Wdh

1. Klausur-Termin: 15.2. 8-10 A151
22.2. 8-10

+ Maxwell-Gleichungen der Magnetostatik

$\nabla \cdot B = 0$ "keine magn. Monopole"
$\nabla \times B = \frac{4\pi}{c} \cdot j$ 2. Ampèresches Gesetz
} lokale Form

$\oiint \underline{B} \cdot d\underline{F} = 0$
$\oint_{\partial F} \underline{B} \cdot d\underline{r} = \frac{4\pi}{c} \cdot \underline{I}_F$
} integrale Form

Rückblick Draht

$\underline{B} = B(\varrho) \cdot e_\varrho$

$2\pi \varrho \cdot B(\varrho) = \frac{4\pi}{c} \cdot I$

$\longrightarrow B(\varrho) = \frac{2I}{c\varrho}$

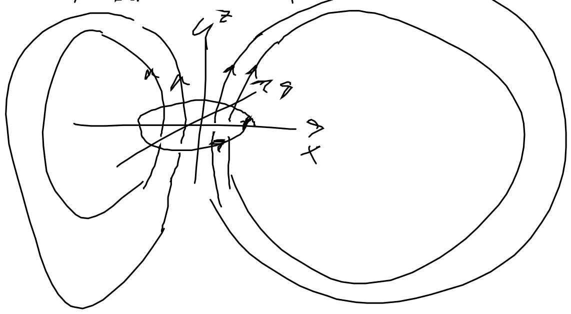
+ Vektorpotential $\underline{B} = \nabla \times \underline{A}$ $A \to A + \nabla \varphi$ Eichfreiheit

Coulomb-Eichung $\nabla \cdot A = 0 \longrightarrow \Delta A_i = -\frac{4\pi}{c} \cdot j_i$ $\hat{=}$ Poisson-Gleichung

$\longrightarrow \boxed{\underline{A}(r) = \frac{1}{c} \int \frac{j(r')}{|r - r'|} d^3 r'}$ in Coulomb-Eichung

für endliche stat. SV

+ Bsp. Leiterschleife



## 4.7. Magnet. Moment stat. lokalis. Ströme

$$\underline{A}(\underline{r}) = \frac{1}{c}\int \frac{\underline{j}(\underline{r}')}{|\underline{r}-\underline{r}'|}\, d^3r' \qquad r \gg r'$$

$$= \frac{1}{c}\int j(r')\Big[\underbrace{\frac{1}{r}}_{\text{trägt nicht bei}} + \overbrace{\frac{r\cdot r'}{r^3}}^{\text{magnet. Moment}} + \mathcal{O}\Big(\frac{1}{r^3}\Big)\Big]\, d^3r'$$

2 Skalare Felder $g, f$

1 Vektorfeld $\underline{j}$ : $\underline{j}(\infty) \to 0$

$$\int\big[f\cdot(j\cdot\nabla)g + g(j\cdot\nabla)f + g\cdot f(\nabla\cdot\underline{j})\big]\, d^3r = 0$$

$$\int g\,(j\cdot\nabla)\cdot f\, d^3r = \sum_i \int g\,(j_i\,\partial_i)\cdot f\, d^3r$$

$$= -\sum_i \int (\partial_i\, g\cdot j_i)\cdot f\, d^3r$$

$$= -\sum_i \int f\; j_i\partial_i\, g\, d^3r - \sum_i \int f\cdot g\; \partial_i j_i\, d^3r$$

$$= -\int f\,(j\cdot\nabla)\,g\, d^3r - \int f\cdot g\,(\nabla\cdot j)\, d^3r$$

MS: $\nabla\cdot j = 0$

$$\boxed{\int\big[f\cdot(j\cdot\nabla)\cdot g + g(j\cdot\nabla)\cdot f\big]\, d^3r = 0}$$

