Wdh + Magnetostatik $\rightsquigarrow \frac{\partial \rho}{\partial t} = 0 \Rightarrow \boxed{\nabla \cdot j = 0}$

$$\frac{\partial \rho}{\partial t} + \nabla j = 0$$

+ mikroskop. Def. der SD

$$j = \rho(r) \cdot v(r)$$

Ladungsdichte, mittl. Geschwindigkeit

+ Konzept Stromfaden ≙ Analogon zur PL in der ES

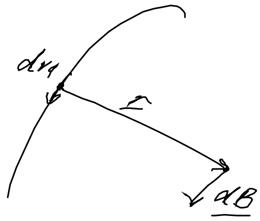
Stromfaden-Substitution

$$j\, d^3r \Leftrightarrow I\, d\underline{r}$$

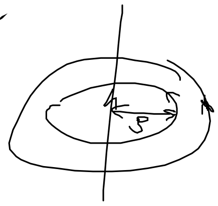
Volumenintegral über Stromdichte ≙ Konturintegral entlang Stromfaden

+ Biot-Savart

$$\boxed{d\underline{B} = \frac{I}{c} \frac{d\underline{r}_1 \times \underline{r}}{|\underline{r}|^3}}$$



unendlich langer Draht



$$\underline{B} = \frac{2I}{c \cdot \rho} \cdot \underline{e}_\varphi$$

+ Ampère'sches Gesetz

Kraft auf ein stromdurchflossenes Linienelement $d\underline{r}_1$ in ext. Feld

$$d\underline{F} = \frac{I_1}{c}\, d\underline{r}_1 \times \underline{B}$$

$$I_{SI} = I \sqrt{4\pi\varepsilon_0}$$

$$\mu_0 \cdot \varepsilon_0 = \frac{1}{c^2}$$

=> Kraft zwischen 2 geschlossenen Leitern

$$F_{12} = -\frac{I_1 \cdot I_2}{c^2} \oint_{C_1} \oint_{C_2} \frac{\underline{r}_1 - \underline{r}_2}{|r_1 - r_2|^3}\, d\underline{r}_1 \cdot d\underline{r}_2$$

$\int j \, d^3r$

4.4. Maxwell-Gleichungen der Magnetostatik

hatten $\quad d\underline{B} = \frac{I}{c} \frac{d\underline{r}_1 \times \underline{r}}{|\underline{r}|^3}$ $\quad r$ = Vektor von dr_1 zum Beobachtungspunkt

$$B(r) = \int dB = \frac{I}{c} \oint \frac{d\underline{r}' \times (\underline{r} - \underline{r}')}{|r - r'|^3} = \boxed{\frac{1}{c} \int \frac{j(r') \times (\underline{r} - \underline{r}')}{|r - r'|^3} d^3r' = B(r)}$$

Beobachtungspunkt

Gesetz von Biot-Savart (globale Form)

ES $\quad \underline{E}(r) = \int \rho(r') \frac{\underline{r} - \underline{r}'}{|r - r'|^3} d^3r'$

umschreiben

$$B(r) = \frac{1}{c} \int d^3r' \, j(r') \times \left(-\nabla \frac{1}{|r - r'|} \right) = +\frac{1}{c} \int d^3r' \left(\nabla \frac{1}{|r - r'|} \right) \times j(r')$$

$$= \nabla \times \left(\frac{1}{c} \int \frac{j(r')}{|r - r'|} d^3r' \right)$$ B ist ein Rotationsfeld

$\Rightarrow \boxed{\nabla \cdot B = 0}$ Maxwell-Gleichung $\quad \Rightarrow \quad \oiint_{\partial V} \underline{B} \cdot d\underline{F} = 0$

komponentenweise

$\nabla \times (\nabla \times \underline{A}) = \nabla (\nabla \cdot \underline{A}) - \Delta \underline{A}$

$= \mathrm{grad}(\mathrm{div}\, A) - \text{Laplace } A$

$$\nabla \times B = \nabla \times \left(\nabla \times \left(\frac{1}{c} \int \frac{j(r')}{|r - r'|} d^3r' \right) \right)$$

$$= \frac{1}{c} \int \nabla \left(\nabla \frac{j(r')}{|r - r'|} \right) d^3r' - \frac{1}{c} \int \Delta \frac{j(r')}{|r - r'|} d^3r'$$

$$= -\frac{\nabla}{c} \int j(r') \left(\nabla' \frac{1}{|r - r'|} \right) d^3r' + \frac{4\pi}{c} j(r) \qquad \Delta \frac{1}{|r - r'|} = -4\pi \delta(r - r')$$

