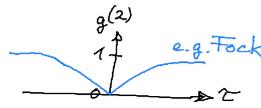


English Summary:

Conditions for nonclassical light

$$g^{(2)}(\tau) > g^{(2)}(0), \quad g^{(2)}(0) < 1 \quad (\text{sub-Poissonian correlations})$$



4.6 Quantum Master equations

$$\rho_{SR}(t) = \rho_S(t) \otimes \rho_R(0) + \rho_c(t), \quad \rho_S(t) = \text{tr}_R \rho_{SR} \quad \text{reduced density matrix}$$

system reservoir coupling

$$\dot{\rho}_S(t) = -\frac{i}{\hbar^2} \text{tr}_R \int_0^t dt' [V(t), [V(t'), \rho_S(t')] \otimes \rho_R(0)]$$

Master eq. in interaction picture; interaction Ham. V ; Born-Markov approx.

Beispiel: gedämpfter harmon. Oszillator

Ham. op. des System $S \otimes R$: $H = \underbrace{H_S + H_R}_{H_0} + \underbrace{H_{SR}}_V$

$H_S = \hbar \omega_0 a^\dagger a$ harmon. Osz. $[a, a^\dagger] = 1$

$H_R = \sum_j \hbar \omega_j r_j^\dagger r_j$ bosonische Reservoirmoden $[r_j, r_l^\dagger] = \delta_{jl}$

im therm. gl. $\rho_R = \prod_j e^{-\frac{\hbar \omega_j r_j^\dagger r_j}{kT}} (1 - e^{-\frac{\hbar \omega_j}{kT}})$

(z.B. Vakuum-Strahlungsfeld oder Phononen-Moden in Festkörpern)

↑
Zerfall einer Mode ω_0
in einer opt. Kavität

↑
Zerfall eines angeregten Atoms
durch spontane Emission

$$H_{SR} = \sum_j \hbar (\kappa_j^* a r_j^\dagger + \kappa_j a^\dagger r_j) \quad \kappa_j \in \mathbb{C} \quad \text{Dämpfungsrate}$$

$$= \hbar (a \Gamma^\dagger + a^\dagger \Gamma) \quad \Gamma, \Gamma^\dagger \quad \text{Kopplungsop.}$$

im WW-Bild:

$$a_W(t) = e^{i\omega_0 a^\dagger a t} a e^{-i\omega_0 a^\dagger a t} \otimes e^{-i\omega_0 t} = a e^{-i\omega_0 t}$$

$$a_W^\dagger(t) = e^{i\omega_0 a^\dagger a t} a^\dagger e^{-i\omega_0 a^\dagger a t} = a^\dagger e^{i\omega_0 t}$$

Schröd.
Bild

⊗ denn $\dot{a} = -i\omega_0 [a, a^\dagger] = -i\omega_0 (a^\dagger a - a a^\dagger) = -i\omega_0 a$
 (Heisenberg-gln.) $[a, a^\dagger] = 1$

$$\Gamma_w^+(t) = e^{i \sum_n \omega_n r_n^+ t} \sum_j k_j^* r_j^+ e^{-i \sum_m \omega_m r_m^+ t} = \sum_j k_j^* r_j^+ e^{i\omega_j t} \Rightarrow \langle \Gamma_w^+(t) \rangle_R = 0$$

$$\Gamma_w(t) = e^{i \sum_n \dots} \sum_j k_j r_j e^{-i \sum_m \dots} = \sum_j k_j r_j e^{-i\omega_j t} \Rightarrow \langle \Gamma_w(t) \rangle_R = 0$$

Mastergl. (Born-Markov) im WW-Bild

$$\dot{\hat{\rho}}_S = - \int_0^t dt' \{ (a a \rho_S(t') - \rho_S(t') a) e^{-i\omega_0(t+t')} \underbrace{\langle \Gamma_w^+(t) \Gamma_w^+(t') \rangle_R}_0 + \text{h.c.} \}$$

