Wdh

- benchmark mit exakt lösbaren Modellen

i.) spin-boson: pure dephasing

$$H = \underbrace{\frac{\omega}{2}\sigma^z}_{H_S} + \underbrace{\sigma^z}_{A}\otimes\underbrace{\sum_k\left(h_k b_k + h_k^* b_k^\dagger\right)}_{B} + \underbrace{\sum_k \omega_k b_k^\dagger b_k}_{H_B}$$

=> BMS-Mastergleichung
$$\frac{d}{dt}\begin{pmatrix}\rho_{00}\\ \rho_{11}\\ \rho_{01}\\ \rho_{10}\end{pmatrix} = \begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&\eta&0\\0&0&0&\eta^*\end{pmatrix}\begin{pmatrix}\rho_{00}\\ \rho_{11}\\ \rho_{01}\\ \rho_{10}\end{pmatrix}$$

$Re(\eta) \leq 0$ "Dekohärenz"

$$\rho_{01}(t) = e^{\eta\cdot t}\rho_{01}(0)$$

ii.) SRL $f(\varepsilon) = \frac{1}{e^{\beta(\varepsilon-\mu)}+1}$ — (ε)

$$H = \varepsilon d^\dagger d + \left(d^\dagger\sum_k t_k c_k + h.c.\right) + \sum_k \varepsilon_k c_k^\dagger c_k$$
quadratisch in d, c_k

⇝ exakt lösbar über Heisenberg-BWGL

BMS-MG $f = f(\varepsilon)$

dot leer

$$\frac{d}{dt}\begin{pmatrix}\rho_{00}\\ \rho_{11}\end{pmatrix} = \underbrace{\Gamma(\varepsilon)\begin{pmatrix}-f & +(1-f)\\ +f & -(1-f)\end{pmatrix}}_{\mathcal{L}_{pop}}\underbrace{\begin{pmatrix}\rho_{00}\\ \rho_{11}\end{pmatrix}}_{\vec\rho} \quad \Rightarrow \vec\rho(t) = e^{\mathcal{L}_{pop}t}\vec\rho(0)$$

besetzt

BMS wird gut für schwache Kopplung $\Gamma(\varepsilon)\to 0$

DCG wird gut für $t\to 0$, ist aber immer besser als BMS

1.4. Super-Operator Notation

BMS-MG: $$\frac{d}{dt}\begin{pmatrix}\rho_{pop}\\ \rho_{coh}\end{pmatrix} = \underbrace{\begin{pmatrix}\mathcal{L}_{pop} & 0\\ 0 & \mathcal{L}_{coh}\end{pmatrix}}_{\mathcal{L}}\begin{pmatrix}\rho_{pop}\\ \rho_{coh}\end{pmatrix} \rightsquigarrow \rho(t) = e^{\mathcal{L}\cdot t}\rho(0)$$

Problem: Matrix von $\mathcal{L}$ per Hand zu konstruieren ist aufwändig

=> generelle Vektorisierung:

$$\rho = \sum_{ij}\rho_{ij}|i\rangle\langle j| \quad \Longleftrightarrow \quad vec(\rho) = \sum_{ij}\rho_{ij}\,|i\rangle\otimes|j\rangle$$

für eine bestimmte Ordnung der Basisvektoren kann das Tensorprodukt durch das Kronecker-Produkt dargestellt werden

$$A\otimes B = \begin{pmatrix}A_{11}B & \dots & A_{1N_A}\cdot B\\ \vdots & & \\ A_{M_A 1}B & \dots & A_{M_A N_A}\cdot B\end{pmatrix}$$

Zahlen Matrix: $M_B\times N_B$

Vektoren: $N_A = 1$, $N_B = 1$

$$z.B.: \rho = \begin{pmatrix}\rho_{00} & \rho_{01}\\ \rho_{10} & \rho_{11}\end{pmatrix} \quad\Longleftrightarrow\quad vec(\rho) = \begin{pmatrix}\rho_{00}\\ \rho_{01}\\ \rho_{10}\\ \rho_{11}\end{pmatrix}$$

