Lecture 22: summary

4. Sync in the presence of noise

4.1 What is sync? Classical example of a pendulum clock.

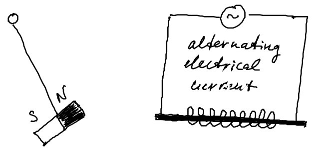
1) What is a self-sustained oscillator? Give examples.
2) What are the properties of self-sustained oscillators?
3) What are the factors which define whether the systems can sync or not? Use the example of a pendulum clock.
4) Which types of sync do you know?

Sync is an adjustment of rhythms of oscillating objects due to their weak interaction

L23 { 5) What are the counter-examples? What is not sync?
6) To call a phenomenon sync we must be sure certain conditions are fulfilled. What are these conditions?

Counter-examples

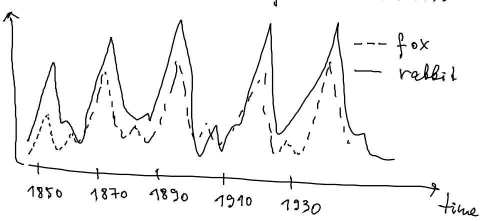
1) Magnetic pendulum



it oscillates in the electromagnetic field with the frequency of the electric current; this is an example of a forced system that has no rhythm on its own: if the pendulum is isolated then there is no oscillations.

Resonance is not sync!

2) Population dynamics: foxes and rabbits



This ecological system can not be decomposed into two oscillators. If foxes and rabbits were separated, there would be no oscillations at all. We can not consider them as two subsystems having their own rhythms ⟹ we can not speak about sync!

3) Too strong coupling is not sync!

To call a phenomenon sync, we must be sure that:

* self-sustained oscillators – they have their own rhythm;
  – the form, period and freq. do not depend on I.C., but on the internal parameters of the system

* weak interaction

* certain range of mismatch; in particular if the freq. of one oscillator is slowly varied, the second system follows this variation

4.2 Synchronization of periodic self-sustained oscillations

Forced sync of van der Pol oscillator: truncated equations for amplitude and phase

$$\ddot{x} - (\varepsilon - x^2)\dot{x} + x = b \sin(\omega t) \quad (4.1)$$

ε – control parameter

b and ω – amplitude and frequency of the external force

We rewrite (4.1):

$$\ddot{x} + x = (\varepsilon - x^2)\dot{x} + b \sin(\omega t)$$

$$\ddot{x} + \omega^2 x = (\omega^2 - 1)x + (\varepsilon - x^2)\dot{x} + b \sin(\omega t) \quad (4.2)$$

Assumptions: – freq. of the external force ω is close to the natural frequency of autonomous oscillator $(\omega_0 = 1)$: $\omega \sim 1$