$\langle r_n r_m \rangle = \langle r_n^+ r_m^+ \rangle = 0$
 $\langle r_n^+ r_m \rangle = \bar{n}_n \delta_{nm}$
 $\langle r_n r_m^+ \rangle = (\bar{n}_n + 1) \delta_{nm}$

$$+ (a^\dagger a \rho_S(t') - \rho_S(t') a^\dagger) e^{i\omega_0(t+t')} \underbrace{\langle \Gamma_w(t) \Gamma_w(t') \rangle_R}_0$$

$$+ (a a \rho_S(t') - \rho_S(t') a) e^{-i\omega_0(t-t')} \underbrace{\langle \Gamma_w^+(t) \Gamma_w(t') \rangle_R}_{\sum_j |k_j|^2 e^{i\omega_j(t-t')} \bar{n}(\omega_j, T)}$$

$$+ (a^\dagger a \rho_S(t') - \rho_S(t') a^\dagger) e^{i\omega_0(t-t')} \underbrace{\langle \Gamma_w(t) \Gamma_w^+(t') \rangle_R}_{\sum_j |k_j|^2 e^{-i\omega_j(t-t')} (\bar{n}(\omega_j, T) + 1)}$$

+ h.c. }

mit $\bar{n}(\omega_j, T) = \frac{1}{\mathcal{Z}_R} \text{tr}_R (e^{-\frac{\hbar\omega_j}{kT}}) = \frac{e^{-\frac{\hbar\omega_j}{kT}}}{1 - e^{-\frac{\hbar\omega_j}{kT}}}$ Bose-Verteilung

$\tau = t - t'$

$$\dot{\hat{\rho}}_S = - (a a \rho_S - \rho_S a) \int_0^t dt' e^{-i\omega_0 \tau} \langle \Gamma_w^+(t) \Gamma_w^+(t-\tau) \rangle_R + \text{h.c.}$$

$$- (a^\dagger a \rho_S - \rho_S a^\dagger) \int_0^t dt' e^{i\omega_0 \tau} \langle \Gamma_w(t) \Gamma_w^+(t-\tau) \rangle_R + \text{h.c.}$$

Kontinuumslimes der Reservoir-Korrelationsfkt. es:

$$\langle \Gamma_w^+(t) \Gamma_w^+(t-\tau) \rangle_R = \int_0^\infty d\omega e^{i\omega\tau} g(\omega) |k(\omega)|^2 \bar{n}(\omega, T) \approx \delta(\tau)$$

$$\langle \Gamma_w(t) \Gamma_w^+(t-\tau) \rangle_R = \int_0^\infty d\omega e^{-i\omega\tau} g(\omega) |k(\omega)|^2 [\bar{n}(\omega, T) + 1] \approx \delta(\tau)$$

⏟
Schnell
osill.
⏟
langsam variabel

$\int \approx 0$ für $\tau \gg t_R = \frac{\hbar}{kT}$ Reservoir-Korrelationszeit

$$\Leftrightarrow \omega \tau = \frac{\hbar \omega}{kT} \gg 1$$

Def. Zerfallsraten:

$$\gamma_1 := \int_0^t d\tau \int_0^\infty d\omega e^{-i(\omega - \omega_0)\tau} g(\omega) |K(\omega)|^2$$

$$\gamma_2 := \int_0^t d\tau \int_0^\infty d\omega e^{-i(\omega - \omega_0)\tau} g(\omega) |K(\omega)|^2 \bar{n}(\omega, T)$$

Hauptbeitrag der τ -Integration: kleine $\tau \Rightarrow$ ersetze $\int_0^t \rightarrow \int_0^\infty d\tau$

$$\lim_{t \rightarrow \infty} \int_0^t d\tau e^{-i(\omega - \omega_0)\tau} = \pi \delta(\omega - \omega_0) + i \frac{P}{\omega_0 - \omega}$$

Cauchy-Hauptwert

$$\Rightarrow \gamma_1 = \frac{\gamma}{2} + i\Delta$$

$$\gamma_2 = \frac{\gamma}{2} \bar{n} + i\Delta'$$

mit $\gamma := 2\pi g(\omega_0) |K(\omega_0)|^2$

$$\Delta := P \int_0^\infty d\omega \frac{g(\omega) |K(\omega)|^2}{\omega_0 - \omega}$$

$$\Delta' := P \int_0^\infty d\omega \frac{g(\omega) |K(\omega)|^2}{\omega_0 - \omega} \bar{n}$$

Mastergl.

