

## Nachtrag zur EIT: Slow light

für schwache Probe-Felder, die durch ein nichtlineares Materialien propagieren

$$E(z,t) = \mathcal{E}(z,t) e^{i[\omega t - kz + \phi(z,t)]}$$

$$\underline{\text{SVE}}: \frac{\partial \mathcal{E}}{\partial z} + \frac{1}{c} \frac{\partial \mathcal{E}}{\partial t} = -\frac{\sigma}{2\epsilon_0} \mathcal{E} - \frac{1}{2\epsilon_0} \chi \text{Im}[P]$$

$$\frac{\partial \phi}{\partial z} + \frac{1}{c} \frac{\partial \phi}{\partial t} = -\frac{\Delta}{2\epsilon_0} \chi \frac{1}{\mathcal{E}} \text{Re}[P]$$

- im wesentlichen 3 Effekte

Amplitude ( $\mathcal{E}$ ): gain & Absorption

Phase ( $\phi$ ): Gruppengeschwindigkeit

$$P \propto d_{ab} S_{ab}, \quad \chi = \frac{P}{\epsilon_0 \omega P}$$

$$\chi = \frac{i d_{ab}}{\epsilon_0} \frac{\delta_3 + i(\omega_{ab} - \omega)}{(\delta_1 + i\Delta)(\delta_3 + i\Delta) + \Omega_{\mu}^2/4} = \chi(\omega)$$

$$v_g = \frac{c}{n_g} = \frac{c}{n + \omega \frac{\partial n}{\partial \omega}} \quad (\text{SVE})$$



$$n(\omega) = 1 + \frac{1}{2} \text{Re}[\chi]$$

$$\frac{\partial n}{\partial \omega} \stackrel{\Delta \ll \delta_1, \delta_3}{=} -\frac{d_{ab}}{2\epsilon_0} \frac{1}{\delta_1 \delta_3 + \Omega_{\mu}^2/4} \frac{\partial}{\partial \omega} \Delta(\omega)$$

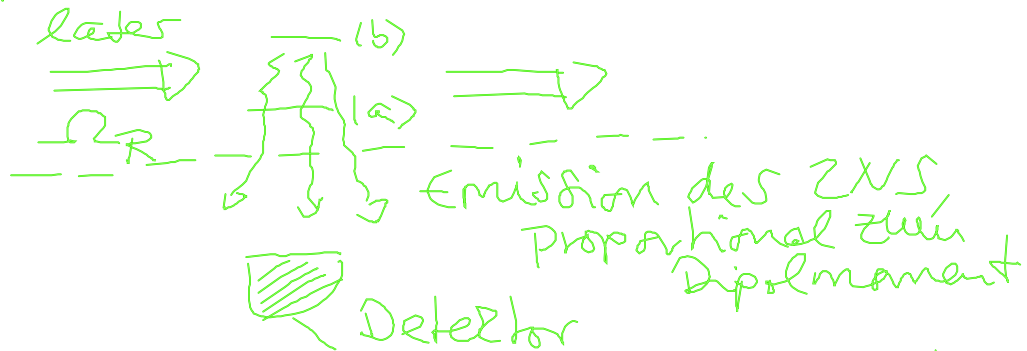
$$= A \frac{1}{\delta_1 \delta_3 + \Omega_{\mu}^2/4} \quad \begin{matrix} \Omega_{\mu} \ll 1 \\ \delta_1, \delta_3 \ll 1 \end{matrix}$$

→ ∞

$V_g \rightarrow 0$  (Puls hält an)  
 Anwendung: Wigner-Kristallisation  
 (LURIM)

Resonanz-Fluoreszenz: (Skully, Chapter 10)

experimentelle Situation



Identifizieren das emittierte Licht mit einer rein quantenelektromagnetischen Größe (Photoelektronen)

$$\begin{aligned}
 S(\omega) &= \lim_{T \rightarrow \infty} \frac{1}{T} \sqrt{T} \int_0^T dt' \langle E^{(-)}(t) E^{(+)}(t') \rangle e^{-i\omega(t-t')} \\
 &\quad \text{(Steady state)} \\
 &= \lim_{T \rightarrow \infty} \frac{1}{T} \sqrt{dt} \left[ \int_0^t dt' + \int_t^T dt' \right] \langle \dots \rangle e^{-i\omega(t-t')} \\
 &\quad \frac{t'}{T} = \frac{t}{T} - \tau, \quad t - t' = \tau \\
 &= \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^t dt \left[ \int_0^{t-t} d\tau \langle E^{(+)}(t) E^{(+)}(0) \rangle e^{-i\omega\tau} \right. \\
 &\quad \left. + \int_0^t d\tau \langle E^{(-)}(0) E^{(+)}(t) \rangle e^{-i\omega\tau} \right]
 \end{aligned}$$

$$\int_0^+ d\tau f(\tau) = \int_0^{T-t} d\tau f(\tau) = \int_0^{\infty} d\tau f(\tau)$$

