Kontrolle rauschinduzierter Oszillationen im VdP-Modell

Analytische Näherung der spektralen Leistungsdichte $S(\omega)$

MEAN-FIELD GL (VdP) (SDDE)

$$\dot{x} = y(t)$$

$$\dot{y} = \tilde{\varepsilon} y - \omega_0^2 x + D\,\xi(t) + K\,[y(t-\tau) - y(t)]$$

FOURIER TRANSFORM $\int_{-\infty}^{\infty} d\omega\, e^{-i\omega t}$

$$-i\omega\,\hat{x}(\omega) = \hat{y}(\omega) \iff \hat{x} = \frac{i}{\omega}\hat{y} \qquad (1)$$

$$-i\omega\,\hat{y}(\omega) = \varepsilon\,\hat{y}(\omega) - \omega_0^2\,\hat{x}(\omega) + D\,\hat{\xi}(\omega) + K\,\hat{y}(\omega)\left(e^{i\omega\tau} - 1\right) \qquad (2)$$

(1) → (2) | iω :

$$\hat{y} = \frac{i\omega D\,\hat{\xi}(\omega)}{\omega^2 - \omega_0^2 - i\omega\varepsilon - i\omega K\left(e^{i\omega\tau} - 1\right)}$$

$$\langle \hat{y}(\omega)\,\hat{y}^*(\omega') \rangle = \frac{(i\omega D)(-i\omega' D)\,\langle \hat{\xi}(\omega)\,\hat{\xi}(\omega') \rangle}{\left[\omega^2 - \omega_0^2 - i\omega\varepsilon - i\omega K\left(e^{i\omega\tau} - 1\right)\right]\left[\omega'^2 - \omega_0^2 + i\omega'\varepsilon + i\omega' K\left(e^{-i\omega'\tau} - 1\right)\right]}$$

$$\langle \hat{\xi}(\omega)\, \hat{\xi}(\omega')\rangle = \left(\frac{1}{2\pi}\right)^2 \int_{-\infty}^{\infty} dt\, e^{i\omega t} \int_{-\infty}^{\infty} dt'\, e^{-i\omega' t'} \underbrace{\langle \xi(t)\, \xi(t')\rangle}_{\delta(t-t')}$$

$$= \frac{1}{2\pi} \underbrace{\frac{1}{2\pi}\int_{-\infty}^{\infty} dt\, e^{i(\omega-\omega')t}}_{\delta(\omega-\omega')}$$

$\omega = \omega'$

$$\longrightarrow \langle \hat{y}(\omega)\, \hat{y}^*(\omega')\rangle = \frac{D^2}{2\pi} \frac{\omega^2\, \delta(\omega-\omega')}{(\omega^2-\omega_0^2+\omega K \sin\omega\tau)^2 + \omega^2(\varepsilon - K(1-\cos\omega\tau))^2}$$

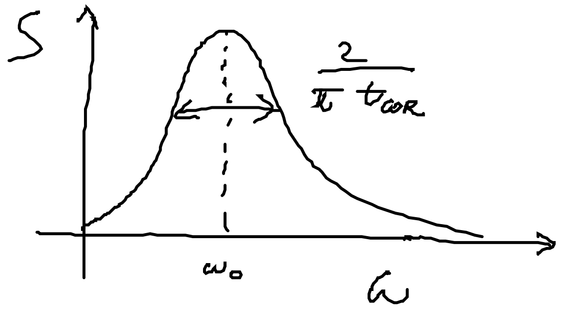
Wiener - Khinchine - Theorem $\qquad t' = t + s$

$$\langle \hat{y}(\omega)\, \hat{y}^*(\omega')\rangle = \left(\frac{1}{2\pi}\right)^2 \int_{-\infty}^{\infty} dt\, e^{i\omega t} \int_{-\infty}^{\infty} dt'\, e^{-i\omega' t'} \langle y(t)\, y(t')\rangle$$

