L 24 Summary

4.2 Sync of periodic self-sustained oscillations

Forced sync of van der Pol oscillator:

truncated equations for amplitude and phase.

Questions:

1) Write the equation of van der Pol oscillator with external periodic force and explain all the terms.

2) What are the assumptions used to derive truncated equations for amplitude and phase?

3) Write the truncated eq. for ampl. and phase; explain all the terms.

4) What is phase approximation?
What is the assumption for considering sync in phase approx.? Give eq. for the phase and explain all the terms.

5) Temporal phase dynamics $\varphi(t)$ for different values of detuning Δ: give the sketch and explain (ε and β are fixed):

- What is the meaning of the sign of Δ? Why do we have (observe) pairs of curves?
- What do the intervals of $\varphi(t) = \text{const}$ mean?
- What is the dynamics outside the sync region?

6) How is sync related to steady states?

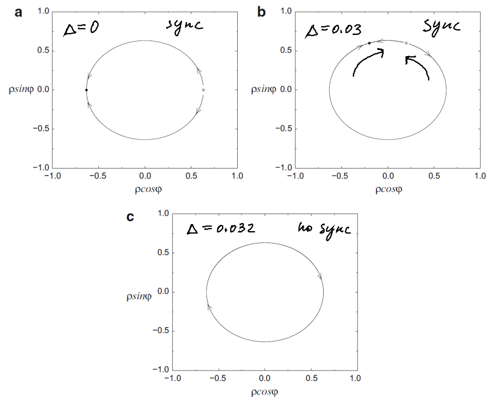
What is Arnold tongue? Provide a sketch for fixed ε.

7) Provide stability analysis for the steady states (fixed points) of the system:

$$\frac{d\varphi}{dt} = -\Delta + \frac{\beta}{2\sqrt{\varepsilon}} \sin\varphi \quad (4.9)$$

When are these states stable / unstable?

Fixed points of the system (4.9) are considered in a one-dimensional phase space on the circle with radius $\rho = 2\sqrt{\varepsilon}$, which is constructed in the plane with coordinates $\rho \sin\varphi$, $\rho \cos\varphi$ (see figure).



Phase portraits of system (4.9) for $\varepsilon = 0.1$, $\beta = 0.01$ and for different values of the detuning Δ:

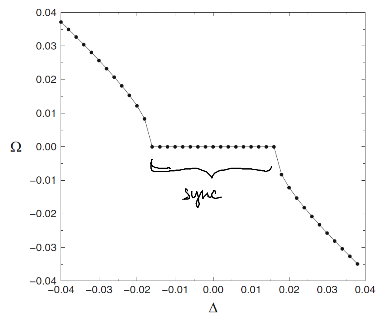
a: $\Delta = 0$ (sync)

b: $\Delta = 0.03$ (sync)

c: $\Delta = 0.032$ (no sync)

Stable and unstable fixed points are marked by • and x.

As can be seen from (4.11) and (4.12), for $\Delta = 0$, the coordinate of the unstable fixed point is $\varphi_1 = 0$, while that of the stable fixed point is $\varphi_2 = \pi$ (see panel a). As the detuning increases, when $2\Delta\sqrt{\varepsilon}/\beta$ tends to unity, the stable and unstable fixed points move towards each other along the circle: the unstable point rotates counter-clockwise and the stable one clockwise. For $\varphi = \pi/2$, they collide and disappear when $2\Delta\sqrt{\varepsilon}/\beta$ exceeds unity. (see panel c). The fixed points disappear when we exit sync region. The representative point rotates along the circle with average velocity $\langle \dot{\varphi}(t) \rangle$, which defined beat frequency.



Beat frequency $\Omega = \langle \dot{\varphi}(t) \rangle$ as a function of detuning Δ of system (4.9) for $\varepsilon = 0.1$ and $\beta = 0.01$.

The frequency $\Omega(t)$ is the difference between the mean frequency of self-sustained oscillations and the frequency of the external force. The interval of Δ values $\Omega = 0$ corresponds to sync region. Outside this region there are beats with frequency Ω.