Schaller

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Inhalt: Elektrostatik, Multipolentw. / Dielektrika
Magnetostatik, Maxwell-Gleichungen
Ebene Wellen, Strahlung

Bibel: Jackson

päd. Einf.: Greiner / Nolting / Rebhan / Fließbach / Griffiths / Schwinger

Mathe: Arfken "Math. methods for physicists"

Vorbemerkungen

Notation & Konventionen

Gradient
$$\nabla = \begin{pmatrix} \partial_x \\ \partial_y \\ \partial_z \end{pmatrix} = \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix}$$
"Nabla"

$$\longrightarrow \nabla \varphi = \operatorname{grad} \varphi = \begin{pmatrix} \partial_x \varphi \\ \partial_y \varphi \\ \partial_z \varphi \end{pmatrix}$$

Divergenz
$$\nabla \cdot \underline{A} = \operatorname{div} \underline{A} = \partial_x A_x + \partial_y A_y + \partial_z A_z$$

Rotation
$$\nabla \times \underline{A} = \operatorname{rot} \underline{A} = \begin{vmatrix} e_x & e_y & e_z \\ \partial_x & \partial_y & \partial_z \\ A_x & A_y & A_z \end{vmatrix}$$

$$= \begin{pmatrix} \partial_y A_z - \partial_z A_y \\ \partial_z A_x - \partial_x A_z \\ \partial_x A_y - \partial_y A_x \end{pmatrix}$$

Grundgleichungen der ED

Gauss-Einheitensystem

Maxwell-Gleichungen

$$\boxed{\begin{aligned} \nabla \cdot \underline{D} &= 4\pi \cdot \rho \\ \nabla \cdot \underline{B} &= 0 \\ \nabla \times \underline{H} &= \frac{4\pi}{c} \underline{j} + \frac{1}{c} \frac{\partial \underline{D}}{\partial t} \\ \nabla \times \underline{E} &= -\frac{1}{c} \frac{\partial \underline{B}}{\partial t} \end{aligned}}$$

Lösungen

$\underline{D}$ – dielektr. Verschiebung

ρ – Ladungsdichte

$\underline{B}$ – magn. Flussdichte

$\underline{H}$ – magn. Feldstärke

$\underline{j}$ – Stromdichte

(ρ, $\underline{j}$: vorgeg.)

c – Lichtgeschwindigkeit

$\underline{E}$ – elektr. Feldstärke

$$\underline{D} = \underline{\underline{\epsilon}}\, \underline{E} \qquad \underline{B} = \underline{\underline{\mu}}\, \underline{H}$$

meist homogene & isotrope Medien $\quad \underline{\underline{\epsilon}} \rightarrow \epsilon = \epsilon_r \cdot \epsilon_0$

$\underline{\underline{\mu}} \rightarrow \mu = \mu_r \cdot \mu_0$

Lorentz-Kraft $\quad \underline{F} = q\left(\underline{E} + \frac{1}{c} \cdot \underline{v} \times \underline{B}\right)$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \underline{j} = 0 \qquad \text{"Kontinuitätsgleichung"}$$

1. Elektrostatik

- $\underline{H} = 0 \quad \underline{B} = 0 \qquad \partial_t E = \partial_t D = 0$
- $\underline{D} = \epsilon\, \underline{E} \qquad \rightarrow$ im Vakuum

$\underline{D} = \epsilon_0 \cdot \underline{E}$ → in Gauss-Einh. $\epsilon_0 = 1$

$$\boxed{\begin{aligned} \nabla \cdot \underline{E} &= 4\pi \rho \\ \nabla \times E &= 0 \end{aligned}}$$

1.1. Coulomb-Gesetz (1785)

Kraft zwischen 2 gel. Körpern

+ in der Verb.-linie
+ prop. zu beiden Ladungen
+ fällt mit Quadrat des Abst. ab

$$\underline{F}_{12} = k\, q_1 \cdot q_2 \frac{\underline{r}_1 - \underline{r}_2}{|\underline{r}_1 - \underline{r}_2|^3} = k\, q_1 q_2 \frac{\underline{e}_{12}}{|\underline{r}_1 - \underline{r}_2|^2}$$

Gauss-System $k = 1 \longrightarrow |F| = \frac{q_1 q_2}{r_{12}^2}$

Einh. der Ladung $[q] = 1\sqrt{N \cdot m^2} = 1\sqrt{\frac{kg \cdot m^3}{s^2}}$

"elektrostat. Einheit"