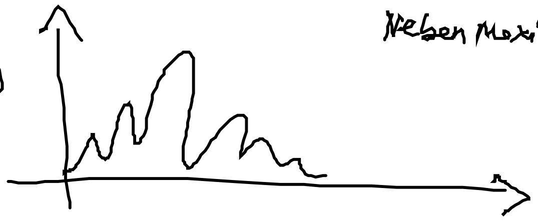
$$= \underbrace{\frac{1}{2\pi}\int_{-\infty}^{\infty} dt\, e^{i(\omega-\omega')t}}_{\delta(\omega-\omega')} \underbrace{\frac{1}{2\pi}\int_{-\infty}^{\infty} dt'\, e^{i\omega'\overbrace{(t-t')}^{-s}} \langle y(t)\, y(t')\rangle}_{S(\omega')}$$

$$\Rightarrow S_{yy}(\omega) = \frac{D^2}{2\pi} \frac{\omega^2}{(\omega^2-\omega_0^2+\omega K \sin\omega\tau)^2 + \omega^2(\varepsilon - K(1-\cos\omega\tau))^2}$$

$\boxed{K=0}$ $$S(\omega) = \frac{D^2}{2\pi}\,\frac{\omega^2}{(\omega^2-\omega_0^2)^2+\omega^2\varepsilon^2}$$



$\boxed{K \neq 0}$



Neben Maxima ~ τ

Generisches Modell für Anregbarkeit Typ I

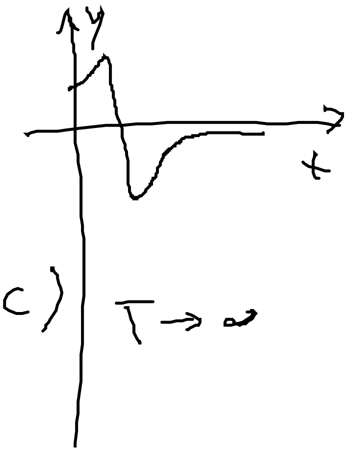
(SNIPER-Bif)

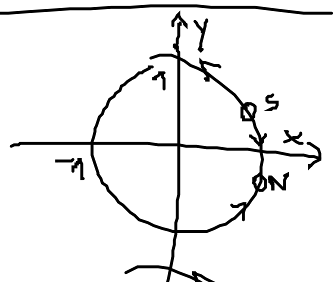
(AUS 10)

$$\boxed{\begin{aligned}\dot{x} &= x(1-x^2-y^2) + y(x-b) + D\,\xi(t) + K[x(t-\tau)-x(t)]\\ \dot{y} &= y(1-x^2-y^2) - x(x-b) + D\,\xi(t) + K[y(t-\tau)-y(t)]\end{aligned}}$$

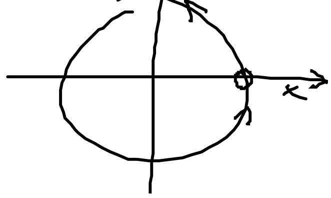
$\boxed{K=D=0}$

$b<1$ (anregbar) regime

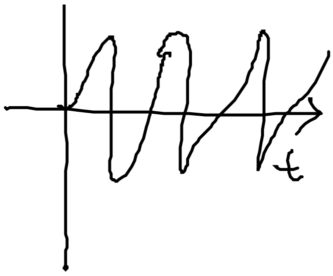


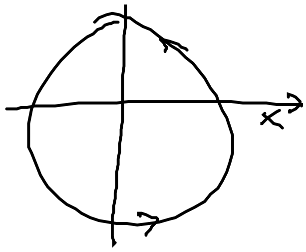


$b=1$ (SN Bif on LC) $T \to \infty$

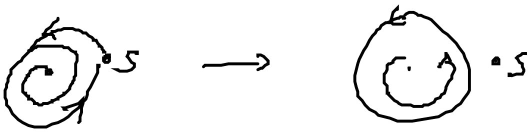


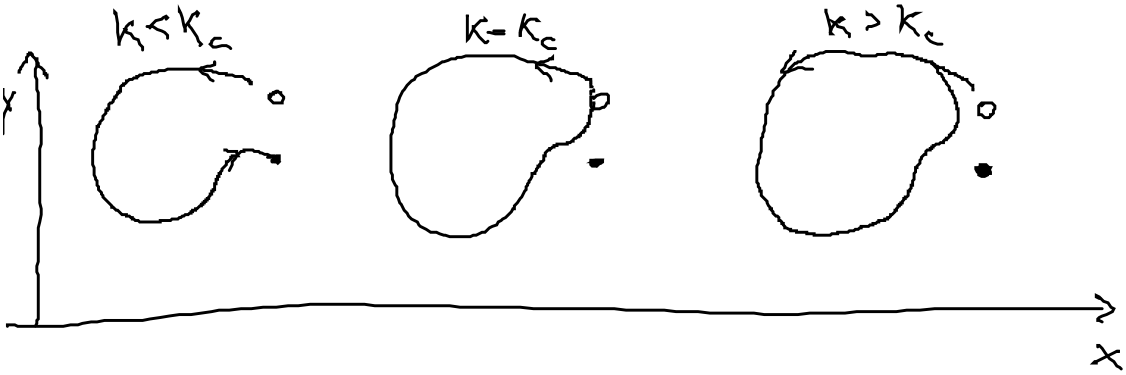
$b > 1$ (osc.)





