Wdh

$$S = \frac{c}{4\pi} E \times H \quad \hat{=} \text{ Energie-Flussdichte}$$

Poynting-Vektor

el. Energiedichte

$$\frac{\partial w}{\partial t} + \underset{\uparrow}{j \cdot E} = -\nabla \cdot S = -\text{div}\, S \qquad \text{Poynting-Theorem}$$

mech. Energiedichte

$$w = \frac{1}{8\pi}\left[\underline{E}\cdot\underline{D} + \underline{B}\cdot\underline{H}\right]$$

$S \neq 0$ heißt nicht immer Energiefluss, man braucht $\nabla \cdot S \neq 0$

$$\frac{d}{dt}\left[W_V^{Feld} + W_V^{mech}\right] = -\oint_{\partial V} \underline{S}\cdot d\underline{F}$$

Impulsbilanz im Vakuum: $E = D \quad B = H \longrightarrow S = \frac{c}{4\pi} E \times B$

$$\underline{P}_V^{Feld} = \frac{1}{4\pi c}\int_V (\underline{E}\times\underline{B})\, d^3r = \frac{1}{c^2}\int_V \underline{S}\, d^3r$$

$$\frac{d}{dt}\left(P_V^{Feld} + P_V^{mech}\right)_i = \oint_{\partial V}\left(\sum_j T_{ij} n_j\right)\cdot dF$$

Maxwellscher Spannungstensor

$$\boxed{T_{ij} = \frac{1}{4\pi}\left[E_i E_j + B_i B_j - \frac{1}{2}\delta_{ij}\left[\underline{E}\cdot\underline{E} + \underline{B}\cdot\underline{B}\right]\right]}$$

$$\text{Tr}\{T\} = \text{Sp}\{T\} = \sum_i T_{ii} = -w$$

$$P = -\sum_{ij} T_{ij} n_i n_j$$

5.5. Lösungen im Vakuum

einfach $B = H \quad D = E \qquad \rho = 0 \quad j = 0$

$$\nabla \cdot E = 0 \qquad \nabla \times E = -\frac{1}{c}\partial_t B$$

$$\nabla \cdot B = 0 \qquad \nabla \times B = +\frac{1}{c}\partial_t E$$

$$\nabla \times (\nabla \times E) = -\frac{1}{c}\partial_t \nabla \times B = -\frac{1}{c^2}\partial_t^2 E = \nabla(\nabla \cdot E) - \Delta E$$

$$\Rightarrow -\Delta E = -\frac{1}{c^2}\partial_t^2 E \quad \rightsquigarrow \quad \boxed{\begin{array}{l}\Box E_i = 0 \\ \Box B_i = 0\end{array}}$$

analog

z.B. 1D: $\left\{\partial_x^2 - \frac{1}{c^2}\partial_t^2\right\} E_i(x,t) = 0$

Ansatz: $f_\pm(\underbrace{\underline{k}\cdot\underline{r} \pm \omega t}_{x})$ (skalare Fkt)

z.B. $\Delta f_\pm(x) = f''(x)\cdot \underline{k}\cdot\underline{k}$

$\partial_t^2 f_\pm(x) = f''(x)\cdot \omega^2$

$$\left[\underline{k}\cdot\underline{k} - \frac{1}{c^2}\cdot\omega^2\right] = 0$$

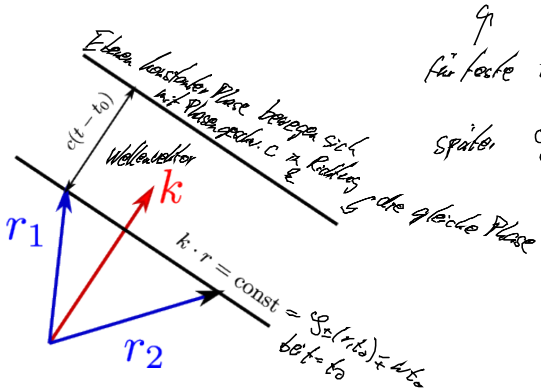
$\omega = \pm c|\underline{k}|$ "Dispersionsrelation"

5.5.1 Ebene Wellen

$f_\pm(\underline{k}\cdot\underline{r} \pm \omega t)$ skalare Lsg der Wellengl.