SI-System $k_{SI} = \frac{1}{4\pi \epsilon_0}$ $\epsilon_0 = 8.854\ldots \frac{C^2}{N \cdot m^2}$

Dielektrizitätskonst. des Vakuums

N Punktladungen: (Gauss)

$$\underline{F}_i = q_i \cdot \sum_{j \neq i} q_j \frac{\underline{r}_i - \underline{r}_j}{|\underline{r}_i - \underline{r}_j|^3}$$

Feldstärke $\underline{F} = q \cdot \underline{E}$

$$E_j(r) = q_j \cdot \frac{r - r_j}{|r - r_j|^3}$$

am Orte $\underline{r}$ $\underline{E}(\underline{r}) = \sum_{j=1}^{N} E_j(r) = \sum_j q_j \frac{\underline{r} - \underline{r}_j}{|\underline{r} - \underline{r}_j|^3}$

Kontinuumsbeschreibung $q_j \longrightarrow \rho(\underline{r}')\, d^3 r'$

$$\underline{E}(\underline{r}) = \int d^3 r'\, \rho(\underline{r}') \frac{r - r'}{|r - r'|^3}$$

zurück $\rho(r) = \sum_j q_j\, \delta(\underline{r} - \underline{r}_j)$

1.2. Integralsätze

1.2.1. Integralsatz von Stokes

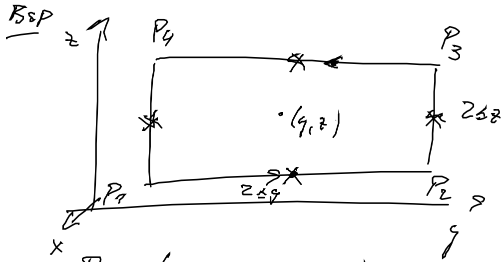
Normalenvektor

$$\oint_{\partial S} \underline{A}\cdot d\underline{r} = \iint_{S} (\nabla \times A)\, dS$$

$S \leftarrow$ Fläche

∂S: bel. geschl. Kurve in $\mathbb{R}^3$

Bsp



$P_1 = (y-\Delta y, z-\Delta z)$ kleine Δy Δz

$P_2 = (y+\Delta y, z-\Delta z)$

$P_3 = (y+\Delta y, z+\Delta z)$

$P_4 = (y-\Delta y, z+\Delta z)$

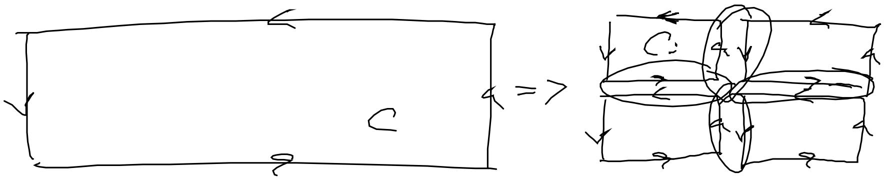
$$\oint_{P_1 \dots P_4} A\, dr = \int_{P_1 P_2} A_y\, dy + \int_{P_2 P_3} A_z\, dz + \int_{P_3 P_4} A_y\, dy + \int_{P_4 P_1} A_z\, dz$$

$$\begin{aligned}
&= A_y(x, y, z-\Delta z)(+2\Delta y) \\
&\quad + A_z(x, y+\Delta z, z)(+2\Delta z) \\
&\quad + A_y(x, y, z+\Delta z)(-2\Delta y) \\
&\quad + A_z(x, y-\Delta y, z)(-2\Delta z) \\
&= (\cancel{A_y} - \Delta z \cdot \partial_z A_y)(+2\Delta y) \\
&\quad + (\cancel{A_z} + \Delta y \cdot \partial_y \cdot A_z)(+2\Delta z) \\
&\quad + (\cancel{A_y} + \Delta z\, \partial_z A_y)(-2\Delta y) \\
&\quad + (\cancel{A_z} - \Delta y\, \partial_y A_z)(-2\Delta z) \\
&= \underbrace{4\Delta y \cdot \Delta z}_{\Delta S} (\partial_y A_z - \partial_z A_y) = \Delta S\, (\nabla \times A)_x
\end{aligned}$$

