Nonequilibrium Statistical Physics

Lectures Anna Zakharova Winter term 2019/20

DE Statistische Physik im Nichtgleichgewicht

Lectures Mo $12^{15} - 13^{45}$ EW 203
We $10^{15} - 11^{45}$ EW 203

Tutorials We $16^{15} - 17^{45}$ EW 114 [first tutorial on 23.10]

10 ECTS points → lecture (4 SWS) + tutorial (2 SWS)

12 ECTS points → lecture + tutorial
+
seminar (2 SWS) or sp. lecture (2 SWS)

Seminar "Complex Networks and their applications"
Tu 16^{15} EW 731 (tomorrow is the intro)

Lecture course

- Classical statistics in non-equilibrium
- Noise-induced oscillations and patterns
- Constructive role of noise: stochastic resonance, coherence resonance
- Stochastic effects in networks
- Synchronization in the presence of noise

2019 – tutorials

2020 – Jan + Feb → work on projects

Project presentations Mo 10.02 $12^{15} - 13^{45}$
We 12.02 $10^{15} - 11^{45}$

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1. Stochastic processes (random fluctuations, noise)

Elementary statistics and probability theory (basics)

Experiment, sample set, event

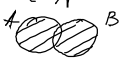
Experiment: any process of observation or procedure that
a) can be repeated (theoretically) an infinite number of times
b) has a well-defined set of possible (events) outcomes

Sample set: set of all possible outcomes

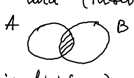
Event: subset of the sample space of an experiment
(microstate, the result of the measurement of an observable)

Mathematical description is given by Boolean algebra $\mathcal{A}$

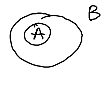
$\cup$ "or" (union) $A \cup B$: the elements that are in either set, or both



$\cap$ "and" (intersection) $A \cap B$: the elements that are in both sets

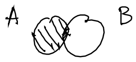


$\subseteq$ (inclusion) $A \subseteq B$



Axioms for A, B, C (sets or events)

- $A \cap B = B \cap A$, $A \cup B = B \cup A$ commutative law
- $A \cap (B \cap C) = (A \cap B) \cap C$ associative law
  $A \cup (B \cup C) = (A \cup B) \cup C$
- $A \cap (A \cup B) = A$, $A \cup (A \cap B) = A$



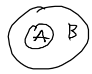
- $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
  $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ distributive law

$\exists S$ (certain event) : $A \cap S = A$

$\exists \emptyset$ (empty set) : $A \cup \emptyset = A$

$\forall A \ \exists B$: $A \cap B = \emptyset$; $A \cup B = S$: Complement $(B = \bar{A})$ "not A"
(complementary event / set)

$A \subseteq B$, if $A \cap B = A$



A and B are disjoint (or mutually exclusive)
if $A \cap B = \emptyset$ (the intersection of A and B is an empty set).

Sample set $\{A_1, A_2, \dots, A_n\}$, $A_i \cap A_j = A_i \delta_{ij}$ (pairwise disjoint)

$\bigcup_{i=1}^{n} A_i = S$ (the union of all events is a certain event)

Example : $\{1, 2, 3, 4, 5, 6\}$ a dice that has 6 sides

Probability (Kolmogorov)

Axioms for probability

A – event, S – certain event

$P(A)$ – probability of A , $P \ [0, 1]$

- $P(A) \geq 0, \ \forall A$
- $P(S) = 1$

- if $A \cap B = \emptyset \Rightarrow P(A \cup B) = P(A) + P(B)$
  disjoint (mutually exclusive)

These axioms lead to

- $P(A) + P(\bar{A}) = P(A \cup \bar{A}) = P(S) = 1$ ("not A")

  $\Rightarrow P(A) \leq 1$

- $P(\emptyset) = 0$, since $\underbrace{P(S)}_{1} = P(\emptyset \cup S) = P(\emptyset) + \underbrace{P(S)}_{1}$

  $\emptyset$ and S are mutually exclusive

Intuitive understanding of probability

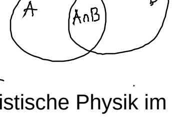
relative frequency $\lim_{N \to \infty} \frac{N_A}{N}$

Conditional probability

$P(A/B) = \frac{P(A \cap B)}{P(B)}$ probability that A occurs given that B has occurred

"prob-ty of A given B"

$P(A \cap B)$ – joint probability



· Two events A_1 and A_2 are called independent (uncorrelated)

if $P(A_2 / A_1) = P(A_2) \Leftrightarrow$

$\Leftrightarrow \boxed{P(A_1 \cap A_2) = P(A_1) \cdot P(A_2)}$

and $P(A_1 / A_2) = P(A_1)$

Random variable $x : \tilde{M} \to M$

($\tilde{M}$ ↓ event, M ↓ realization)

(i) Set M of mutually exclusive events x_i (sample set)

(ii) probability distribution $P(x_i)$

Normalization $\sum_i P(x_i) = 1$ [since $\sum_i P(x_i) = P(\bigcup_i x_i) = P(S) = 1$]