betr. Argument $\varphi_\pm(r,t_0) = \text{const} = \underset{\uparrow\varphi}{\underline{k}\cdot\underline{r}} \pm \omega\cdot t_0$

für feste Zeiten ist dies eine Ebenen-Gleichung



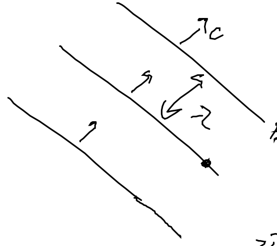
später $\varphi_\pm(r,t) = \underline{k}\cdot\underline{r} \pm \omega\cdot t = \varphi_\pm(r,t_0)$

$$\frac{\varphi_\pm(r,t_0)}{|\underline{k}|} = \frac{\underline{k}\cdot\underline{r}}{|\underline{k}|} \pm \underbrace{\frac{\omega}{|\underline{k}|}}_{c}\cdot t$$

falls $f_\pm(\underline{k}\cdot\underline{r} \pm \omega t)$ periodische Fkt. sind, spricht man von ebenen Wellen

z.B. $f_\pm(\underline{k}\cdot\underline{r} \pm \omega t) = A\, e^{i(\underline{k}\cdot\underline{r} \pm \omega t)}$

- Abstand zw. benachbarten Wellenfronten $\underline{\Delta r}\cdot\underline{k} = 2\pi\cdot 1$



$$\Rightarrow \Delta r_{min} = \boxed{\lambda = \frac{2\pi}{k} \quad \text{Wellenlänge}}$$

- Periode $\tau = \frac{2\pi}{\omega}$ $\qquad e^{i\omega\cdot(t_0+\tau)} \overset{!}{=} e^{i\omega\cdot t_0}$

Frequenz $= \nu = \frac{\omega}{2\pi} = \frac{1}{\tau}$ $\qquad$ Kreisfrequenz $\omega = 2\pi\cdot\nu$

$$\Rightarrow \nu\cdot\lambda = \frac{\omega}{2\pi}\cdot\frac{2\pi}{k} = c$$

Wellenlänge · Frequenz = Phasengeschw.

haben $\nabla \underline{E} = 0 \qquad \nabla B = 0$

$E(r,t) = \text{Re}\, \underline{E}_0\, e^{i(\underline{k}\cdot\underline{r} - \omega t)}$ $\qquad B(r,t) = \text{Re}\, \underline{B}_0\, e^{i(\underline{k}'\cdot\underline{r} - \omega t)}$

im Vakuum $\nabla \cdot \underline{E} = 0 \qquad \nabla \cdot B = 0$

$\Rightarrow \underline{k} \cdot \underline{E_0} = 0 \qquad \underline{k}' \cdot \underline{B_0} = 0 \qquad \underline{E_0} \perp \underline{k} \qquad \underline{B_0} \perp \underline{k}'$

$\nabla \times \underline{E} = -\frac{1}{c} \partial_t B \qquad$ z.B. $\nabla \times \underline{E} = \sum_{ijk} \varepsilon_{ijk} (\partial_i E_j) \cdot e_k \xrightarrow{\quad} \sum_{ijk} \varepsilon_{ijk}\, i k_i E_j \cdot e_k \rightarrow i(\underline{k} \times \underline{E})$

$\rightarrow \partial_x E_0^x e^{i(kr-\omega t)} + \partial_y E_0^y e^{i(kr-\omega t)} + \partial_z E_0^z e^{i(kr-\omega t)}$

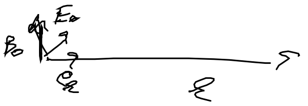
$= i k_x \cdot E_0^x e^{i(..)} + i k_y E_0^y e^{i(...)} + i k_z E_0^z e^{i(...)}$

$= i(\underline{k} \cdot \underline{E_0}) \cdot e^{i(kr-\omega t)} = 0$

$i(k \times E_0) \cdot e^{i(kr-\omega t)} = i \frac{\omega'}{c} B_0 e^{i(k' \cdot r - \omega' t)} \qquad \forall r, \forall t$

$\longrightarrow \omega' = \omega \qquad k' = k$

$\Rightarrow \underline{k} \times \underline{E_0} = |k| \cdot \underline{B_0} \longrightarrow B_0 = e_k \times E_0$