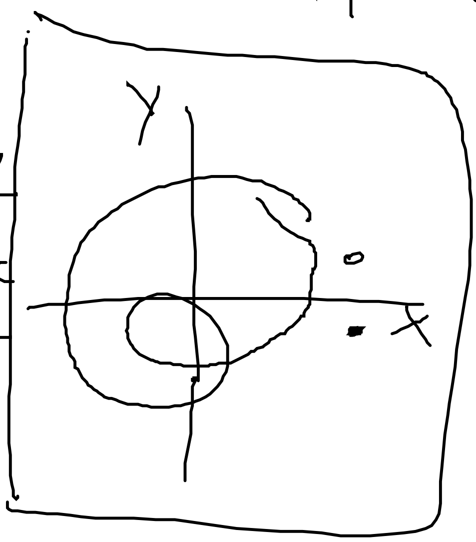
$\boxed{D=0}$ Delay-induzierten globalen (homocl.) Bif.
(Hiz 08)

LC 



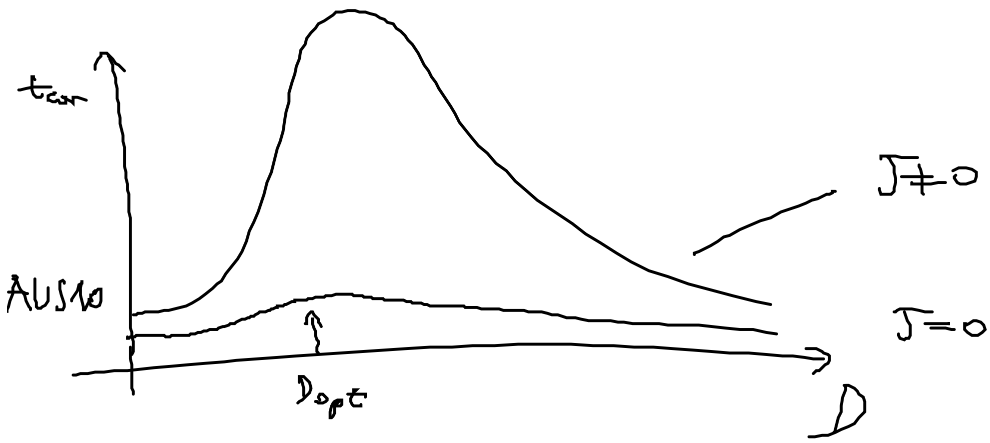
$T \sim \ln|k - k_c|$

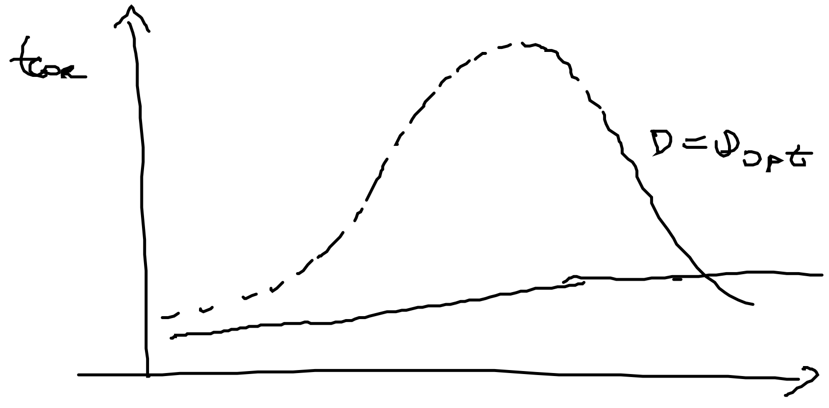
$D \neq 0$
$k \neq 0$ Bif. Period doubling



$\boxed{D \neq 0}$ Kontrolle rauschinduzierter + rausch beinflusster } Oszillationen

(delay-induzierte deterministische LC)





K