dann gilt für hinreichend

großes  $T$   $\int_0^{T-t} d\tau f(\tau) = \int_0^{\infty} d\tau f(\tau)$

$$S(\omega) = \text{Re} \left[ \int_0^{\infty} d\tau \langle E^{(+)}(0) E^{(+)}(\tau) \rangle e^{i\omega\tau} \right]$$

$$\langle E^{(+)}(t) \rangle = \langle \sigma_{-}(t) \rangle$$

$$S(\omega) = \text{Re} \left[ \int_0^{\infty} d\tau \langle \sigma_{+}(0) \sigma_{-}(\tau) \rangle e^{i\omega\tau} \right]$$

nicht zeit aufgelöstes

Spektrum

$$\sigma_{-} \equiv |a\rangle\langle b|$$



$$\dot{g} = -\frac{i}{\hbar} \left[ \frac{\hbar \Omega_R}{2} (\sigma_{-} + \sigma_{+}), g \right]$$

$$+ \frac{\Gamma}{2} \{ 2\sigma_{-} g \sigma_{+} - \sigma_{+} \sigma_{-} g - g \sigma_{+} \sigma_{-} \}$$

Amplituden - Noise

$$H|\pm\rangle = \pm \epsilon |\pm\rangle$$

$$|\pm\rangle = \frac{1}{\sqrt{2}} \{ |a\rangle \pm |b\rangle \}$$

$$\begin{aligned} \frac{\hbar \Omega_R}{2} (|a\rangle\langle b| + |b\rangle\langle a|) \frac{1}{\sqrt{2}} (|a\rangle \pm |b\rangle) \\ = \frac{\hbar \Omega_R}{2\sqrt{2}} \{ |a\rangle\langle b| \underbrace{\langle a|a\rangle}_1 \pm |a\rangle\langle b| \underbrace{\langle b|b\rangle}_1 + \dots \} \\ = \pm \frac{\hbar \Omega_R}{2} |\pm\rangle \end{aligned}$$

Wir haben die Eigenzustände!

$$|a\rangle = \frac{1}{\sqrt{2}} (|+\rangle + |-\rangle), \quad |b\rangle = \frac{1}{\sqrt{2}} (|+\rangle - |-\rangle)$$

$$|a\rangle\langle b| = \sigma_- = \frac{1}{2} \{ |+\rangle\langle +| + |-\rangle\langle +| + |+\rangle\langle -| - |-\rangle\langle -| \}$$

so schreiben wir die  $\hat{g} = \frac{\Omega}{\hbar} [\sigma_x, \hat{H}]$

$$H = \frac{\hbar \Omega_R}{2} (|+\rangle\langle +| - |-\rangle\langle -|) \quad + \frac{\hbar \Omega}{2} \sigma_x$$

$$\langle +|\dot{g}|+\rangle = \dot{g}_{++} = -\frac{\hbar}{2} g_{++} + \frac{\hbar}{4}$$

$$g_{++}(t) = e^{-\frac{\hbar}{2}t} g_{++}(0) + \frac{1}{2} (1 - e^{-\frac{\hbar}{2}t})$$

$$\langle +|\dot{g}|-\rangle = \dot{g}_{+-} = -\left(\frac{3\hbar}{4} + \frac{\hbar}{2}\Omega_R\right) g_{+-} - \frac{\hbar}{4} g_{-+}$$

$$\langle -|\dot{g}|+\rangle = \dot{g}_{-+} = (g_{+-})^*$$

$$\frac{d}{dt} \begin{pmatrix} \mathcal{S}_{+-} \\ \mathcal{S}_{-+} \end{pmatrix} = \underbrace{\begin{pmatrix} 3\Omega_R/4 + i\Omega_R & \Omega_R/4 \\ \Omega_R/4 & 3\Omega_R/4 - i\Omega_R \end{pmatrix}}_{=iM} \begin{pmatrix} \mathcal{S}_{+-} \\ \mathcal{S}_{-+} \end{pmatrix} + \underbrace{\begin{pmatrix} 1 \\ 1 \end{pmatrix}}_{=iB}$$

$$V^{-1} M V = D$$

$$\Omega_R = \text{const}; V^{-1} \dot{R} = \frac{d}{dt} (V^{-1} R) = \underbrace{V^{-1} M V V^{-1} R}_{=D} + V^{-1} B$$

$$[V^{-1} R](t) = e^{-Dt} [V^{-1} R] + \int_0^t dt' e^{-D(t-t')} V^{-1} B$$

$$D = \text{diag}(\lambda_1, \lambda_2)$$

$$e^{-Dt} = \begin{pmatrix} e^{-\lambda_1 t} & 0 \\ 0 & e^{-\lambda_2 t} \end{pmatrix}$$

$$R(t) = \{ V e^{-Dt} V^{-1} \} R(0) + V D^{-1} (1 - e^{-Dt}) V^{-1} B$$

$$V = \begin{pmatrix} \cos \Theta & -\sin \Theta \\ \sin \Theta & \cos \Theta \end{pmatrix}, \quad V V^{-1} = 1$$

