

Wdh

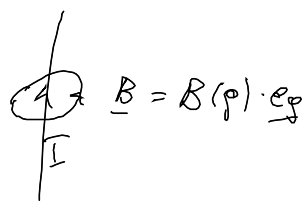
1. Klausur-Termin: 15.2.	8-10	A151
22.2.	8-10	

+ Maxwell-Gleichungen der Magnetostatik

$$\left. \begin{aligned} \nabla \cdot \underline{B} &= 0 && \text{"keine magn. Monopole"} \\ \nabla \times \underline{B} &= \frac{\mu_0}{c} \cdot \underline{j} && \text{2. Ampèresches Gesetz} \end{aligned} \right\} \text{Differentialform}$$

$$\left. \begin{aligned} \oint \underline{B} \cdot d\underline{F} &= 0 \\ \oint \underline{B} \cdot d\underline{r} &= \frac{\mu_0}{c} \cdot I_F \end{aligned} \right\} \text{integrierte Form}$$

Richtlich Draht



$$\begin{aligned} \text{z.B. } \oint \underline{B}(\rho) \cdot d\underline{r} &= \frac{\mu_0}{c} \cdot I \\ \rightarrow B(\rho) &= \frac{2I}{c\rho} \end{aligned}$$

+ Vektorpotential $\underline{B} = \nabla \times \underline{A}$ $A \rightarrow A + \nabla \varphi$ Eichfreiheit

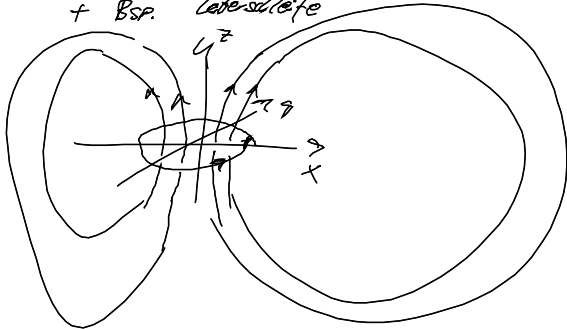
Coulomb-Forderung $\nabla \cdot \underline{A} = 0 \rightarrow \Delta A_i = -\frac{\mu_0}{c} \cdot j_i \hat{=} \text{Poisson-Gleichung}$

$$\rightarrow \underline{A}(\underline{r}) = \frac{\mu_0}{c} \int \frac{\underline{j}(\underline{r}')}{|\underline{r} - \underline{r}'|} d^3 r'$$

in Coulomb-Forderung

für endliche stat. SV

+ Bsp. Leiter-schleife



4.7. Magnet. Moment stat. lokalis. Ströme

$$\underline{A}(\underline{r}) = \frac{1}{c} \int \frac{\underline{j}(\underline{r}')}{|\underline{r} - \underline{r}'|} d^3 r' \quad \underline{r} \gg \underline{r}'$$

$$= \frac{1}{c} \int \underline{j}(\underline{r}') \left[\frac{1}{r} + \frac{\underline{r} \cdot \underline{r}'}{r^3} + \mathcal{O}\left(\frac{1}{r^3}\right) \right] d^3 r'$$

↑
trägt nicht bei

2 Skalare Felder g, f
1 Vektorfeld $\underline{j} = \underline{j}(\underline{r}) \rightarrow 0$

$$\int [\underline{j} \cdot (\underline{j} \cdot \underline{\nabla}) g + \underline{j} \cdot (\underline{j} \cdot \underline{\nabla}) f + g \cdot \underline{\nabla} (\underline{j} \cdot \underline{j})] d^3 r = 0$$

$$\int g (\underline{j} \cdot \underline{\nabla}) \cdot \underline{j} d^3 r = \sum_i \int g (\underline{j} \cdot \underline{\nabla})_i \cdot \underline{j} d^3 r$$

$$= - \sum_i \int (\underline{\nabla} \cdot \underline{j})_i \cdot \underline{j} d^3 r$$

$$= - \sum_i \int \underline{j} \cdot \underline{\nabla} \cdot \underline{j} d^3 r - \sum_i \int \underline{j} \cdot \underline{\nabla} \cdot \underline{j} d^3 r$$

$$= - \int \underline{j} \cdot (\underline{j} \cdot \underline{\nabla}) g d^3 r - \int \underline{j} \cdot \underline{\nabla} (g \cdot \underline{j}) d^3 r$$