- $$vec(A\rho) = \sum_{ijk}\underbrace{A_{ik}\cdot\rho_{kj}}_{(A\rho)_{ij}}|i\rangle\otimes|j\rangle$$

$$\Rightarrow A\otimes \mathbb{1}\; \mathrm{vec}(\rho) = \sum_{ij} \rho_{ij} \Big(\sum_{kl} A_{kl}\, |k\rangle\underbrace{\langle l|i\rangle}_{\delta_{il}}\Big)\otimes|j\rangle$$

$$= \sum_{ijk} \rho_{ij} A_{ki}\, |k\rangle\otimes|j\rangle \overset{k\leftrightarrow i}{=} \sum_{ijk} \rho_{kj} A_{ik}\, |i\rangle\otimes|j\rangle \quad \Big] =$$

- $$\mathrm{vec}(\rho B) = \sum_{ijk} \rho_{ik} B_{kj}\, |i\rangle\otimes|j\rangle$$

$$\mathbb{1}\otimes B^T\, \mathrm{vec}(\rho) = \sum_{ij} \rho_{ij}\, |i\rangle\otimes\Big(\sum_{kl} B_{lk}\, |k\rangle\langle l|j\rangle\Big) \quad \Big\} =$$

transponiert

$$\Rightarrow \boxed{\mathrm{vec}(A\rho B) = A\otimes B^T\cdot \mathrm{vec}(\rho)}$$

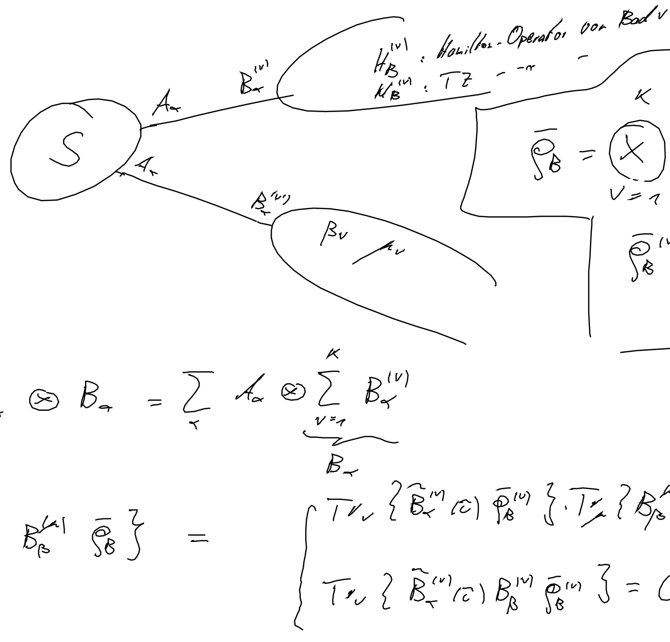
$$\dot\rho = -i[H_S,\rho] + \sum_\alpha \Big[L_\alpha \rho L_\alpha^\dagger - \frac{1}{2}\{L_\alpha^\dagger L_\alpha, \rho\}\Big] \Leftrightarrow \frac{d}{dt}\mathrm{vec}(\rho) = \mathcal{L}\, \mathrm{vec}(\rho)$$

$$\mathcal{L} = -i H\otimes\mathbb{1} + i\,\mathbb{1}\otimes H^T + \sum_\alpha \Big[L_\alpha\otimes(L_\alpha^\dagger)^T - \frac{1}{2} L_\alpha^\dagger L_\alpha\otimes\mathbb{1} - \frac{1}{2}\mathbb{1}\otimes(L_\alpha^\dagger L_\alpha)^T\Big]$$