$$V^{-1} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$$

$$\lambda_{1,2} = \frac{3n}{4} \pm \underbrace{\sqrt{\Omega_R^2 - \left(\frac{n}{4}\right)^2}}_{=: \mu}$$

$$\tan \theta = -\frac{4i}{n} (\Omega_R - \mu) = \frac{\sin \theta}{\cos \theta}$$

$$\begin{pmatrix} \frac{3n}{4} + i\Omega_R & \frac{n}{4} \\ \frac{n}{4} & \frac{3n}{4} - i\Omega_R \end{pmatrix} \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix} = \lambda_1 \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix}$$

$$= \begin{pmatrix} \cos \theta \left[ \frac{3n}{4} + i\Omega_R - \frac{n}{4} \right] \\ \sin \theta \left[ -\left(\frac{n}{4}\right) \frac{1}{\Omega_R - \mu} + \frac{3n}{4} - i\Omega_R \right] \end{pmatrix}$$

$$\Rightarrow -\frac{n^2}{16i} \frac{1}{\Omega_R - \mu} - i\mu - i\Omega_R = 0$$

$$\tan \theta = x$$

$$\operatorname{atan} \tan \theta = \theta = \operatorname{atan} x$$

$$\cos \theta = \cos (\operatorname{atan} x) = \frac{1}{\sqrt{1+x^2}}$$

$$\sin \theta = \frac{x}{\sqrt{1+x^2}}$$

$$g_{\pm}(t) = \frac{1}{\mu} (2\mu \cos \mu t - 2i\Omega_R \sin \mu t) e^{-3nt/4} g_{\pm}(0) - \frac{n}{4\mu} \sin \mu t e^{-3nt/4} g_{\mp}(0)$$

$$\begin{aligned}
& + \frac{r}{z_M} \left\{ \frac{z_M (\Gamma_z - i\Omega_R)}{r^2 + 2\Omega_R^2} \cos \mu t \right. \\
& \quad \left. - \left[ 1 - \frac{(3r/2) (\Gamma_z - i\Omega_R)}{r^2 + 2\Omega_R^2} \right] \sin \mu t \right\} \\
& \quad - \frac{r (\Gamma_z - i\Omega_R)}{r^2 + 2\Omega_R^2} e^{-3\mu t/4}
\end{aligned}$$

$$S_{++}(t) = f(S_{++}(0), t)$$

$$S_{+-}(t) = g(S_{-+}(0), S_{+-}(0), t)$$

$$\langle \sigma^- \rangle = \frac{1}{2} \{ 2S_{++} - S_{+-} + (S_{+-})^* - 1 \}$$

$$\approx \langle 1a \rangle \langle b \rangle$$

$$S_{++} + S_{--} = 1$$

$$S_{++}(t) = \bar{F} \left( \langle \sigma^- \rangle(0), \langle \sigma^+ \rangle(0), \langle \sigma_z \rangle(0) \right)$$

$$S_{++} = \frac{1}{2} (1 + S_{ab} + S_{ba}), \quad S_{ab} = \langle \sigma^- \rangle$$

$$S_{+-} = \frac{1}{2} (2S_{aa} - 1 - S_{ab} + S_{ba})$$

und damit

$$\langle \sigma^- (t) \rangle e^{-i\omega_0 t} =$$

( $a_i$  in  
Scully 10.5)

$$= a_1(t) + a_2(t) \langle \sigma_- (0) \rangle$$

$$+ a_3(t) \langle \sigma_+ (0) \rangle + a_4(t) \{ \langle \sigma_z (0) \rangle - 1 \} / 2$$

$$\left\{ S(\omega) = \frac{1}{\pi} \operatorname{Re} \left\{ \int_0^{\infty} d\tau \langle \sigma_+(t) \sigma_-(t+\tau) \rangle e^{i\omega\tau} \right\} \right\}$$

$$[t \rightarrow \tau, 0 \rightarrow t]$$

$$\langle \sigma_-(t+\tau) \rangle e^{-i\omega_s \tau} =$$

$$a_1(\tau) + a_2(\tau) \langle \sigma_-(t) \rangle e^{i\omega t}$$

$$+ a_3(\tau) \langle \sigma_+(t) \rangle e^{-i\omega t}$$

$$+ a_4(\tau) \{ \langle \sigma_z(t) \rangle + 1 \} / 2$$

$$\begin{aligned} \langle \sigma_-(t) \rangle &\stackrel{t \rightarrow \infty}{=} \frac{1}{2} - \frac{1}{2} - i \operatorname{Im} \left[ -\frac{r(r - i\Omega_R)}{\Omega_R^2 + r^2} \right] \\ &= -i \frac{r \Omega_R}{r^2 + 2\Omega_R^2} // \end{aligned}$$

$$\begin{aligned} \langle \sigma_z(t) \rangle &\stackrel{t \rightarrow \infty}{=} \langle |a\rangle \langle a| - |b\rangle \langle b| \rangle \\ &= 2 \operatorname{Re} [S_{+-}] \\ &= -r \frac{r/2}{r^2 + 2\Omega_R^2} \end{aligned}$$