

lecture 11 summary

(i) Wiener process (no drift)

$$\dot{x} = \sqrt{2D} \xi(t)$$

$$\frac{\partial}{\partial t} P = D \frac{\partial^2}{\partial x^2} P$$

- has independent increments
- nonstationary (probability density explicitly depends on time)
- variance grows linearly in time $\langle (\Delta X(t))^2 \rangle = 2D(t-t_0)$
- mean value is defined by I.C.
 $\langle x(t) \rangle = x_0$
- Gaussian process, Markov process

(ii) Ornstein-Uhlenbeck process

$$\dot{x} = \underbrace{-kx}_{\text{linear drift}} + \sqrt{2D} \xi(t)$$

$$\frac{\partial}{\partial t} P = \frac{\partial}{\partial x} (kxP) + D \frac{\partial^2}{\partial x^2} P$$

- stationary
- variance is bounded $\langle \Delta X^2 \rangle = \frac{D}{k}$
- Gaussian process, Markov process
- ACF depends only on time differences
 $\langle X(t+\tau)X(t) \rangle = \frac{D}{k} e^{-k|\tau|} = D \tau_c e^{-|\tau|/\tau_c}$
- $\tau_c = \frac{1}{k}$ correlation time

3. Noise-induced oscillations and patterns,

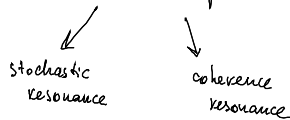
noise \rightarrow undesired, destructive, makes deterministic dynamics irregular

Noisy sources in nonlinear dynamical systems \rightarrow induce new regime

\rightarrow more ordered behaviour

\rightarrow formation of more regular temporal dynamics (increase of degree of coherence)

Constructive role of noise



3.1 Stochastic resonance (SR)

Enhancement of the weak periodic signal with the help of noise.

The term SR was introduced in 1981-1982

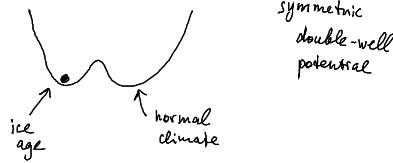
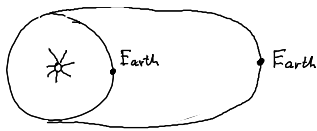
[Benzi et al. J. Phys. A 14 (1981)

Benzi et al. Tellus 34 (1982), p. 10

Nicolis, Tellus 34 (1982), p. 1-9]

Example 1. Periodicity of the ice age on the Earth

a model to explain the almost periodic recurrences (repetitions) of the Earth's ice ages.



A particle in a double-well potential driven by a periodic force.
 Periodic force \rightarrow tiny oscillations of the eccentricity of Earth's orbit:
 period $\sim 10^5$ years; the orbit changes from the nearly circular to more elliptical.

The switching was achieved by adding random force into the model
 \downarrow
 fluctuations of the atmosphere

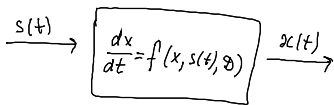
The fundamental result: temporally ordered transitions;
 the climate almost followed the vanishing small external periodic perturbation
 for some finite noise strength of the atmosphere.

Stochastic resonance

- nonlinear system
- periodic force
- noise

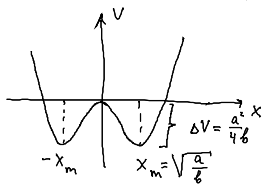
SR is a mechanism by which a nonlinear system in a noisy environment gets an enhanced sensitivity towards small external periodic force for an intern./optimal noise intensity.

General scheme of SR



The response of the nonlinear system to weak input signal $s(t)$ is significantly increased for some optimal non-zero noise.