Hilfssatz der MS

a) $f(r') = 1$, $g(r') = x'_i$ $\qquad \int j_i(r')\, d^3r' = 0 \;\longrightarrow$ keine Monopolbeiträge

b.) $f(r') = x'_i$, $g(r') = x'_j$ $\;\Rightarrow\; \int\big(x'_i\, j_j + x'_j\, j_i\big)\, d^3r' = 0$

$$\underline{r}\cdot\Big(\int \underline{r}'\cdot j_i(r')\, d^3r'\Big) = \sum_j x_j \int x'_j\, j_i\, d^3r'$$

$$= -\frac{1}{2}\sum_j x_j \int\big(x'_i\, j_j - x'_j\, j_i\big)\, d^3r'$$

$a\times b = \sum_{ijk} \varepsilon_{ijk}\, a_i\, b_j\, e_k$

$(a\times b)_k = \sum_{ij} \varepsilon_{ijk}\, a_i\, b_j$

$$= -\frac{1}{2} \sum_{jk} \varepsilon_{ijk}\, x_j \int (\underline{r}' \times \underline{j})_k \, d^3r'$$

$$= -\frac{1}{2} \left[ \underline{r} \times \int \left( \underline{r}' \times \underline{j}(r') \right) d^3r' \right]_i$$

$$\boxed{\int \underline{j}(r)(\underline{r}\cdot\underline{r}')\, d^3r' = -\frac{1}{2}\, \underline{r} \times \left( \int \underline{r}' \times \underline{j}(r')\, d^3r' \right)}$$

$$\boxed{\underline{m} = \frac{1}{2c} \int \underline{r}' \times \underline{j}(r') \, d^3r'}$$

magnet. Moment

Vektorpot $\quad \underline{A}(r) = \frac{\underline{m} \times \underline{r}}{r^3}$

MF mittels $\quad B = \nabla \times A$

$$B = \nabla \times A = \nabla \times \frac{\underline{m} \times \underline{r}}{r^3} = \frac{1}{r^3} \nabla \times (\underline{m} \times \underline{r}) + \left( \nabla \frac{1}{r^3} \right) \times (\underline{m} \times \underline{r})$$

$$= \frac{1}{r^3} \left[ (\nabla \cdot \underline{r}) \cdot \underline{m} - (\nabla \cdot \underline{m}) \cdot \underline{r} \right] + \left( -\frac{3}{2} \frac{2 \cdot \underline{r}}{r^5} \right) \times (\underline{m} \times \underline{r})$$

$$= \frac{1}{r^3} \left[ 3 \cdot \underline{m} - \underline{m} \right] - 3 \frac{\underline{r} \times (\underline{m} \times \underline{r})}{r^5}$$

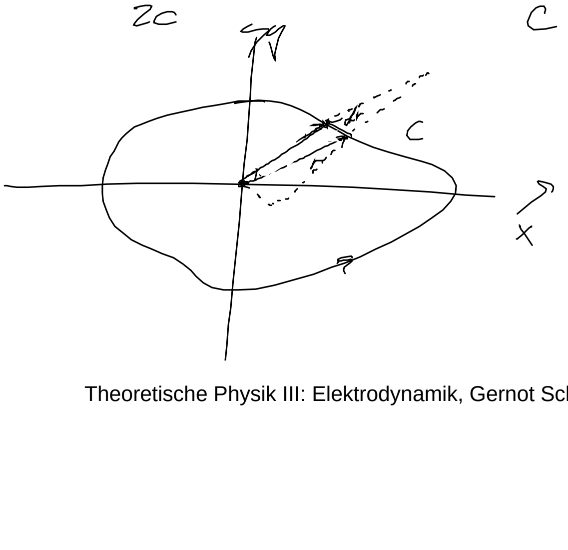
$$= \boxed{\frac{3\underline{r}\,(\underline{m}\cdot\underline{r}) - \underline{m} \cdot r^2}{r^5} = \underline{B}}$$

