Lecture 26 summary

4.3 Sync in the presence of noise, Effective sync.

Questions:

1) What is the math. condition for phase locking and frequency locking in the determ. case?

2) What are the mechanisms of sync without noise?

3) What is the role of noise for sync?

4) What are the main difficulties in defining sync in the presence of noise?

5) What are phase slips?

6) Write the equation of van der Pol oscillator with noise and periodic force and corresponding reduced eqs. Can we consider the phase equation separately? When?

7) What is the dynamics of phase difference $\varphi(t)$ in the presence of noise? Provide a sketch.

8) How can the phase be defined for noisy periodic (and deterministic chaotic) systems?

9) What is the math. condition of phase sync in the presence of noise?

The conditions of sync can be defined in a statistical way by using the notion of effective synchronization. This can be done by imposing a certain restriction on some statistical measures of corresponding stochastic process. In particular, a definition of effective sync can be based on the following items:

* stationary probability density of the phase difference:
in this case the peak of $p(\varphi)$ should be well expressed in comparison with the uniform distribution.

* the mean frequency: this should match the driving frequency (up to some small statistical error),

* the effective diffusion (coefficient) constant:

$$D_{eff} = \frac{1}{2} \frac{d}{dt} \left[\langle \varphi^2(t) \rangle - \langle \varphi(t) \rangle^2 \right]$$

this measure should be small enough so that phase locking segments are much longer than the period of the external force. In other words, this restriction requires that the oscillator phase is locked during a considerable number of periods of the external force:

$$D_{eff} \leq \frac{\Omega}{n}, \quad n \gg 1$$

n – number of periods of the external force; Ω – frequency of the external force.