- Spur $\hat{=}$ Summe der Diagonal-Elemente

$$\mathrm{Tr}\{\rho\} = (\mathrm{vec}(\mathbb{1}))^T\cdot \mathrm{vec}(\rho) \quad \rightsquigarrow \text{Bsp: } (\mathrm{vec}(\mathbb{1}))^T = (1,0,0,1)$$

2. Stationärer Quantentransport



$$\bar\rho_B = \bigotimes_{\nu=1}^{K} \bar\rho_B^{(\nu)}$$

$$\bar\rho_B^{(\nu)} = \frac{e^{-\beta_\nu(H_B^{(\nu)} - \mu_\nu N_B^{(\nu)})}}{Z_B^{(\nu)}}$$

$$H_I = \sum_\alpha A_\alpha\otimes B_\alpha = \sum_\alpha A_\alpha\otimes \underbrace{\sum_{\nu=1}^{K} B_\alpha^{(\nu)}}_{B_\alpha}$$

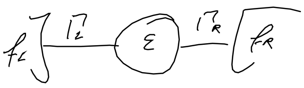
$$\mathrm{Tr}\{\tilde B_\alpha^{(\nu)}(\tau)\, B_\beta^{(\mu)}\, \bar\rho_B\} = \begin{cases} \mathrm{Tr}_\nu\{\tilde B_\alpha^{(\nu)}(\tau)\, \bar\rho_B^{(\nu)}\}\cdot \mathrm{Tr}_\mu\{B_\beta^{(\mu)}\, \bar\rho_B^{(\mu)}\} = 0 & : \text{falls } \mu\neq\nu \\ \mathrm{Tr}_\nu\{\tilde B_\alpha^{(\nu)}(\tau)\, B_\beta^{(\nu)}\, \bar\rho_B^{(\nu)}\} = C_{\alpha\beta}^{(\nu)}(\tau) & : \text{falls } \nu=\mu \end{cases}$$

$\leadsto$ keine Korrelationen zwischen verschiedenen Reservoiren (schwache Kopplung)

$$C_{\alpha\beta}(\tau) = \sum_{\nu=1}^{K} C^{(\nu)}_{\alpha\beta}(\tau)$$

$$\leadsto \boxed{\mathcal{L} = \mathcal{L}^{(0)} + \sum_\nu \mathcal{L}^{(\nu)} \qquad \mathcal{L}^{(0)}\rho \stackrel{!}{=} -i[H_S, \rho]}$$

2.1. Bsp: SET



$$H = \varepsilon d^\dagger d + \sum_{k,\nu} \left(d^\dagger t_{k\nu} c_{k\nu} + h.c. \right) + \sum_{k\nu} \varepsilon_{k\nu} a_{k\nu}^\dagger a_{k\nu}$$

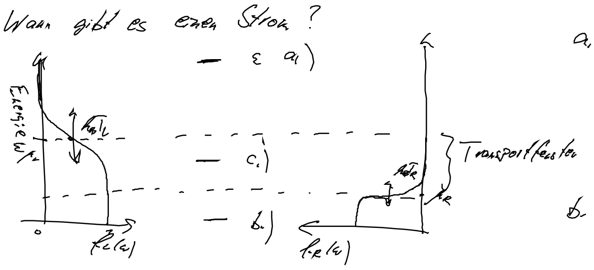
$$\nu \in \{L, R\}$$

$$\mathcal{L}_{BAS} = \underbrace{\begin{pmatrix} -\Gamma_L \cdot f_L & +\Gamma_L(1-f_L) \\ +\Gamma_L \cdot f_L & -\Gamma_L(1-f_L) \end{pmatrix}}_{\mathcal{L}^{(L)}} + \underbrace{\begin{pmatrix} -\Gamma_R \cdot f_R & +\Gamma_R(1-f_R) \\ +\Gamma_R \cdot f_R & -\Gamma_R(1-f_R) \end{pmatrix}}_{\mathcal{L}^{(R)}}$$

$$\Gamma_\nu = \Gamma_\nu(\varepsilon) \qquad \Gamma_\nu(\omega) = 2\pi \sum_k |t_{k\nu}|^2 \delta(\omega - \varepsilon_{k\nu})$$

$$f_\nu(\omega) = \frac{1}{e^{\beta_\nu(\omega-\mu_\nu)}+1}$$

Wann gibt es einen Strom?



