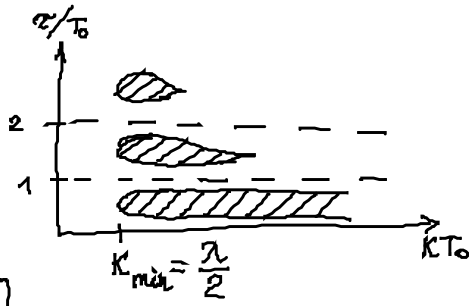
English Summary

Stabilization of unstable fixed point by time-delayed feedback

Stab. boundaries $\mathrm{Re}\,\Lambda = 0 \Rightarrow$



Multiple feedback (ETDAS)

$$K \sum_{n=0}^{\infty} R^n \left[x(t - n\tau) - x(t-(n+1)\tau) \right]$$

$$\Rightarrow \boxed{\Lambda + K \frac{1 - e^{-\Lambda\tau}}{1 - R e^{-\Lambda\tau}} = \lambda + i\omega} \qquad (0 \leq R < 1)$$

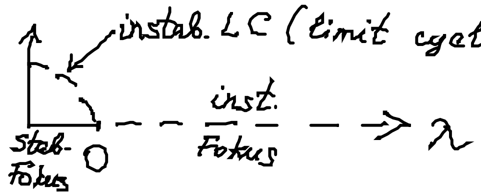
Stabilisierung instabiler periodischer Orbits

Normalform einer subkrit. Hopf-Bif.:

$$\boxed{\dot{z} = \left(\lambda + i\omega + (1 + i\gamma)|z|^2\right) z + b\left(z(t-\tau) - z(t)\right)} \quad z \in \mathbb{C}$$

$$\lambda < 0, \quad \omega = 1, \quad \gamma > 0, \quad b = b_0 e^{i\beta} \in \mathbb{C}$$

ohne Kontrolle



$z = r e^{i\phi}$: $\quad \dot{r} = (\lambda + r^2) r$

$\qquad \dot{\phi} = \omega + \gamma r^2$

LC: $r^2 = -\lambda$, ex. für $\lambda < 0$

$\dot{\phi} = \omega - \gamma\lambda \Rightarrow T = \dfrac{2\pi}{\omega - \gamma\lambda}$ Periode des UPO (unstable periodic orbit)

nichtinvasive Kontrolle!

Wähle $\tau \stackrel{!}{=} nT = \dfrac{2\pi n}{1-\gamma\lambda}$ (T = Periode des UPO)
$n \in \mathbb{N}$

(Pyragas-Kurve in der (τ, λ)-Ebene)

Hopf-Kurve: lin. Stab. des Fixp. $z(t) \sim e^{\eta t}$

$$\eta + b\,(e^{-\eta\tau}) = \lambda + i$$

Hopf-Bif: $\eta = i\omega$:

$$0 = \lambda + b_0\left[\cos(\beta - \omega\tau) - \cos\beta\right] \quad (1)$$

$$\omega - 1 = b_0\left[\sin(\beta - \omega\tau) - \sin\beta\right] \quad (2)$$

$$(1) \Rightarrow \omega\tau = \pm \arccos\left(\frac{b_0\cos\beta - \lambda}{b_0}\right) + \beta + 2\pi n$$

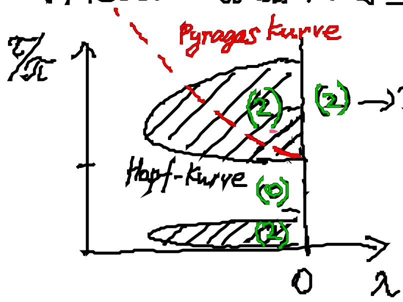
$$\left.\begin{aligned} \left(\frac{b_0\cos\beta - \lambda}{b_0}\right)^2 &= \cos^2(\beta - \omega\tau) \\ \left(\frac{\omega - 1 + b_0\sin\beta}{b_0}\right)^2 &= \sin^2(\beta - \omega\tau) \end{aligned}\right\} \cos^2 + \sin^2 = 1 \qquad (3)$$

