Nachtrag zur EIT: Slow light

für schwache Probe-Felder, die durch ein nichtlineare Materialien propagieren

$$E(z,t) = \mathcal{E}(z,t)\, e^{i[\omega t - kz + \phi(z,t)]}$$

SVE:
$$\frac{\partial \mathcal{E}}{\partial z} + \frac{1}{c}\frac{\partial \mathcal{E}}{\partial t} = -\frac{\sigma}{2\varepsilon_0}\mathcal{E} - \frac{1}{2\varepsilon_0} k \operatorname{Im}[P]$$

$$\frac{\partial \phi}{\partial z} + \frac{1}{c}\frac{\partial \phi}{\partial t} = -\frac{\Delta}{2\varepsilon_0} k \frac{1}{\mathcal{E}} \operatorname{Re}[P]$$

– im wesentlichen 3 Effekte
Amplitude ($\mathcal{E}$): Gain & Absorption
Phase (ϕ): Gruppengeschwindigkeit

$$P \propto d_{ab}\rho_{ab}, \quad \chi = \frac{P}{\varepsilon_0 \Omega_p}$$

$\Delta = (\omega_{ab} - \omega)$

$$\chi = \frac{i\, d_{ab}}{\varepsilon_0}\frac{\gamma_3 + i(\omega_{ab}-\omega)}{(\gamma_1 + i\Delta)(\gamma_3 + i\Delta) + \Omega_\mu^2/4} = \chi(\omega)$$

$$v_g = \frac{c}{n_g} = \frac{c}{n + \omega\frac{\partial n}{\partial \omega}} \quad (SVE)$$

$\rightsquigarrow$ $\boxed{\text{Medium}}$

$$n(\omega) = 1 + \frac{1}{2}\operatorname{Re}[\chi]$$

$$\frac{\partial n}{\partial \omega} \overset{\Delta \ll \gamma_1,\gamma_3}{=} -\frac{d_{ab}}{2\varepsilon_0}\frac{1}{\gamma_1\gamma_3 + \Omega_\mu^2/4}\frac{\partial}{\partial \omega}\Delta(\omega)$$

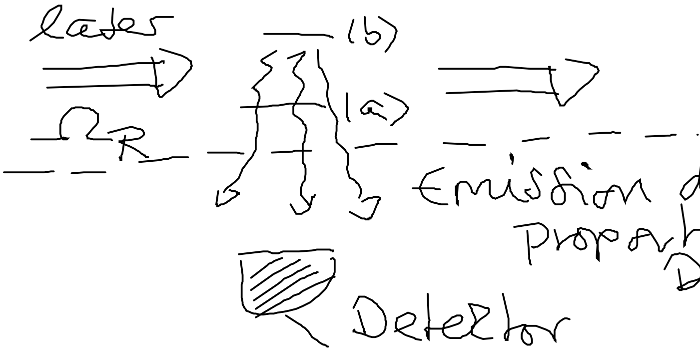
$$= A\frac{1}{\gamma_1\gamma_3 + \Omega_\mu^2/4} \longrightarrow \infty \qquad \Omega_\mu \ll 1,\ \gamma_1,\gamma_3 \ll 1$$

$v_g \to 0$ (Puls hält an)

Anwendung: Wigner-Kristallisation (LUKIN)

Resonanz-Fluoreszenz: (Scully, chapter 10)

experimentelle Situation



Emission des ZNS proportional zum Dipolmoment

Identifizieren das emittierte Licht mit einer rein quantenelektronischen Größe (Photoelektronen)

$$S(\omega) = \lim_{T\to\infty} \frac{1}{T} \int_0^T dt \int_0^T dt' \langle E^{(-)}(t) E^{(+)}(t') \rangle e^{i\omega(t-t')}$$