$$= +\frac{\nabla}{c} \int \frac{(\nabla' j(r'))}{|r - r'|} d^3r' - \frac{\nabla}{c} \int \nabla' \left(\frac{j(r')}{|r - r'|} \right) d^3r' + \frac{4\pi}{c} j(r)$$

$$= 0 - \underbrace{\frac{\nabla}{c} \oiint_{\partial V} \frac{j(r')}{|r - r'|} dF'}_{0} + \frac{4\pi}{c} j$$

$$\nabla\times B = \frac{4\pi}{c}\cdot j(r)$$ zweites Ampèresches Gesetz

$$\iint(\nabla\times B)\,dF = \oint_{C=\partial F} \underline{B}\cdot d\underline{r} \overset{\text{Maxwell}}{=} \frac{4\pi}{c}\iint j\,dF = \frac{4\pi}{c}\cdot I_F$$

Maxwell-Gleichungen der MS

$$\nabla\cdot\underline{B} = 0$$
$$\nabla\times\underline{B} = \frac{4\pi}{c}\cdot j(r)$$

lokale Form

integrale Form

$$\oiint \underline{B}\cdot d\underline{F} = 0$$
$$\oint_{\partial F} \underline{B}\,d\underline{r} = \frac{4\pi}{c}\cdot I_F$$

4.5. Das Vektorpotential

$$B = \nabla\times\underline{A}$$ $\underline{A}$ "Vektorpotential"

$$A \to A + \underbrace{\nabla\cdot\varphi(r)}_{\varphi\text{ skalare Funktion}}$$ "Eichfreiheit"

$$\nabla\times\underline{B} = \nabla\times(\nabla\times\underline{A}) = \nabla(\underbrace{\nabla\cdot\underline{A}}_{0}) - \Delta\underline{A}$$

Coulomb-Eichung: Wähle $\varphi(r)$ so, dass $\nabla\cdot A = 0$

$$\nabla\times B = -\Delta A$$ in Coulomb-Eichung

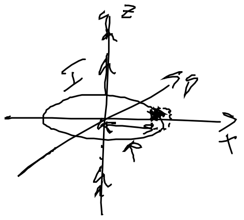
$$\Rightarrow \Delta\underline{A} = -\frac{4\pi}{c}\cdot\underline{j}$$ $$\Delta A_i = -\frac{4\pi}{c}\cdot j_i$$

Poisson-Gleichung für jede Komponente

ohne RB im Endlichen folgt

$$A(r) = \frac{1}{c}\int\frac{j(r')}{|r-r'|}\,d^3r'$$
$$B(r) = \nabla\times\frac{1}{c}\int\frac{j(r')}{|r-r'|}\,d^3r' = \nabla\times A$$

4.6. Magnetfeld einer Stromschleife



in Zylinder-Koord.

$$\underline{e}_\rho = \begin{pmatrix} \cos\varphi \\ \sin\varphi \\ 0 \end{pmatrix} \qquad \underline{e}_\varphi = \begin{pmatrix} -\sin\varphi \\ \cos\varphi \\ 0 \end{pmatrix} \qquad \underline{e}_z = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\Rightarrow \underline{e}_\varphi \times \underline{e}_z = \underline{e}_\rho \qquad \underline{e}_\varphi \times \underline{e}_\rho = -\underline{e}_z$$

$$\underline{j}(r') = \underbrace{I}_{\text{Stromstärke}} \cdot \delta(z') \cdot \delta(\rho' - R) \cdot \underline{e}_\varphi$$

MF $\quad B(\underline{r}) = \frac{1}{c} \int j(\underline{r}') \times \frac{\underline{r} - \underline{r}'}{|r - r'|^3} \, d^3r'$

$$B(z \cdot e_z) = \frac{I}{c} \int_0^\infty \rho' d\rho' \int_0^{2\pi} d\varphi' \int_{-\infty}^{\infty} dz' \, \delta(\rho' - R) \delta(z') \frac{e_{\varphi'} \times \left((z - z') \cdot e_{z'} - \rho' \cdot e_{\rho'}\right)}{\left[(z - z')^2 + (\rho')^2\right]^{3/2}}$$

$$= \frac{I}{c} \left[\quad '' \quad \right] \frac{(z - z') e_{\rho'} + \rho' e_{z'}}{[\ \ldots\ ]^{3/2}}$$

$$= \frac{I}{c} \int_0^{2\pi} d\varphi' \frac{z \cdot e_{\rho'} + R \cdot e_z}{(z^2 + R^2)^{3/2}} \cdot R \qquad \int_0^{2\pi} \sin(\varphi) \, d\varphi = 0$$

