

SUMMARY

Networks of excitable systems

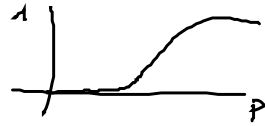
Excitability type I SNIPER
 " II FHN

$$\begin{cases} \dot{r} = r(1-r^2) \\ \dot{\varphi} = b - r \cos \varphi \end{cases}$$

$$\begin{cases} \epsilon \dot{u} = u - \frac{u^3}{3} \\ \dot{v} = u + a \end{cases}$$

Desync in small-world networks:

o with additional links p



o SNIPER with small τ



5. WECHSELSPIEL VON DELAY UND RAUSCHEN

BISHAR : DETERMINIST. } dyn. Systeme
 NUN : STOCHAST. }

STOCHAST. PROZESS

Zufallsvariable $X(t)$ ← ZEITENTWICKL.

(↔ thermodyn. Gleichgewicht)
 =
 Zeitunabhängig

$$P(x_1, t_1; x_2, t_2; \dots) = P(x_1, t_1) P(x_2, t_2 | \dots)$$

↑
Realisationen von $X(t)$

MARKOFF PROZESS

$$P(x_1, t_1 | x_2, t_2; \dots) := \frac{P(x_1, t_1; x_2, t_2; \dots)}{P(x_2, t_2; \dots)}$$

↑
bedingte Wahrscheinl.

↙ Verbundwahrscheinl.

$$t_1 \geq t_2$$

AMNESIE

$$:= P(x_1, t_1 | x_2, t_2)$$

LANGEVIN - GLEICHUNG

$$m\ddot{x} = -\eta\dot{x} + \xi(t)$$

Reibung

Rauschen

Gauß'sches white noise

$\langle \dots \rangle$
stat. Mittel

- $\langle \xi(t) \rangle = 0$
- $\langle \xi(t) \xi(t') \rangle = \delta(t-t')$ unkorrel.
- zentraler Grenzwertsatz: $\xi(t) \rightarrow$ Gauß. verteilung

AUTOKORREL. FKT

$$\Psi(s) := \langle (x(t) - \langle x \rangle) (x(t+s) - \langle x \rangle) \rangle \quad \langle x \rangle = 0$$

ergod. System

$$= \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T dt x(t) x(t+s)$$

WIENER - KHINCHIN - THEOREM

FOURIER
TRAPFO
von

$\psi(s)$

Spektrale
Leistungsichte

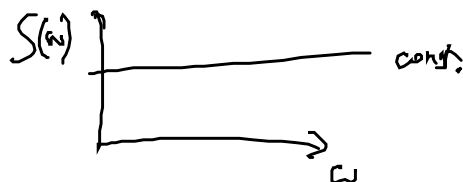
$$S(\omega) := \lim_{T \rightarrow \infty} \frac{\pi}{T} |\hat{X}(\omega, T)|^2$$

$$\hat{X}(\omega, T) = \frac{1}{2\pi} \int_{-T}^T dt e^{i\omega t} x(t)$$

$$\Rightarrow S(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \langle x(t) x(t+s) \rangle e^{i\omega s} ds$$

white noise

$$\longrightarrow S(\omega) \stackrel{\delta(s)}{=} \frac{1}{2\pi}$$



Rausch induzierte Oszillationen

1. Bsp.

$$\begin{cases} \dot{x} = y \\ \dot{y} = (\varepsilon - x^2)y - \omega_0^2 x + D \xi(t) \end{cases}$$

$$D=0 \begin{cases} \varepsilon = -0.01, \omega_0 = 1 \\ \lambda = \frac{\varepsilon}{2} \pm i \sqrt{\omega_0^2 - \left(\frac{\varepsilon}{2}\right)^2} \approx \frac{\varepsilon}{2} \pm i\omega_0 \end{cases}$$

VdP:

- $\varepsilon > 0$ inst. Folos + LC
- $\varepsilon = 0$ Hopf Bif.
- $\varepsilon < 0$ stab. Folos