(steady state)

$$= \lim_{T\to\infty} \frac{1}{T} \int_0^T dt \left[\int_0^t dt' + \int_t^T dt' \right] \langle \ldots \rangle e^{i\omega(t-t')}$$

$$t' = t - \tau, \quad t - t' = \tau$$

$$= \lim_{T\to\infty} \frac{1}{T} \int_0^T dt \Big[\int_0^{\tau=t} d\tau \langle E^{(-)}(\tau) E^{(+)}(0) \rangle e^{-i\omega\tau}$$

$$+ \int d\tau \langle E^{(-)}(0) E^{(+)}(\tau) \rangle e^{i\omega\tau} \Big]$$

$$\int_0^t d\tau\, f(\tau) = \int_0^{2\tau} d\tau\, f(\tau) = \int_0^\infty d\tau\, f(\tau)$$

dann gilt für hinreichend großes T

$$\int_0^{T-t} d\tau\, f(\tau) = \int_0^\infty d\tau\, f(\tau)$$

$$S(\omega) = \mathrm{Re}\left[\int_0^\infty d\tau\, \langle E^{(-)}(0)\, E^{(+)}(\tau)\rangle\, e^{i\omega\tau}\right]$$

$$\langle E^{(+)}(t)\rangle \propto \langle \sigma_-(t)\rangle$$

$$S(\omega) = \mathrm{Re}\left[\int_0^\infty d\tau\, \langle \sigma_+(0)\, \sigma_-(\tau)\rangle\, e^{i\omega\tau}\right]$$

nicht zeitaufgelöstes Spektrum

$$\sigma_- \equiv |a\rangle\langle b|$$

$$\dot{\rho} = -\frac{i}{\hbar}\left[\frac{\hbar\Omega_R}{2}(\sigma_- + \sigma_+), \rho\right] + \frac{\Gamma}{2}\left\{2\sigma^-\rho\,\sigma^+ - \sigma^+\sigma^-\rho - \rho\,\sigma^+\sigma^-\right\}$$

Amplituden-Noise

$$H|\pm\rangle = \pm\varepsilon|\pm\rangle$$

$$|\pm\rangle = \frac{1}{\sqrt{2}}\{|a\rangle \pm |b\rangle\}$$

$$\frac{\hbar\Omega_R}{2}\left(|a\rangle\langle b| + |b\rangle\langle a|\right)\frac{1}{\sqrt{2}}\left(|a\rangle \pm |b\rangle\right)$$

$$= \frac{\hbar\Omega_R}{2\sqrt{2}}\{|a\rangle\underbrace{\langle b|a\rangle}_{=0} \pm |a\rangle\underbrace{\langle b|b\rangle}_{=1} + \dots\}$$

$$= \pm\frac{\hbar\Omega_R}{2}|\pm\rangle$$

Wir haben die Eigenzustände!

$$|a\rangle = \frac{1}{\sqrt{2}}\left(|+\rangle + |-\rangle\right), \quad |b\rangle = \frac{1}{\sqrt{2}}\left(|+\rangle - |-\rangle\right)$$

$$|a\rangle\langle b| = \sigma_- = \frac{1}{2}\{|+\rangle\langle+| + |-\rangle\langle+| - |+\rangle\langle-| - |-\rangle\langle-|\}$$

so drücken wir die $\dot{\rho} = \frac{i}{\hbar}[\rho, H] + \frac{\Gamma}{2}\mathcal{D}[\sigma_-]\rho$

$$H = \frac{\hbar\Omega_R}{2}\left(|+\rangle\langle+| - |-\rangle\langle-|\right)$$

$$\langle+|\dot{\rho}|+\rangle = \dot{\rho}_{++} = -\frac{\Gamma}{2}\rho_{++} + \frac{\Gamma}{4}$$

$$\rho_{++}(t) = e^{-\frac{\Gamma}{2}t}\rho_{++}(0) + \frac{1}{2}\left(1 - e^{-\frac{\Gamma}{2}t}\right)$$