<u>Beispiel</u> Dipolmoment einer Leiterschleife (magn.) $\qquad \underline{j}(r') = \underline{I}\, \delta(R - \rho')\, \delta(z')\, \underline{e}_{\varphi'}$

$$\underline{m} = \frac{1}{2c} \int r' \times j(r')\, d^3r'$$

$$= \frac{I}{2c} \int \rho' d\rho' \int dz' \int d\varphi' \; \delta(R - \rho')\, \delta(z') \, \overbrace{\left[ \rho' \cdot e_{\rho'} + z' \cdot e_{z'} \right] \times \underline{e}_{\varphi'}}^{e_z}$$

$$= \frac{I}{2c} R^2 \cdot 2\pi \cdot e_z = \frac{I \cdot \pi \cdot R^2}{c} \cdot \underline{e}_z$$




$$\underline{dF} = \frac{1}{2}\, \underline{r} \times \underline{dr}$$

$$\underline{m} = \frac{1}{2c} \int r \times j(r)\, d^3r = \frac{I}{2c} \oint_C \underline{r} \times \underline{dr} = \frac{I}{c} \int \underline{dF}$$
$$= \frac{I}{c} \cdot F_c \cdot \underline{n}$$

b) Menge von PL mit gleicher Masse $m$ und Ladung $q$

$$j(r) = q \sum_i v_i \cdot \delta(\underline{r} - \underline{R}_i)$$

Geschw. ↗ Ort ↗

$$\underline{m} = \frac{q}{2cm} \sum_i \underline{R}_i \times m\underline{v}_i = \frac{q}{2mc} \sum_i \underline{R}_i \times \underline{p}_i = \frac{q}{2mc} \sum_i \underline{L}_i = \frac{q}{2mc} \underline{L}$$

$$G = \frac{q}{2mc}$$

"gyromagnet. Verhältnis"

## 4.8. Dipol im MF

### 4.8.1. Kraft auf Dipol

$$d\underline{F} = \frac{I}{c} \underline{dr} \times \underline{B}$$
$$\underline{F} = \frac{I}{c} \oint_C \underline{dr} \times \underline{B} \longrightarrow \frac{1}{c} \int (j \times B)\, d^3r$$
$$B(r) = B_0 + (r\cdot\nabla) B(r)\Big|_{r=0} + \dots$$
$$= B_0 + \sum_i (r\cdot\nabla) B_i(r)\Big|_{r=0} \underline{e}_i$$
$$F = -\frac{1}{c} \underline{B}_0 \times \underbrace{\int j(r)\, d^3r}_{0} - \frac{1}{c} \int \sum_i \left(r \cdot \nabla B_i^0\right) \cdot \underline{e}_i \times j(r)\, d^3r$$

beh. $\int j(r') (r\cdot r')\, d^3r' = -\frac{1}{2} r \times \int [r' \times j(r')]\, d^3r'$

$$F_i = -\frac{1}{c} \int d^3r \left[((r\cdot\nabla)\cdot \underline{B}_0) \times j(r)\right]_i$$
$$= -\frac{1}{c} \sum_{jk} \varepsilon_{ijk} (\nabla \cdot B_j^0) \cdot \int r' \cdot j_k(r')\, d^3r'$$
$$= \frac{1}{2c} \sum_{jk} \varepsilon_{ijk} \left\{ (\nabla B_j^0) \times \left[\int r' \times j(r')\, d^3r'\right] \right\}_k$$
$$= -\sum_{jk} \varepsilon_{ijk} \left[\underline{m} \times (\nabla B_j^0)\right]_k = -\sum_{jk} \varepsilon_{ijk} [m \times \nabla]_k \cdot B_j^0$$
$$= +\sum_{jk} \varepsilon_{ijk} (m \times \nabla)_j \cdot B_k^0$$
$$= +\left[(m \times \nabla) \times B_0\right]_i$$
$$\underline{F} = (\underline{m} \times \nabla) \times \underline{B}_0 = -\underline{m}(\nabla\cdot \underline{B}_0) + \nabla(\underline{m} \cdot \underline{B}_0)$$