$$= \frac{2\pi I}{c} \cdot \frac{R^2}{(z^2 + R^2)^{3/2}} \cdot e_z$$

oder $\quad B(z \cdot e_z) = \frac{I}{c} \oint \underline{dr}' \times \frac{z \cdot \underline{e}_z - R \cdot \underline{e}_{\rho'}}{(z^2 + R^2)^{3/2}} \qquad \underline{dr}' = R \cdot \underline{e}_{\varphi'} \cdot d\varphi'$

$$= \frac{I}{c} \int_0^{2\pi} \frac{R \cdot e_{\varphi'} \times (z \cdot e_z - R \cdot e_{\rho'})}{(z^2 + R^2)^{3/2}} d\varphi' = \frac{2\pi I}{c} \frac{R^2}{(z^2 + R^2)^{3/2}} \cdot \underline{e}_z$$

oder Vektorpotential

$$A(z \cdot e_z + \rho \cdot e_\rho) = \frac{1}{c} \int \frac{j(r')}{|r - r'|} d^3r' = \frac{I}{c} \oint_C \frac{dr}{|r - r'|}$$

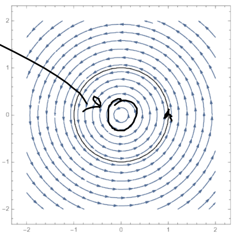
$$= \frac{I \cdot R}{c} \int_0^{2\pi} \frac{e_{\varphi'} d\varphi'}{|\rho \cdot e_\rho + z \cdot e_z - R \cdot e_{\rho'}|}$$

$$= \frac{I \cdot R}{c} \int_0^{2\pi} \frac{d\varphi' \, e_{\varphi'}}{\sqrt{\rho^2 + z^2 + R^2 - 2R\rho(\cos\varphi \cos\varphi' + \sin\varphi \sin\varphi')}}$$

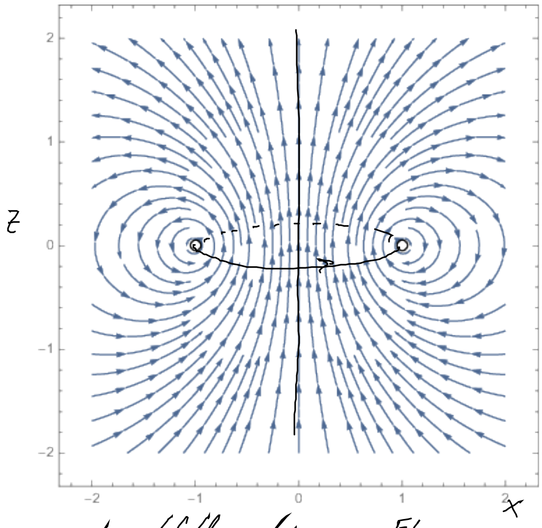
$- \quad . \quad .$

Entwicklung für kleine ϱ

$$A = \frac{\pi \cdot I}{c} \frac{R^2 \varrho}{[R^2 + z^2]^{3/2}} \cdot e_\varphi + \mathcal{O}(\varrho^2)$$



Vektorpot. in xy-Ebene



Magnetfeld in der xz-Ebene

Spule

$$B(z \cdot e_z) = \frac{2\pi I}{c} \cdot e_z \sum_{n=-N/2}^{N/2} \frac{R^2 \Delta n}{[(z - n \cdot \underbrace{\tfrac{L}{N}}_{\Delta z})^2 + R^2]^{3/2}}, \quad \Delta n = 1$$

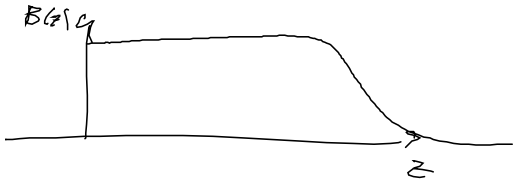
$$\approx \frac{2\pi I}{c} \cdot e_z \int_{-N/2}^{+N/2} \frac{R^2 \, dn}{[(z - n \cdot \frac{L}{N})^2 + R^2]^{3/2}}$$

$$dz' = dn \cdot \frac{L}{N}$$

$$z' = n \cdot \frac{L}{N}$$

$$= \frac{2\pi I}{c} \cdot e_z \frac{N}{L} \int_{-L/2}^{+L/2} \frac{R^2 \, dz'}{[(z-z')^2 + R^2]^{3/2}}$$

$$\xrightarrow[L \to \infty]{} \frac{4\pi I}{c} \cdot \frac{N}{L} \cdot e_z$$



$$B = \frac{1}{c} \int j(r') \times \frac{r - r'}{|r - r'|^3} \, d^3 r'$$