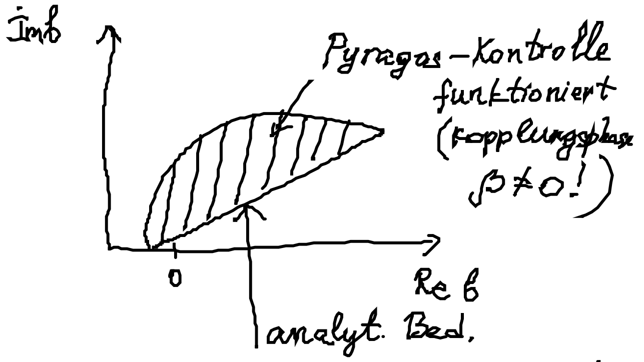
$$\stackrel{(3)}{\Rightarrow} \omega = g(\lambda, b_0, \beta) \Rightarrow \tau = h(\lambda, b_0, \beta)$$ Hopf-Kurven in (τ, λ)-Ebene

ohne Kontrolle $(b = 0)$ — subkr. Hopf

mit Kontrolle $(b = b_0 e^{i\beta})$ — superkrit. Hopf entlang der Pyragaskurve

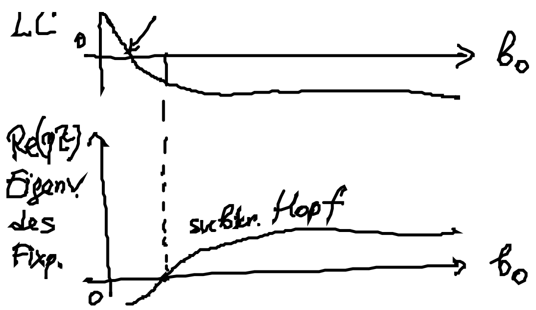
. Fiedler et al. PRL 98, 114101 (2007)





Re$(\lambda\tau)$
Floques exp des

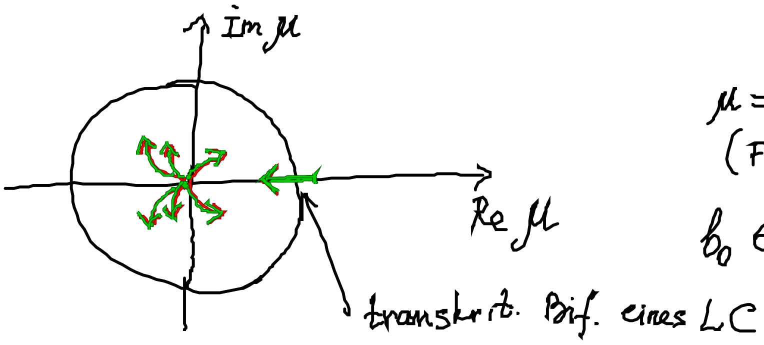
transkr. Bif von LC



$\tau_{Hopf}(\lambda) < \tau_{Pyragas}(\lambda)$

$\beta = \frac{\pi}{4}$

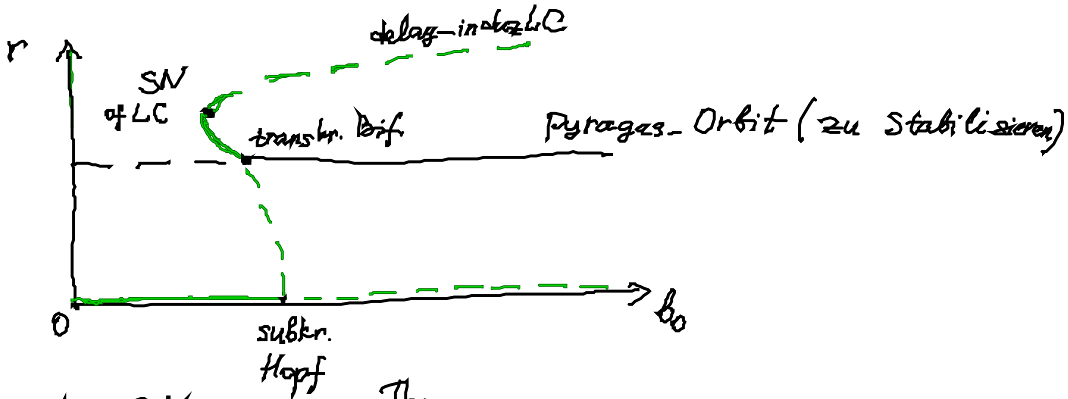
$\gamma = -10, \quad \tau = \frac{2\pi}{1-\gamma\lambda}, \quad \lambda = -0{,}005$



$\mu = e^{\Lambda\tau}$

(Floquet Mult.)

