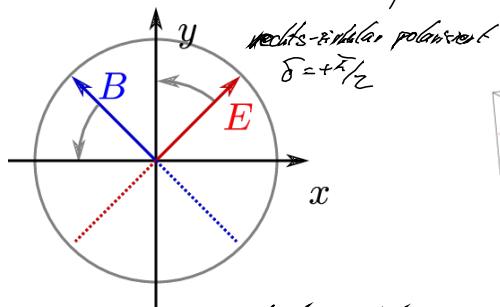
• linear polarisierte Welle  $\delta = 0$  oder  $\delta = \pm \pi$

$E$  &  $B$  schwingen in einer konstanten Ebene



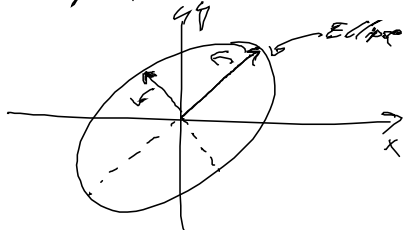
◦ zirkular polarisierte Welle  $\delta = \pm \frac{\pi}{2}$   $|E_{0x}| = |E_{0y}| = E_0$

$$E = E_0 \begin{pmatrix} \cos(kz - \omega t + \varphi) \\ \mp \sin(kz - \omega t + \varphi) \\ 0 \end{pmatrix} \quad B = e_z \times E$$



Anwendung: Polarisationsfilter

◦ allgemein "elliptisch polarisiert"



jede ebene EA-Welle kann als Überlagerung von  
entweder 2 transversalen linear polarisierten Wellen  
oder 2 zirkular pol. Wellen

betrachte nur el. Feld (B geht analog)

$$\left( \frac{E_x}{|E_{0x}|} \cos \delta - \frac{E_y}{|E_{0y}|} \frac{1}{\sin \delta} \right)^2 + \left( \frac{E_x}{|E_{0x}|} \right)^2 = 1 \quad \hat{=} \text{Ellipsengleichung}$$

$$\hat{=} \left( \frac{E_x'}{a} \right)^2 + \left( \frac{E_y'}{b} \right)^2 = 1$$