

## Mollow Triplett: (Resonanzfluoreszenz II)

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$$\text{Messung: } S(\omega) = \text{Re} \left[ \int_0^{\infty} dt \langle \sigma_+(0) \sigma_-(t) \rangle_{ss} e^{i\omega t} \right]$$

im Steady State  $\xrightarrow{\Omega_R}$   $\begin{array}{c} |b\rangle \\ \uparrow \downarrow \\ |a\rangle \end{array}$  (radiativer Zerfall)

$$\lim_{t \rightarrow \infty} \langle \sigma_-(t) \rangle = -j \frac{\Gamma \Omega_R}{\Gamma^2 + 2\Omega_R^2}$$

$$\lim_{t \rightarrow \infty} \langle \sigma_z(t) \rangle = \frac{-\Omega_R^2}{\Gamma^2 + 2\Omega_R^2}$$

Ziel: Zwei-Zeiten-Korrelationen

$$\lim_{t \rightarrow \infty} \langle \sigma_+(t) \sigma_-(t+\tau) \rangle = \langle \sigma_+(0) \sigma_-(\tau) \rangle_{ss}$$

$$\begin{aligned} \text{Lösung } \langle \sigma_-(t) \rangle = & a_1(t) + a_2(t) \langle \sigma_-(0) \rangle \\ & + a_3(t) \langle \sigma_+(0) \rangle \\ & + a_4(t) \frac{1}{2} \{ \langle \sigma_z(0) \rangle + 1 \} \end{aligned}$$

$$\underline{a_1(t)} = -j \frac{\Omega_R \Gamma}{\Gamma^2 + 2\Omega_R^2}$$

$$\left\{ 1 - e^{-\frac{3}{4}\Gamma t} \left[ \cos \mu t \left( \frac{4\Omega_R^2 - \Gamma^2}{4\mu\Gamma} \right) \sin \mu t \right] \right\}$$

Hilft uns das für  $\langle \sigma_+(0) \sigma_-(\tau) \rangle$ ?  
 Quantenregression (einfach!)

$$\frac{d}{d\tau} \langle A(\tau) \rangle = \left\langle \frac{d}{d\tau} A(\tau) \right\rangle$$

$$= -\frac{i}{\hbar} \langle [H, A] \rangle = a_1 \langle B(\tau) \rangle + a_2 \mathbb{1}$$

$$\frac{d}{d\tau} \langle B(\tau) \rangle = b_1 \langle A(\tau) \rangle + b_2$$

→ Lösung wäre dann

$$\langle A(\tau) \rangle = c_1(t) \langle A(0) \rangle + c_2(t) \langle B(0) \rangle + c_3(t)$$

$$\frac{d}{d\tau} \langle C(t) A(\tau) \rangle = \langle C(t) \frac{d}{d\tau} A(\tau) \rangle$$

$$= a_1 \langle C(t) B(\tau) \rangle + a_2 \langle C(t) \rangle$$

$$\frac{d}{d\tau} \langle C(t) B(\tau) \rangle = b_1 \langle C(t) A(\tau) \rangle + b_2 \langle C(t) \rangle$$

benenne nun  $C(t)A(\tau) = A'(\tau)$

$$\langle A'(\tau) \rangle = c_1(\tau) \langle A'(0) \rangle + c_2 \langle \mathbb{1} \rangle$$

$$A'(0) = C(t)A(0)$$

$$\langle A(t+\tau) \rangle = a_1(\tau) \langle A(t) \rangle + a_2(\tau) \langle B(t) \rangle + a_3(\tau)$$

$$\begin{aligned} \langle A'(t) \rangle &= \langle C(t) A(t+\tau) \rangle \\ &= \tilde{a}_1(t) \langle C(t) A(t) \rangle + \tilde{a}_2 \langle C(t) B(t) \rangle \\ &\quad + \tilde{a}_3(t) \langle C(t) \rangle \end{aligned}$$

alg. Onwager-Lex-Regression  
Theorem

existiert  $\langle \hat{O}(t+\tau) \rangle = \sum_j a_j(\tau) \langle \hat{O}_j(t) \rangle$