$b_0 \in [0, 0.3]$



korrektes Odd-Number-Theorem:

- E.W. Hooton, A. Amann, PRL 109, 154101 (2012)
  Analytical limitation for time-delayed feedback control in autonomous systems
- A. Amann, E.W. Hooton, Phil. Trans. R. Soc. A. (2013)

Exp. Verifikation durch Laserexperiment

- Schikora, Wünsche, Henneberger PRE 83, 026203 (2011)
- Schikora et al. PRL 97, 213902 (2006)

- Stabilisierung des Odd-Number Orbits
- Nichtinvasive Kontrolle
- Stab. möglich für endl. Kontrollphase $\beta \neq \phi$

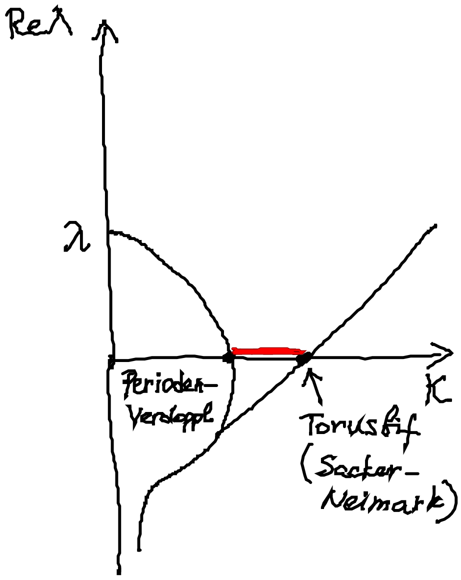
3.2.3. Chaos-Kontrolle durch zeitverzögerte Rückkopplung

Pyragas, Phys. Lett. A 170, 421 (1992)

$$\boxed{\dot{x} = f(x) + KA\,[x(t-\tau) - x(t)]} \quad x \in \mathbb{R}^n$$

$n \times n$ Kopplungsmatrix A

$\dot{x} = f(x)$: chaot. Attraktor mit unendlich vielen UPOs



Unkontroll. Floquet-Problem

$$\delta x = e^{\Lambda t} u(t)$$

$$u(t) = u(t+T)$$

$$(\lambda + i\omega)u + \dot{u} = Dfu$$

Kontrolle (diagonale)

$$\Lambda u + \dot{u} = Dfu + K\left[e^{-\Lambda\tau} - 1\right]u$$

$$\Rightarrow \boxed{\Lambda + K\left(1 - e^{-\Lambda\tau}\right) = \lambda + i\omega}$$

Schöll und Schuster (eds): Handbook of Chaos Control (2008)

3.2.4 Kontrolle raum-zeitliche Systeme

- Kyrychko, Blyuss, Hogan, Schöll: Chaos 19, 043126 (2009)

Gray-Scott-Modell (Reaktions-Diff.-System)

$U + 2V \to 3V$
$V \to P$

$A \rightleftarrows U$

Activator U
Inhibitor V

$$\begin{pmatrix} \dot{u} \\ \dot{v} \end{pmatrix} = \begin{pmatrix} -uv^2 + a(1-u) + D_u \nabla^2 u \\ uv^2 - (a+b)v + D_v \nabla^2 v \end{pmatrix} + KA \begin{pmatrix} u(t-\tau) - u(t) \\ v(t-\tau) - v(t) \end{pmatrix}$$

bis zu 3 räumlich-homogene Fixpunkte

$E_0 = (1,0)$ immer stabil

$E_{1,2}$ nichttrivial

Aktivator-Kontrolle: $A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

Inhibitor-Kontrolle: $A = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$