$= \underline{dS} \cdot (\nabla \times \underline{A}) \qquad \underline{dS} = \Delta S \cdot \underline{n}_S$

$$\underline{n}_S \cdot (\nabla \times A) = \lim_{\Delta S \to 0} \frac{1}{\Delta S} \oint \underline{A} \cdot d\underline{r}$$

Verallgemeinerung



$$\oint_C \underline{A} \cdot d\underline{r} = \lim_{\Delta S_i \to 0} \sum_i \oint_{C_i} \underline{A}\, d\underline{r}_i$$

$$= \lim_{\Delta S_i \to 0} \sum_i \underline{n}_{S_i} (\nabla \times A) \Delta S_i$$

$$= \iint_S \underline{n}_S (\nabla \times \underline{A}) dS = \iint_S (\nabla \times \underline{A}) \cdot d\underline{S}$$

Satz von Stokes

1.2.2. Integralsatz von Gauss

$$\int_V (\nabla \cdot A) d^3r = \oiint_{\partial V} \underline{A} \cdot d\underline{S} \qquad \text{"Satz von Gauss"}$$

Volumen   Vektorfeld   Rand von V

Fluss eines Vektorfeldes durch eine Fläche S

$$\varphi_S[A] = \iint \underline{A}(\underline{r}) \cdot d\underline{S}$$

$$\approx \sum_i A(\underline{r}_i) \cdot \Delta \underline{S}_i \qquad = \Delta S_i \cdot \underline{n}_i$$

Normalenvektor (zeigt nach außen)

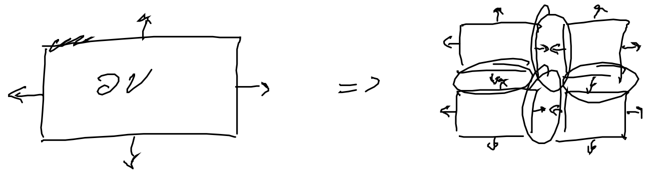
repräsentativer Wert von $\underline{A}$ im Flächenstück i

<u>Bsp</u> räumlich homog. Vektorfeld

$$\underline{A} = \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix}$$

$$\Delta \underline{S}_1 = \Delta y \cdot \Delta z \cdot \underline{e}_x = -\Delta \underline{S}_2$$
$$\Delta \underline{S}_3 = \Delta x \cdot \Delta z \cdot e_y = -\Delta \underline{S}_4$$
$$\Delta \underline{S}_5 = \Delta x \cdot \Delta y \cdot e_z = -\Delta S_6$$

$$\oiint_{\partial V} \underline{A}\, d\underline{S} = \iint dy\, dz \left[A_x(x_0 + \tfrac{\Delta x}{2}, y_0, z_0) - A_x(x_0 - \tfrac{\Delta x}{2}, y_0, z_0)\right]$$
$$+ \iint dx\, dz \left[A_y(x_0, y_0 + \tfrac{\Delta y}{2}, z_0) - A_y(x_0, y_0 - \tfrac{\Delta y}{2}, z_0)\right]$$
$$+ \iint dx\, dy \left[A_z(x_0, y_0, z_0 + \tfrac{\Delta z}{2}) - A_z(x_0, y_0, - \tfrac{\Delta z}{2})\right]$$
$$= \iint dy\, dz\ \partial_x A_x^{(x_0,y_0,z_0)} \cdot \Delta x + \iint dx\, dz \cdot \partial_y A_y \cdot \Delta y + \iint_{dx\,dy} (\partial_z A_z) \Delta z$$
$$\longrightarrow \Delta V \left(\partial_x A_x + \partial_y A_y + \partial_z A_z\right)$$
$$\Delta V \cdot (\nabla \underline{A})(x_0, y_0, z_0)$$

Strategie (2D) 

$$\lim_{\Delta V \to 0} \frac{1}{\Delta V} \oiint \underline{A}\, d\underline{S} = \nabla \underline{A}(r_0)$$

$$\oiint_{\partial V} \underline{A}\, d\underline{S} = \sum_i \oiint_{\partial V_i} \underline{A}\, d\underline{S}_i = \sum_i \Delta V_i \left(\nabla \underline{A}(\underline{r}_i)\right)$$
$$= \int_V (\nabla A)\, d^3 r$$

"verallg. Integralsatz von Stokes"