$$\underline{F} = \nabla(m \cdot \underline{B}_0)$$

Potential

$$\boxed{V = -m \cdot B_0}$$

Kompass nadel

## 4.8.2. Drehmoment $M$

nur führende Ordnung

$$dM = r \times dF = \frac{I}{c} r \times (dr \times B_0)$$

$$M = \frac{1}{c} \int r \times (j \times B_0)\, d^3r$$

$$= \frac{1}{c} \int d^3r \left[ j(\underline{r} \cdot B_0) - \underline{B_0}(\underline{r} \cdot j) \right]$$

$$g = f = |r| \quad (\nabla j = 0)$$

$$0 = 2 \int r(j \cdot \nabla) \cdot r\, d^3r = 2 \int r \cdot (j \cdot e_r) = 2 \cdot \int j(r) \cdot \underline{r}\, d^3r$$

$$M = \frac{1}{c} \int d^3r\, (r \cdot B_0) \cdot j(r) = -\frac{1}{2c} B_0 \times \int d^3r \left( r \times j(r) \right)$$

$$= \underline{m} \times B_0$$

# 4.9 MS in Materie

## 4.9.1. Maxwell-Gleich. der MS

$$j_{mak}(r) = \frac{1}{\Delta V} \int j_{mik}(r + r')\, d^3r' = \langle j_{mik} \rangle$$

$$j_{mik}(r) = j_{mak}(r) + j_{mol}(r)$$

$$\langle B_{mik} \rangle = \langle \nabla \times A_{mik} \rangle = B_{mak} = \nabla \times A_{mak}$$

$$A_{mol} = \frac{1}{c} \int \frac{j_{mol}(r')}{|r - r'|}\, d^3r' \approx \sum_i \frac{m_i \times (r - r_i)}{|r - r_i|^3}$$

Zahl der Dipole im Volumen

analog zur Polarisation $\quad M = N \cdot \langle m \rangle \qquad \langle m \rangle = \frac{1}{N} \sum_i m_i$

"Magnetisierung"

$$A_{mak}(r) = \int \frac{j_{mak}(r')}{|r-r'|} d^3r' + \int M(r') \times \left(\nabla' \frac{1}{|r-r'|}\right) d^3r'$$

$$\nabla \times (\varphi \cdot M) = (\nabla\varphi) \times M + \varphi (\nabla \times M)$$

$$M(r') \times \left(\nabla' \frac{1}{|r-r'|}\right) = \underbrace{-\nabla' \times \left(\frac{M(r')}{|r-r'|}\right)}_{\text{Trägt nicht bei}} + \frac{(\nabla' \times M(r'))}{|r-r'|}$$

$$\int_V (\nabla \times \underline{b})\, d^3r = \oint_{\partial V} (\underline{n} \times \underline{b})\, dF$$

Normalen Vektor

$$\int_V \nabla' \times \frac{M(r')}{|r-r'|} d^3r' = 0$$

"Magnetisierungsstrom"

$$\to A_{mak}(r) = \frac{1}{c} \int \frac{j_{mak}(r') + [c \cdot \nabla' \times M(r')]}{|r-r'|}$$

"Leiterstrom"

$$\nabla \times B_{mak} = \frac{4\pi}{c} (j_{mak} + c \cdot \nabla \times M)$$

$$\nabla \times \underbrace{(B_{mak} - 4\pi M)}_{H} = \frac{4\pi}{c} \cdot j_{mak}$$

magnet. FS

makroskopisch

$$\boxed{\nabla \times H = \frac{4\pi}{c} \cdot j \qquad \nabla \cdot B = 0}$$

MWG der MS in Materie