Example 2. Overdamped Brownian particle in bistable potential
 $V(x) = -\frac{a}{2}x^2 + \frac{b}{4}x^4$; periodic force $A_0 \cos(\Omega t)$;
 white noise $g(t)$



Langevin equation (SDE)

$$\dot{x} = -V'(x) + A_0 \cos(\Omega t) + g(t)$$

$$\langle g(t) \rangle = 0$$

$$\langle g(t) g(t') \rangle = 2D \delta(t-t')$$

Overdamped

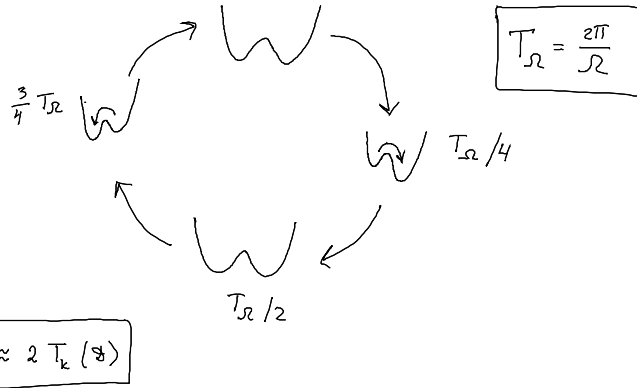
The motion of a particle in a potential field $U(y)$ in the presence of damping is given by
 $\ddot{y} + \underbrace{\gamma \dot{y}}_{\text{damping}} = -U'(y)$
 overdamped $\rightarrow \gamma \gg 1 \Rightarrow$ we neglect second derivative

$$\alpha = \beta = 1 \Rightarrow \dot{x} = -x - x^3 + A_0 \cos(\Omega t) + \xi(x)$$

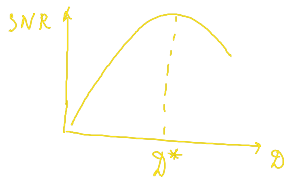
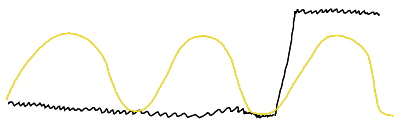
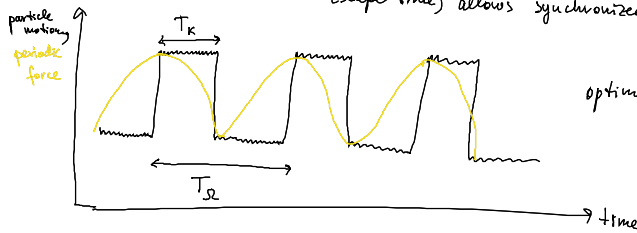
Noise-induced switchings between $-x_m$ and x_m without periodic force are described by

Kramers rate:
$$r_k = \frac{1}{T_k(\beta)} = \frac{1}{2\pi} \exp\left(-\frac{\Delta V}{\beta}\right) \quad (\alpha = \beta = 1)$$

Periodic driving: the double-well potential $V(x,t) = V(x) - A_0 x \cos(\Omega t)$ is tilted back and forth \rightarrow raising and lowering the potential barriers of the right and left well.

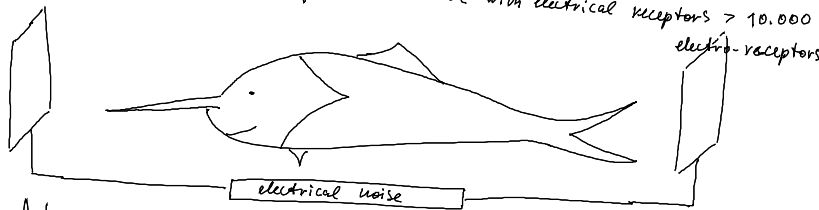


A suitable dose of noise (when the period of driving approx. equals twice the noise-induced escape time) allows synchronized hopping.



Examples: Brownian particles, lasers, tunnel diodes, quantum systems, chemical systems, sociological and biological systems (music as noise for mental activity).

Example 8. Paddle fish ; in North America the rivers are not clear →
 → the fish has developed an antenna with electrical receptors > 10.000
 electrical-receptors



Antenna is used instead of eyes to catch the food (plankton).
 Plankton emits signal of low frequency.

[Russel, Wilkens and Moss,
 Nature 1999]

In the experiment external random
 electrical field was applied and the amount
 of plankton was measured.

low noise:
 a certain amount
 of food

intermediate
 noise:
 the largest
 amount of food

large noise:
 can not find
 plankton