MS: $\underline{\nabla} \cdot \underline{j} = 0$

$$\int [\underline{j} \cdot (\underline{j} \cdot \underline{\nabla}) g + g (\underline{j} \cdot \underline{\nabla}) \cdot \underline{j}] d^3 r = 0$$

Hilfssatz der MS

a) $f(\underline{r}) = 1$
 $g(\underline{r}) = x_i$

$$\int \underline{j}(\underline{r}') d^3 r' = 0 \rightarrow \text{keine Monopolbeiträge}$$

b) $f(\underline{r}') = x'_i$
 $g(\underline{r}') = x'_j$

$$\Rightarrow \int (x'_i \underline{j}_i + x'_j \underline{j}_j) d^3 r' = 0$$

$$\underline{r} \cdot \left(\int \underline{r}' \cdot \underline{j}(\underline{r}') d^3 r' \right) = \sum_i x_j \int x'_i \underline{j}_i d^3 r'$$

$$= - \frac{1}{2} \sum_i x_j \int (x'_i \underline{j}_j - x'_j \underline{j}_i) d^3 r'$$

$a \times b = \sum_{ijk} \epsilon_{ijk} a_i b_j e_k$
 $(a \times b)_k = \sum_{ij} \epsilon_{ijk} a_i b_j$

$$= -\frac{1}{2} \sum_{ijk} \epsilon_{ijk} x_j \int (x_k j_i) d^3r'$$

$$= -\frac{1}{2} \left[\underline{r} \times \left(\int \underline{r}' \times \underline{j}(\underline{r}') d^3r' \right) \right]_i$$

$$\int \underline{j}(\underline{r}') (\underline{r} \cdot \underline{r}') d^3r' = -\frac{1}{2} \underline{r} \times \left(\int \underline{r}' \times \underline{j}(\underline{r}') d^3r' \right)$$

$$\underline{h} = \frac{1}{2c} \int \underline{r}' \times \underline{j}(\underline{r}') d^3r'$$

Magnet. Moment

Vektorpot $\underline{A}(\underline{r}) = \frac{\underline{h} \times \underline{r}}{r^3}$

MF mittels $\underline{B} = \nabla \times \underline{A}$

$$\underline{B} = \nabla \times \underline{A} = \nabla \times \frac{\underline{h} \times \underline{r}}{r^3} = \frac{1}{r^3} \nabla \times (\underline{h} \times \underline{r}) + \left(\nabla \frac{1}{r^3} \right) \times (\underline{h} \times \underline{r})$$

$$= \frac{1}{r^3} \left[(\nabla \cdot \underline{r}) \cdot \underline{h} - (\nabla \cdot \underline{h}) \cdot \underline{r} \right] + \left(-\frac{3}{r^2} \frac{\underline{r} \cdot \underline{r}}{r^3} \right) \times (\underline{h} \times \underline{r})$$

$$= \frac{1}{r^3} \left[3 \cdot \underline{h} - \underline{h} \right] - 3 \frac{\underline{r} \times (\underline{h} \times \underline{r})}{r^5}$$

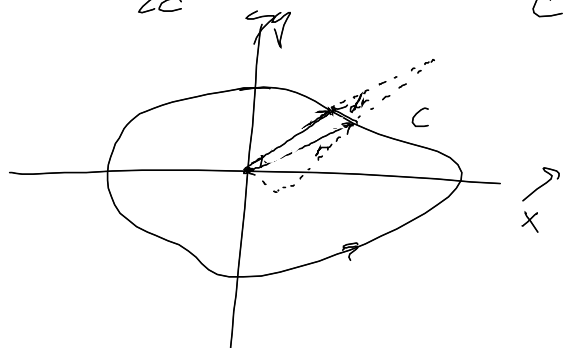
$$= \frac{2 \underline{h} (\underline{r} \cdot \underline{r}) - \underline{h} \cdot \underline{r}^2}{r^5} = \underline{B}$$