$$\begin{aligned} \text{dann } \langle \hat{O}_i(t) \hat{O}(t+\tau) \hat{O}_k(t) \rangle \\ = \sum_j a_j(\tau) \langle \hat{O}_i(t) \hat{O}_j(t) \hat{O}_k(t) \rangle \end{aligned}$$

$$\begin{aligned} \langle \sigma_+(t) \sigma_-(t+\tau) \rangle &= \\ &= a_1(t) \langle \sigma_+(t) \rangle + a_2(\tau) \langle \sigma_+(t) \sigma_-(t) \rangle \\ &\quad + a_3(\tau) \langle \sigma_+(t) \sigma_+(t) \rangle \\ &\quad + a_4(\tau) \frac{1}{2} \{ \langle \sigma_+(t) \sigma_-(t+\tau) \rangle \} \leftarrow = -|b\rangle \langle a| \end{aligned}$$

$$= a_1(t) \langle \sigma_+(t) \rangle + a_2(\tau) \underbrace{\langle \sigma_+(t) \sigma_-(t) \rangle}_{\text{auch bekannt}}$$

$$\begin{aligned} \langle \sigma_+(t) \sigma_-(t) \rangle &= \frac{1}{2} \{ \langle \sigma_z(t) \rangle + 1 \} \\ &= s_1(t) = \frac{\Omega_F^2}{\Omega_F^2 + 2\Omega_R^2} \left\{ 1 - \left( \cos \mu t + \frac{3\Omega_F}{4\Omega_R} \sin \mu t \right) e^{-\frac{\gamma}{2} t} \right\} \end{aligned}$$

für  $\langle \sigma_z(0) \rangle = -1$ ,  $\langle \sigma_-(0) \rangle = 0 = \langle \sigma_+(0) \rangle$

$$\begin{aligned} \lim_{t \rightarrow \infty} \langle \sigma_+(t) \sigma_-(t+\tau) \rangle \\ = [a_1(\tau) a_1^*(\infty) + a_2(\tau) s_1(\infty)] e^{-\frac{\gamma}{2} \tau} \end{aligned}$$

$$\begin{aligned}
& e^{-i\omega_{ba}t} \left( \frac{\Omega_R^2}{r^2 + 2\Omega_R^2} \right) \left\{ \frac{r^2}{r^2 + 2\Omega_R^2} + \frac{e^{-r\tau/2}}{2} + \right. \\
& \left. + \frac{e^{-3r\tau/4}}{4} \left[ e^{-\frac{i\sqrt{\Omega_R^2 - (r/4)^2}\tau}{2}} \left\{ \frac{2\Omega_R^2 - r^2}{2\Omega_R^2 + r^2} + \frac{i\frac{r}{4\mu} (10\Omega_R^2 - r^2)}{2\Omega_R^2 + r^2} \right\} \right. \right. \\
& \left. \left. + e^{\frac{i\mu\tau}{2}} \left\{ \frac{2\Omega_R^2 - r^2}{2\Omega_R^2 + r^2} - \frac{i\frac{r}{4\mu} (10\Omega_R^2 - r^2)}{2\Omega_R^2 + r^2} \right\} \right] \right\}
\end{aligned}$$

Diskussion:

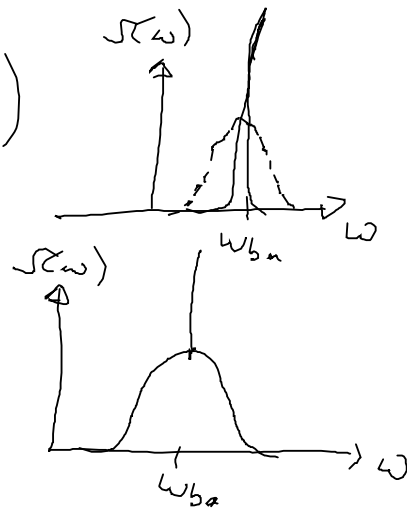
(1) Schwache Anregung  $\Omega_R \ll \frac{r}{4}$ :  
 also für  $\mu = \sqrt{\Omega_R^2 - (r/4)^2} \approx i\frac{r}{4}$