$$\langle+|\dot{\rho}|-\rangle = \dot{\rho}_{+-} = -\left(\frac{3\Gamma}{4} + i\Omega_R\right)\rho_{+-} - \frac{\Gamma}{4}\rho_{-+}$$

$$-\frac{\Gamma}{2}$$

$$\langle-|\dot{\rho}|+\rangle = \dot{\rho}_{-+} = \left(\dot{\rho}_{+-}\right)^*$$

$$\frac{d}{dt}\begin{pmatrix} \rho_{+-} \\ \rho_{-+} \end{pmatrix} = \underbrace{\begin{pmatrix} 3\Gamma/4 + i\Omega_R & \Gamma/4 \\ \Gamma/4 & 3\Gamma/4 - i\Omega_R \end{pmatrix}}_{=: M} \begin{pmatrix} \rho_{+-} \\ \rho_{-+} \end{pmatrix} \underbrace{- \frac{\Gamma}{2}\begin{pmatrix} 1 \\ 1 \end{pmatrix}}_{=: B}$$

$$\dot{R} = MR + B$$

$$V^{-1} M V = D$$

$$\Omega_R = \text{const}: V^{-1}\dot{R} = \frac{d}{dt}\left(V^{-1}R\right) = \underbrace{V^{-1}MV}_{=D} V^{-1}R + V^{-1}B$$

$$[V^{-1}R](t) = e^{-Dt}\,[V^{-1}R] + \\ + \int_0^t dt'\, e^{-D(t-t')}\, V^{-1}B$$

$$D = \text{diag}\,(\lambda_1, \lambda_2)$$

$$e^{-Dt} = \begin{pmatrix} e^{-\lambda_1 t} & 0 \\ 0 & e^{-\lambda_2 t} \end{pmatrix}$$

$$R(t) = \left\{ V e^{-Dt} V^{-1} \right\} R(0) \\ + V D^{-1} \left(\underline{1} - e^{-Dt}\right) V^{-1} B$$

$$V = \begin{pmatrix} \cos\Theta & -\sin\Theta \\ \sin\Theta & \cos\Theta \end{pmatrix}, \quad V V^{-1} = 1$$

$$V^{-1} = \begin{pmatrix} \cos\Theta & \sin\Theta \\ -\sin\Theta & \cos\Theta \end{pmatrix}$$

$$\lambda_{1,2} = \frac{3\Gamma}{4} \pm \underbrace{\sqrt{\Omega_R^2 - \left(\frac{\Gamma}{4}\right)^2}}_{=:\mu}$$

$$\tan\Theta = -\frac{4i}{\Gamma}\left(\Omega_R - \mu\right) = \frac{\sin\Theta}{\cos\Theta}$$

$$\begin{pmatrix} 3\Gamma/4 + i\Omega_R & \Gamma/4 \\ \frac{\Gamma}{4} & \frac{3\Gamma}{4} - i\Omega_R \end{pmatrix} \begin{pmatrix} \cos\Theta \\ \sin\Theta \end{pmatrix} = \lambda_1 \begin{pmatrix} \cos\Theta \\ \sin\Theta \end{pmatrix} \checkmark$$

$$= \begin{pmatrix} \cos\Theta\left[\frac{3\Gamma}{4} + \cancel{i\Omega_R} - \cancel{i\Omega_R} + i\mu\right] \\ \sin\Theta\left[-\left(\frac{\Gamma^2}{16i}\right)\frac{1}{\Omega_R - \mu} + \frac{3\Gamma}{4} - i\Omega_R\right] \end{pmatrix}$$

$$\Rightarrow -\frac{\Gamma^2}{16i}\frac{1}{\Omega_R - \mu} - i\mu - i\Omega_R = 0$$