Beispiele Dipolmoment einer Leiterschleife (Loop) $\underline{j}(\underline{r}') = I \delta(R-\rho') \delta(z') \underline{e}_\phi$

$$\underline{h} = \frac{1}{2c} \int \underline{r}' \times \underline{j}(\underline{r}') d^3r'$$

$$= \frac{I}{2c} \int \rho' d\phi' \int dz' \int d\rho' \delta(R-\rho') \delta(z') \left[\rho' \underline{e}_\phi + z' \underline{e}_z \right] \times \underline{e}_\phi$$

$$= \frac{I}{2c} R^2 \cdot 2\pi \cdot \underline{e}_z = \frac{I \cdot \bar{h} \cdot R^2}{c} \cdot \underline{e}_z$$



$$d\underline{F} = \frac{1}{2} \underline{r} \times d\underline{r}$$

$$\underline{m} = \frac{1}{c} \int \underline{r} \times \underline{j}(\underline{r}) d^3r = \frac{1}{2c} \oint \underline{r} \times d\underline{r} = \frac{1}{c} \int d\underline{F}$$

$$= \frac{1}{c} \cdot F_c \cdot \underline{u}$$

b.) Menge von PL mit gleicher Masse m und Ladung q

$$\underline{j}(\underline{r}) = q \sum_i \underline{v}_i \cdot \delta(\underline{r} - \underline{R}_i)$$

↑ Gesch. ↑ Ort

$$\underline{m} = \frac{q}{2c} \sum_i \underline{R}_i \times \underline{v}_i = \frac{q}{2c} \sum_i \underline{R}_i \times \underline{p}_i = \frac{q}{2c} \sum_i \underline{L}_i = \left(\frac{q}{2c} \right) \underline{L}$$

$$G = \frac{q}{2c}$$

"gyromagnet. Verhältnis"

4.8. Dipol in \underline{A} \underline{F}

4.8.1. Kraft auf Dipol

$$d\underline{F} = \frac{1}{c} d\underline{r} \times \underline{B}$$

$$\underline{F} = \frac{1}{c} \int d\underline{r} \times \underline{B} \rightarrow \frac{1}{c} \int (\underline{j} \times \underline{B}) d^3r$$

$$\underline{B}(\underline{r}) = \underline{B}_0 + (\underline{r} \cdot \nabla) \underline{B}(\underline{r}) \Big|_{r=0} + \dots$$

$$= \underline{B}_0 + \sum_i (\underline{r} \cdot \nabla) B_i(\underline{r}) \Big|_{r=0} \underline{e}_i$$

$$\underline{F} = -\frac{1}{c} \underline{B}_0 \times \underbrace{\int \underline{j}(\underline{r}) d^3r}_0 - \frac{1}{c} \int \sum_i (\underline{r} \cdot \nabla B_i^{\downarrow}) \cdot \underline{e}_i \times \underline{j}(\underline{r}) d^3r$$

$$\underline{F}_i = -\frac{1}{c} \int d^3r \left[(\underline{r} \cdot \nabla) \cdot \underline{B}_0^{\downarrow} \right] \times \underline{j}(\underline{r}) \Big|_i$$

looks $\int \underline{j}(\underline{r}') (\underline{r} \cdot \underline{r}') d^3r'$
 $= -\frac{1}{2} \underline{r} \times \int (\underline{r}' \times \underline{j}(\underline{r}')) d^3r'$

$$= -\frac{1}{c} \sum_{jk} \epsilon_{ijk} (\nabla \cdot \underline{B}_0^{\downarrow}) \cdot \int r'_j \cdot j_k(\underline{r}') d^3r'$$

$$= \frac{1}{c} \sum_{jk} \epsilon_{ijk} \left\{ (\nabla B_i^{\downarrow}) \times \left[\int r'_j \times j(\underline{r}') d^3r' \right] \right\}_k$$

$$= -\sum_{jk} \epsilon_{ijk} \left[\underline{L} \times (\nabla B_i^{\downarrow}) \right]_k - \sum_{jk} \epsilon_{ijk} \left[\underline{L} \times \nabla \right]_k \cdot B_i^{\downarrow}$$

$$= + \sum_{jk} \epsilon_{ijk} (\underline{L} \times \nabla)_j \cdot B_k^{\downarrow}$$

$$= + \left[(\underline{L} \times \nabla) \times \underline{B}_0^{\downarrow} \right]_i$$

$$\underline{F} = (\underline{L} \times \nabla) \times \underline{B}_0^{\downarrow} = -\underline{L} (\nabla \cdot \underline{B}_0) + \nabla (\underline{L} \cdot \underline{B}_0)$$

$$\underline{E} = \nabla(\underline{u} \cdot \underline{B}_0) \quad \boxed{\text{Potential}} \quad \boxed{V = -\underline{u} \cdot \underline{B}_0}$$