$$\begin{aligned}
& \langle \sigma_+(t) \sigma_-(t+\tau) \rangle = \\
& = e^{-i\omega_{ba}t} \left( \frac{\Omega_R^2}{r^2 + 2\Omega_R^2} \right) \\
& \left\{ \frac{r^2}{r^2 + 2\Omega_R^2} + \frac{e^{-r\tau/2}}{2} + \right. \\
& \left. + e^{-3r\tau/4} \left[ e^{r\tau} \left( \frac{2\Omega_R^2 - r^2}{2\Omega_R^2 + r^2} + \frac{i\frac{r}{4\mu} (10\Omega_R^2 - r^2)}{2\Omega_R^2 + r^2} \right) \right. \right. \\
& \left. \left. + e^{-r\tau} \left( \frac{2\Omega_R^2 - r^2}{2\Omega_R^2 + r^2} - \frac{i\frac{r}{4\mu} (10\Omega_R^2 - r^2)}{2\Omega_R^2 + r^2} \right) \right] \right\}
\end{aligned}$$

$$= e^{-i\omega_{ba}t} \left( \frac{\Omega_R^2}{r} \right)^2 \left\{ 1 + \frac{e^{-r\tau/2}}{2} - \frac{e^{-r\tau/2}}{2} \right\}$$

$$S(\omega) = \text{Re} \left[ \int_0^{\infty} dt \langle \sigma_+(t) \sigma_-(t+t) \rangle e^{i(\omega - \omega_0)t} \right]$$

$$= \left( \frac{\Omega_R}{r} \right)^2 \delta(\omega - \omega_0)$$



(2) starke Anregung  $\Omega_R \gg \frac{\Gamma}{4}$

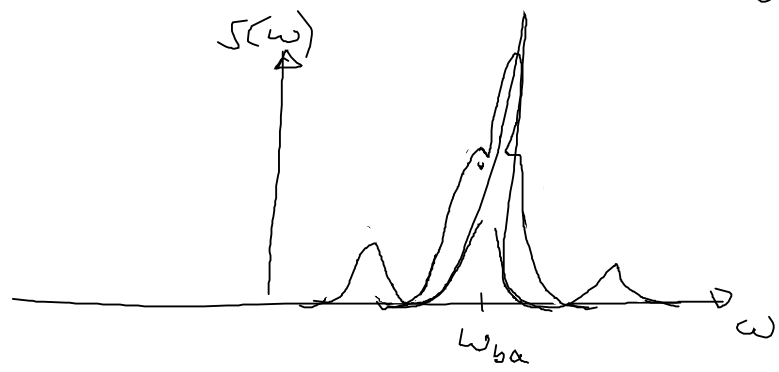
→ die komplette 2-Zeiten-Korrelation muss mitgenommen werden

$S(\omega)$  konvergiert, da  $\mu = \sqrt{\Omega_R^2 - \left(\frac{\Gamma}{4}\right)^2}$  reell,  $\Gamma < 4\Omega_R$  ist vorhanden

$$S(\omega) = \left( \frac{\Omega_R^2}{2\Omega_R^2 + \Gamma^2} \right) \left[ \frac{4\pi\Gamma^2}{\Gamma^2 + 2\Omega_R^2} \delta(\omega_{ba} - \omega) + \frac{\Gamma^2}{(\omega_{ba} - \omega)^2 + \left(\frac{\Gamma}{2}\right)^2} + \frac{3\Gamma/4 \cdot \frac{2\Omega_R^2 - \Gamma^2}{2\Omega_R^2 + \Gamma^2} + (\omega_{ba} + \mu - \omega) \frac{\Gamma}{4\mu} A}{(\omega_{ba} - \omega + \mu)^2 + (3\Gamma/4)^2} + \frac{(3\Gamma/4) \frac{2\Omega_R^2 - \Gamma^2}{2\Omega_R^2 + \Gamma^2} - (\omega_{ba} - \mu - \omega) \frac{\Gamma}{4\mu} \frac{10\Omega_R^2 - \Gamma^2}{4\mu(2\Omega_R^2 + \Gamma^2)}}{(\omega_{ba} - \mu - \omega)^2 + (3\Gamma/4)^2} \right]$$

