

## Nonequilibrium Statistical Physics

lectures Anna Zakharova Winter term 2019/20

DE Statistische Physik im Nichtgleichgewicht

lectures Mo 12<sup>15</sup> - 13<sup>45</sup> EW 203

We 10<sup>15</sup> - 11<sup>45</sup> EW 203

Tutorials We 16<sup>15</sup> - 17<sup>45</sup> EW 114 [first tutorial on 23.10]

10 ECTS points  $\rightarrow$   $\overset{4\text{ SWS}}{\text{lecture}} + \overset{2\text{ SWS}}{\text{tutorial}}$

12 ECTS points  $\rightarrow$   $\overset{2\text{ SWS}}{\text{lecture}} + \overset{2\text{ SWS}}{\text{tutorial}}$   
 $+ \overset{2\text{ SWS}}{\text{seminar}}$  or  $\overset{2\text{ SWS}}{\text{sp. lecture}}$

Seminar "Complex Networks and their applications"

Tu 16<sup>15</sup> EW 731 (tomorrow is the intro)

### lecture course

- Classical statistics in non-equilibrium
- Noise-induced oscillations and patterns
- Constructive role of noise: stochastic resonance  
coherence resonance
- Stochastic effects in networks
- Synchronization in the presence of noise

2019 - tutorials

2020 - Jan + Feb  $\rightarrow$  work on projects

Project presentations Mo 10.02 12<sup>15</sup> - 13<sup>45</sup>

We 12.02 10<sup>15</sup> - 11<sup>45</sup>

## 1. Stochastic processes [random fluctuations, noise]

Elementary statistics and probability theory (basics)

Experiment, sample set, event

Experiment: any process of observation or procedure that

- can be repeated (theoretically) an infinite number of times
- has a well-defined set of possible (events) outcomes

Sample set: set of all possible outcomes

Event: subset of the sample space of an experiment

(microstate, the result of the measurement of an observable)

Mathematical description is given by Boolean algebra  $\mathcal{A}$

$\cup$  "or" (union)  $A \cup B$ : the elements that are in either set, or both

$\cap$  "and" (intersection)  $A \cap B$ : the elements that are in both sets

$\subseteq$  (inclusion)  $A \subseteq B$

Axioms for  $A, B, C$  (sets or events)

- $A \cap B = B \cap A$ ,  $A \cup B = B \cup A$  commutative law
- $A \cap (B \cap C) = (A \cap B) \cap C$  associative law  
 $A \cup (B \cup C) = (A \cup B) \cup C$
- $A \cap (A \cup B) = A$ ,  $A \cup (A \cap B) = A$




- $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$   
 $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$  distributive law

$\exists S$  (certain event):  $A \cap S = A$

$\exists \emptyset$  (empty set):  $A \cup \emptyset = A$

$\forall A \exists B$ :  $A \cap B = \emptyset$ ;  $A \cup B = S$ : Complement ( $B = \bar{A}$ )  
 (complementary event/set) "not A"

$A \subseteq B$ , if  $A \cap B = A$  

A and B are disjoint (or mutually exclusive)  
 if  $A \cap B = \emptyset$  (the intersection of A and B is an empty set).

Sample set  $\{A_1, A_2, \dots, A_n\}$ ,  $A_i \cap A_j = \emptyset$  if  $i \neq j$  (pairwise disjoint)  
 $\bigcup_{i=1}^n A_i = S$  (the union of all events is a certain event)

Example:  $\{1, 2, 3, 4, 5, 6\}$  a dice that has 6 sides

### Probability (Kolmogorov)

#### Axioms for probability

A - event, S - certain event

$P(A)$  - probability of A,  $P \in [0, 1]$

•  $P(A) \geq 0$ ,  $\forall A$

•  $P(S) = 1$

• if  $A \cap B = \emptyset \Rightarrow P(A \cup B) = P(A) + P(B)$   
 disjoint (mutually exclusive)

These axioms lead to

•  $P(A) + P(\bar{A}) = P(A \cup \bar{A}) = P(S) = 1$   
 "not A"

$\Rightarrow P(A) \leq 1$

•  $P(\emptyset) = 0$ , since  $\underbrace{P(S)}_1 = P(\emptyset \cup S) = P(\emptyset) + \underbrace{P(S)}_1$   
 $\emptyset$  and S are mutually exclusive

#### Intuitive understanding of probability

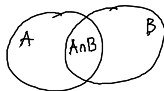
relative frequency  $\lim_{N \rightarrow \infty} \frac{N_A}{N}$

#### Conditional probability

$P(A/B) = \frac{P(A \cap B)}{P(B)}$   
 "prob. of A given B"

probability that A occurs given that B has occurred

$P(A \cap B)$  - joint probability



Two events  $A_1$  and  $A_2$  are called independent (uncorrelated)

$$\text{if } P(A_2 | A_1) = P(A_2) \Leftrightarrow$$

$$\Leftrightarrow \boxed{P(A_1 \cap A_2) = P(A_1) \cdot P(A_2)}$$

$$\text{and } P(A_1 | A_2) = P(A_1)$$

Random variable  $x: \tilde{M} \rightarrow M$   
 $\downarrow$  event       $\downarrow$  realization

(i) set  $M$  of mutually exclusive events  $x_i$  (sample set)

(ii) probability distribution  $P(x_i)$

Normalization  $\sum_i P(x_i) = 1$  [since  $\sum_i P(x_i) = P(\bigcup_i x_i) = P(S) = 1$ ]