Beispiel $\underline{k} = k \cdot \underline{e_z}$

$$\boxed{\begin{aligned} E &= \mathrm{Re}\left[\left(E_x^0 \cdot e_x + E_y^0 \cdot e_y\right) \cdot e^{i(kz-\omega t)}\right] \\ B &= \mathrm{Re}\left[\left(-E_y^0 \cdot e_x + E_x^0 \cdot e_y\right) e^{i(kz-\omega t)}\right] \end{aligned}}$$

$E_x^0 = E_0, \underline{E_y^0 = 0}$

$E_0 \in \mathbb{R} \qquad T = \frac{1}{4\pi} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -E_0^2 \cdot \cos^2(kz-\omega t) \end{pmatrix}$

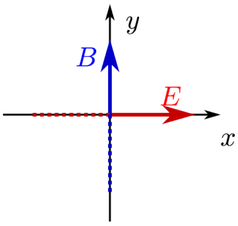
5.5.2. Polarisation ebener Wellen

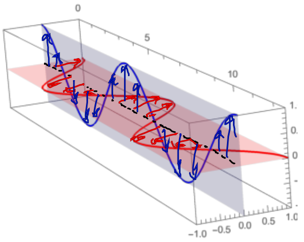
$E_x^0 = |E_x^0| \cdot e^{i\varphi} \qquad E_y^0 = |E_y^0| e^{i(\varphi + \overset{\text{rel. Phase}}{\delta})}$

$\underline{E} = \begin{pmatrix} |E_0^x| \cdot \cos(kz - \omega t + \varphi) \\ |E_0^y| \cos(kz - \omega t + \varphi + \delta) \\ 0 \end{pmatrix} \qquad B = e_k \times E$

- linear polarisierte Welle $\quad \delta = 0 \quad$ oder $\quad \delta = \pm \pi$

E & B schwingen in einer konstanten Ebene





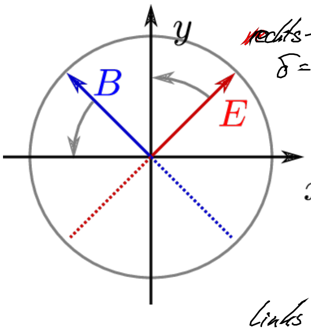
- zirkular polarisierte Welle

$\delta = \pm \frac{\pi}{2}$

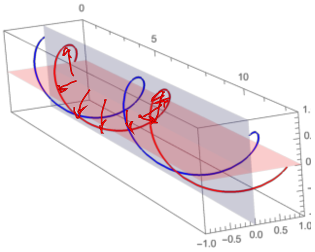
$|E_{0x}| = |E_{0y}| = E_0$

$$E = E_0 \begin{pmatrix} \cos(kz - \omega t + \varphi) \\ \mp \sin(kz - \omega t + \varphi) \\ 0 \end{pmatrix}$$

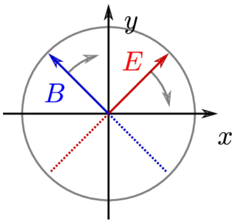
$B = e_k \times E$

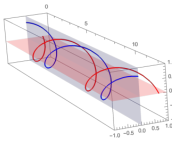


rechts-zirkular polarisiert
$\delta = +\pi/2$



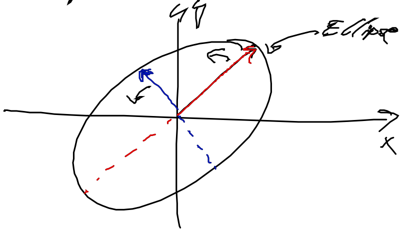
Links-zirkular pol.
$\delta = -\pi/2$





Anwendung: Polarisationsfilter

- allgemein "elliptisch polarisiert"



jede ebene EM-Welle kann als Überlagerung von entweder 2 transversal linear polarisierten Wellen oder 2 zirkular pol. Wellen

betrachte nur el. Feld (B geht analog)

$$\left(\frac{E_x}{|E_{0x}|}\frac{\cos\delta}{\sin\delta} - \frac{E_y}{|E_{0y}|}\frac{1}{\sin\delta}\right)^2 + \left(\frac{E_x}{|E_{0x}|}\right)^2 = 1 \qquad \hat{=} \text{ Ellipsengleichung}$$

$$\hat{=} \left(\frac{E_x'}{a}\right)^2 + \left(\frac{E_y'}{b}\right)^2 = 1$$