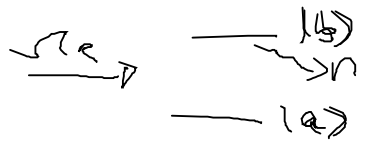
$A = \frac{10\Omega_R^2 - \Gamma^2}{2\Omega_R^2 + \Gamma^2}$

→ zusätzliche Features erscheinen

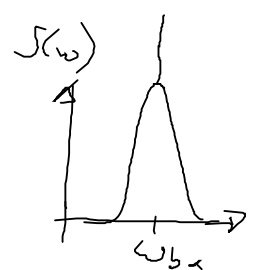


⇒ Mollow-Triplett

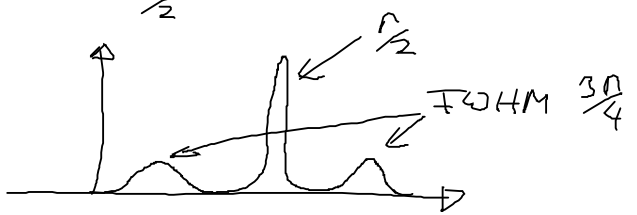
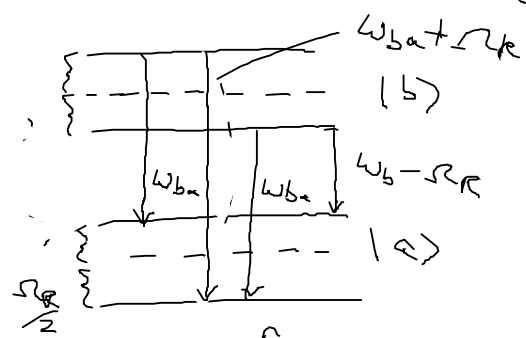
$\Omega_R \gg \frac{\Gamma}{4}$  Peaks (relative) 1 : 3 : 1



lineares  
Spektrum



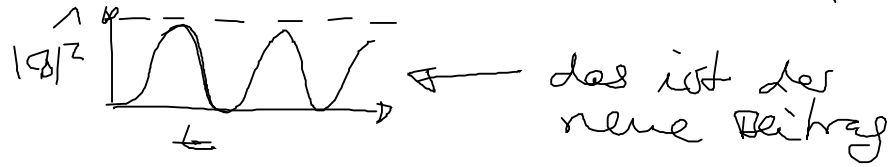
Stark aufheben  
"dressed state"



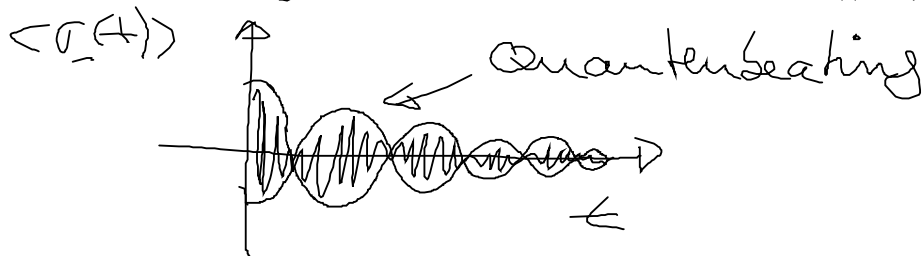
gemessen Frove et al. PRA 19, 227 (1977)

→ Beweis der Rabi-Oszillation,  
da Peaks mit  $\Omega_R$  shiften

Erinnere:  $|c_b|^2 = \sin^2[\Omega_R t]$ ,  $\phi = 0$



$\hat{=}$  AC- Stark-Effekt  
(dynamischer Stark-Effekt)



gemessen (u. a.) Na  $3^2 S_{1/2} \rightarrow 3^2 P_{3/2}$   
Übergang optisch aktiv

$$\frac{\omega_{21}}{2\pi} = 78 \text{ MHz}$$