$D \neq 0$ rauschind. Osz

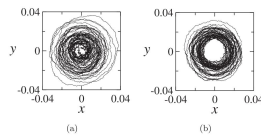
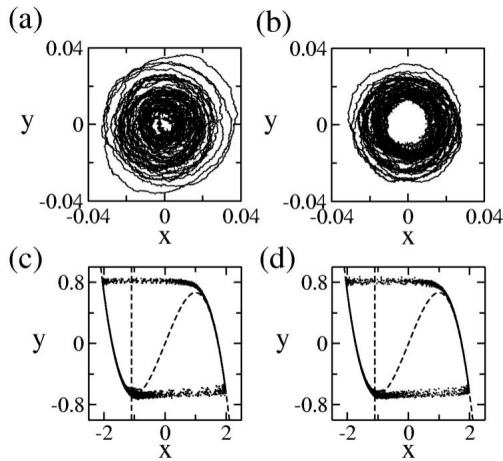


Fig. 1. Numerically simulated phase portraits of noise-induced oscillations of the Van der Pol system at $\omega_0 = 1, \varepsilon = -0.01, D = 0.003$: (a) without feedback $K = 0$; (b) with feedback $K = 0.2, \tau = \tau_0$. In both cases the system was integrated during 300 time units.

2. Bsp. FHN

$$\begin{cases} \dot{x} = x - \frac{x^3}{3} - y \\ \dot{y} = x + a + D \xi(t) \end{cases}$$

$$|a| > 1$$



Vdp

FHN

FIG. 1. Phase portraits of noise-induced motion: (a),(b) Van der Pol oscillator at $D = 0.003$; (c),(d) FitzHugh-Nagumo system at $D = 0.09$ (the dashed lines denote the null isoclines), (a),(c) $K = 0$; (b),(d) $K = 0.2$, $\tau = T_0$.

KOHÄRENZ RESONANZ

(Pikovsky, Kurths, PRL 78 (1997))

konstruktive Einfluss von noise:

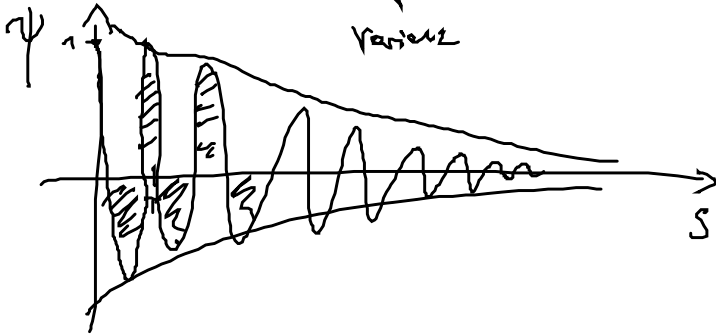
KOHÄRENZ (REGULARITÄT) optimal
für ein $D_{opt} > 0$

Maß für KOHÄRENZ: KORRELATIONZEIT t_{cor}

∞

$$t_{\text{cor}} = \frac{1}{\psi(0)} \int_0^{\infty} |\psi(s)| ds$$

$\underbrace{\hspace{10em}}_{\text{Varienz}}$



Bsp. $\dot{x} = -(\lambda - i\omega_0)x + \xi(t)$

$$\psi(s) = \psi(0) e^{-\lambda s} \cos(\omega_0 s)$$

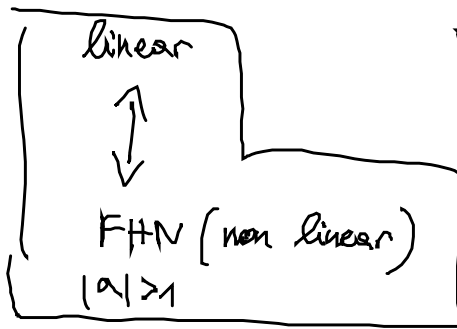
$$t_{\text{cor}} = \int_0^{\infty} e^{-\lambda s} |\cos(\omega_0 s)| ds$$

$$\lambda \ll \omega_0$$

$$\frac{1}{\pi} \int_{-\pi/2}^{+\pi/2} \cos \phi d\phi = \frac{2}{\pi}$$

$$\approx \frac{2}{\pi} \int_0^{\infty} e^{-\lambda s} ds = \frac{2}{\pi \lambda}$$

$$\psi(s) = \psi(0) e^{-\frac{2}{\pi} \frac{s}{t_{\text{cor}}}} \cos(\omega_0 s)$$



Exponentiell abklingende Korrelation