$$\dot{g}_s = (a^\dagger g_s a - a a^\dagger g_s) \gamma_2^* + (a^\dagger g_s a - g_s a a^\dagger) \gamma_2$$

h.c. $g_s = g_s^\dagger$

$$+ (a g_s a^\dagger - a^\dagger a g_s) (\gamma_1 + \gamma_2) + (a g_s a^\dagger - g_s a^\dagger a) (\gamma_1^* + \gamma_2^*)$$

h.c.

$$= a^\dagger g_s a \gamma \bar{n} - (a a^\dagger g_s + g_s a a^\dagger) \frac{\gamma}{2} \bar{n} + (a a^\dagger g_s - g_s a a^\dagger) i \Delta'$$

$a^\dagger a g_s - g_s a^\dagger a$ $[a, a^\dagger] = 1$

$$+ a g_s a^\dagger \gamma (1 + \bar{n}) - (a^\dagger a g_s + g_s a^\dagger a) \frac{\gamma}{2} (1 + \bar{n}) - (a^\dagger a g_s - g_s a^\dagger a) i (\Delta + \Delta')$$

$$= -i\Delta [a^\dagger a, g_s] + \frac{\gamma}{2} (2a g_s a^\dagger - a^\dagger a g_s - g_s a^\dagger a)$$

$$+ \gamma \bar{n} (a g_s a^\dagger + a^\dagger g_s a - a^\dagger a g_s - g_s a a^\dagger)$$

Rücktransform. WW-Bild \rightarrow Schrödinger-Bild $g_s^{(s)}$

$$\dot{g}_s^{(s)} = -\frac{i}{\hbar} [H_s, g_s^{(s)}] + e^{-\frac{i}{\hbar} H_s t} g_s^{(s)} e^{\frac{i}{\hbar} H_s t}$$

$$e^{-i\omega_0 a^\dagger a t} \underbrace{a g_s a^\dagger e^{i\omega_0 a^\dagger a t}}_{\text{Wigner-Bild}} = \underbrace{a g_s^{(s)} a^\dagger}_{\text{Schrodinger-Bild}} \quad H_s = \hbar \omega_0 a^\dagger a$$

Mastergl. für gedämpfte harmon. Osz. =
(Schrodinger-Bild)

$$\dot{g}_s^{(s)} = -i\omega_0' \underbrace{[a^\dagger a, g_s^{(s)}]}_{\substack{\text{renormierte} \\ \text{Frequenz} \\ \text{durch Ankopp.} \\ \text{ans Bad}}} + \frac{\chi}{2} (2a g_s^{(s)} a^\dagger - a^\dagger a g_s^{(s)} - g_s^{(s)} a^\dagger a) + \gamma \bar{n} (a g_s^{(s)} a^\dagger + a^\dagger g_s^{(s)} a - a^\dagger a g_s^{(s)} - g_s^{(s)} a a^\dagger)$$

dissipative Dynamik

Dämpfung durch

- spontane Em.
- stimul. Em. / Abs. ($\sim \bar{n}$)
von Bad-Partikeln

(z.B. Photonen, Phononen)

$$\Leftrightarrow \dot{g}_s^{(s)} = -i\omega_0' [a^\dagger a, g_s^{(s)}] + \frac{\chi}{2} ([a, g_s^{(s)} a^\dagger] + [a g_s^{(s)}, a^\dagger]) + \gamma \bar{n} ([a g_s^{(s)}, a^\dagger] + [a^\dagger, g_s^{(s)} a])$$

$$\begin{aligned} g a a^\dagger &= g a a^\dagger + g \\ a g a^\dagger &= a a^\dagger g \\ &= a^\dagger a g + g \\ &= a^\dagger g a + g \end{aligned}$$

$$\Leftrightarrow \dot{g}_s^{(s)} = -i\omega_0' [a^\dagger a, g_s^{(s)}] + \frac{\chi}{2} (\bar{n}+1) (2a g_s^{(s)} a^\dagger - \{a^\dagger a, g_s^{(s)}\}) + \frac{\chi}{2} \bar{n} (2a^\dagger g_s^{(s)} a - \{a a^\dagger, g_s^{(s)}\})$$

$n+1 \rightarrow n$ Anti-Kommutator $n \rightarrow n-1$
 $n-1 \rightarrow n$ $n \rightarrow n+1$

Lindblad-Form