Kompass nadel

4.8.2. Drehmoment \underline{H}

zur folgende Ordnung

$$d\underline{H} = \underline{r} \times d\underline{F} = \frac{I}{c} \underline{r} \times (d\underline{r} \times \underline{B}_0)$$

$$\begin{aligned} \underline{H} &= \frac{I}{c} \int \underline{r} \times (\underline{j} \times \underline{B}_0) d^3v \\ &= \frac{I}{c} \int d^3v (\underline{j} (\underline{r} \cdot \underline{B}_0) - \underline{B}_0 (\underline{r} \cdot \underline{j})) \end{aligned}$$

$$g = \underline{f} = I \underline{j} \quad (\nabla \cdot \underline{j} = 0)$$

$$\underline{O} = 2 \int \underline{r} (\underline{j} \cdot \nabla) \cdot \underline{r} d^3v = 2 \int \underline{r} (\underline{j} \cdot \underline{e}_z) = 2 \int \underline{j}(\underline{r}) \cdot \underline{r} d^3v$$

$$\underline{H} = \frac{I}{c} \int d^3v (\underline{r} \cdot \underline{B}_0) \underline{j}(\underline{r}) = \frac{I}{2c} \underline{B}_0 \times \int d^3v (\underline{r} \times \underline{j}(\underline{r}))$$

$$= \underline{u} \times \underline{B}_0$$

4.9 MS in Materie

4.9.1. Maxwell-Gleich. der MS

$$\underline{j}_{\text{ext}}(\underline{r}) = \frac{I}{SV} \int \underline{j}_{\text{ext}}(\underline{r} + \underline{r}') d^3r' = \langle \underline{j}_{\text{ext}} \rangle$$

$$\underline{j}_{\text{ext}}(\underline{r}) = \underline{j}_{\text{ext}}(\underline{r}) + \underline{j}_{\text{ext}}(\underline{r})$$

$$\langle \underline{B}_{\text{ext}} \rangle = \langle \nabla \times \underline{A}_{\text{ext}} \rangle = \underline{B}_{\text{ext}} = \nabla \times \underline{A}_{\text{ext}}$$

$$\underline{A}_{\text{ext}} = \frac{I}{c} \int \frac{\underline{j}_{\text{ext}}(\underline{r}')}{|\underline{r} - \underline{r}'|} d^3r' \approx \sum_i \frac{\underline{u}_i \times (\underline{r} - \underline{r}_i)}{|\underline{r} - \underline{r}_i|^3}$$

analog zur Polarisation $\underline{M} = N \cdot \langle \underline{u} \rangle$ $\langle \underline{u} \rangle = \frac{1}{N} \sum_i \underline{u}_i$
 "Magnetisierung"

$$A_{\text{ext}}(r) = \int \frac{j_{\text{ext}}(r')}{|r-r'|} d^3r' + \int dl(r') \times \left(\nabla' \frac{1}{|r-r'|} \right) d^3r'$$

$$\nabla \times (\varphi \cdot \underline{a}) = (\nabla \varphi) \times \underline{a} + \varphi (\nabla \times \underline{a})$$

$$dl(r') \times \left(\nabla' \frac{1}{|r-r'|} \right) = - \nabla' \times \left(\frac{dl(r')}{|r-r'|} \right) + \frac{(\nabla' \times dl(r'))}{|r-r'|}$$

$$\int_V (\nabla \times \underline{b}) d^3r = \oint_{\partial V} (\underline{a} \times \underline{b}) dF$$

∇ ∂V \uparrow ∇
Kontour Normalenvektor

Ergebnis nicht bei

$$\int_V \nabla' \times \frac{dl(r')}{|r-r'|} d^3r' = 0$$

"Magnetostrom"

$$\rightarrow A_{\text{ext}}(r) = \frac{1}{c} \int \frac{j_{\text{ext}}(r') + [c \cdot \nabla' \times dl(r')]}{|r-r'|} d^3r'$$

"Leitstrom"

$$\nabla \times B_{\text{ext}} = \frac{4\pi}{c} (j_{\text{ext}} + c \cdot \nabla \times \underline{A})$$

$$\nabla \times (B_{\text{ext}} - 4\pi \underline{A}) = \frac{4\pi}{c} j_{\text{ext}}$$

$\nabla \times \underline{H} = \frac{4\pi}{c} \cdot \underline{j}$	$\nabla \cdot \underline{B} = 0$
<small>maxim. FS</small> <small>maximaler Wert</small> \downarrow nur 6 der 12 in Materie	