$$\tan\Theta = x$$

$$\operatorname{atan}\tan\Theta = \Theta = \operatorname{atan} x$$

$$\cos\Theta = \cos(\operatorname{atan} x) = \frac{1}{\sqrt{1+x^2}}$$

$$\sin\Theta = \frac{x}{\sqrt{1+x^2}}$$

$$\rho_{+-}(t) = \frac{1}{\mu}\left(2\mu\cos\mu t - 2i\Omega_R\sin\mu t\right) e^{-3\Gamma t/4}\rho_{+-}(0) - \frac{\Gamma}{4\mu}\sin\mu t\, e^{-3\Gamma t/4}\rho_{-+}(0)$$

$$+\frac{\Gamma}{2\mu}\left\{\frac{2\mu\left(\Gamma/2-i\Omega_R\right)}{\Gamma^2+2\Omega_R^2}\cos\mu t\right.$$

$$\left.-\left[1-\frac{(3\Gamma/2)(\Gamma/2-i\Omega_R)}{\Gamma^2+2\Omega_R^2}\right]\sin\mu t\right\}$$

$$e^{-3\Gamma t/4}$$

$$-\frac{\Gamma(\Gamma/2-i\Omega_R)}{\Gamma^2+2\Omega_R^2}$$

$$\rho_{++}(t)=f\left(\rho_{++}(0),t\right)$$

$$\rho_{+-}(t)=g\left(\rho_{-+}(0),\rho_{+-}(0),t\right)$$

$$\langle\sigma_-\rangle=\frac{1}{2}\left\{2\rho_{++}-\rho_{+-}+(\rho_{+-})^*-1\right\}$$

$$=\langle|a\rangle\langle b|\rangle$$

$$\rho_{++}+\rho_{--}=1$$

$$\rho_{++}(t)=\bar{f}\left(\langle\sigma_-\rangle(0),\langle\sigma_+\rangle(0),\langle\sigma_z\rangle(0)\right)$$

$$\rho_{++}=\frac{1}{2}\left(1+\rho_{ab}+\rho_{ba}\right),\quad\rho_{ab}=\langle\sigma_-\rangle$$

$$\rho_{+-}=\frac{1}{2}\left(2\rho_{aa}-1-\rho_{ab}+\rho_{ba}\right)$$

und damit

$$\langle\sigma_-(t)\rangle e^{-i\omega_{ba}t}=$$

(a_i in Scully 10.5)

$$=a_1(t)+a_2(t)\langle\sigma_-(0)\rangle$$

$$+a_3(t)\langle\sigma_+(0)\rangle+a_4(t)\left\{\langle\sigma_z(0)\rangle-1\right\}/2$$

$$S(\omega)=\frac{1}{\pi}\mathrm{Re}\left\{\int_0^\infty d\tau\langle\sigma_+(t)\sigma_-(t+\tau)\rangle e^{i\omega\tau}\right\}$$

$$[t \to \tau,\ 0 \to t]$$

$$\langle \sigma_-(t+\tau)\rangle e^{-i\omega_{ba} t} =$$

$$a_1(\tau) + a_2(\tau)\langle \sigma_-(t)\rangle e^{i\omega t}$$

$$+ a_3(\tau)\langle \sigma_+(t)\rangle e^{-i\omega t}$$

$$+ a_4(\tau)\{\langle \sigma_z(t)\rangle + 1\}/2$$

$$\langle \sigma^-(t)\rangle \overset{t\to\infty}{=} \cancel{\frac{1}{2}} - \cancel{\frac{1}{2}} - i\,\mathrm{Im}\left[-\frac{\Gamma(\Gamma/2 - i\Omega_R)}{2\Omega_R^2 + \Gamma^2}\right]$$

$$= -i\frac{\Gamma\Omega_R}{\Gamma^2 + 2\Omega_R^2} \quad //$$

$$\langle \sigma_z(t)\rangle \overset{t\to\infty}{=} \langle |a\rangle\langle a| - |b\rangle\langle b|\rangle$$

$$= 2\,\mathrm{Re}[\rho_{+-}]$$

$$= -\Gamma\frac{\Gamma/2}{\Gamma^2 + 2\Omega_R^2}$$