a.) $\varepsilon \gg \mu_L, \mu_R$ (tiefe T)

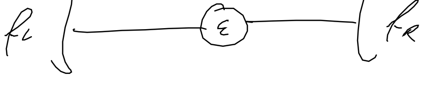
$\leadsto f_L(\varepsilon) = f_R(\varepsilon) = 0$

$\leadsto$ dot entlädt $\leadsto$ kein stat. Strom

b.) $\varepsilon \ll \mu_L, \mu_R$

$\leadsto f_\nu(\varepsilon) = 1$

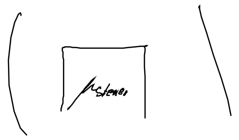
$\leadsto$ dot ist beladen $\leadsto$ kein stat. Strom



c.) $\mu_L > \varepsilon > \mu_R$

$\leadsto f_L \to 1 \quad f_R \to 0$

$\leadsto$ dot wird von links geladen & entlädt nach rechts



$\varepsilon = \varepsilon(\mu_{Steuer})$

$\leadsto$ Transistor

$\Gamma_\nu \geq 0 \qquad 0 \leq f_\nu \leq 1$

$\leadsto$ stat. Lösung $\quad \bar{\rho} = \begin{pmatrix} 1-\bar{f} \\ \bar{f} \end{pmatrix} \qquad \bar{f} = \dfrac{\Gamma_L \cdot f_L + \Gamma_R \cdot f_R}{\Gamma_L + \Gamma_R}$

effektiver GG-Zustand
(weil das System so einfach ist)

geht nur falls

therm. Parameter

$$\mathcal{L}^{(\nu)} = \Gamma^{(\nu)} \left[\mathcal{L}_A + \lambda^{(\nu)} \cdot \mathcal{L}_B \right] \quad \forall \nu$$

Kopplung

Superoperatoren $\rightsquigarrow \bar{\rho}^{(\nu)} = \bar{\rho}^{(\nu)}(\mathcal{L}_A, \mathcal{L}_B, \lambda^{(\nu)})$

$$\mathcal{L}\bar{\rho} = 0 \iff 0 = \left[\sum_\nu \Gamma^{(\nu)}\right]\left[\mathcal{L}_A + \underbrace{\frac{\sum_\nu \Gamma^{(\nu)} \cdot \lambda^{(\nu)}}{\sum_\nu \Gamma^{(\nu)}}}_{\bar{\lambda}} \mathcal{L}_B\right]\bar{\rho} = 0$$

Spezialfall!

2.2. Phänomenologische Def. der Ströme

$$\mathcal{L} = \mathcal{L}^{(0)} + \sum_\nu \mathcal{L}^{(\nu)}$$

Energiebilanz

$$\frac{d}{dt}\langle E\rangle = \mathrm{Tr}\{H_S \dot{\rho}\} = \mathrm{Tr}\{H_S[-iH_S, \rho]\} + \sum_\nu \mathrm{Tr}\{H_S(\mathcal{L}^{(\nu)}\rho)\}$$

$$= -i\,\mathrm{Tr}\{H_S^2\rho - H_S\rho H_S\} + \sum_\nu I_E^{(\nu)} \qquad \left(\sum_\nu \mathcal{L}^{(\nu)}\right)\bar{\rho} = 0$$

$$\rightsquigarrow \boxed{I_E^{(\nu)} = \mathrm{Tr}\{H_S(\mathcal{L}^{(\nu)}\rho)\}} \xrightarrow{t\to\infty} \mathrm{Tr}\{H_S(\mathcal{L}^{(\nu)}\bar{\rho})\}$$

Energiestrom aus Reservoir ν

Teilchenbilanz: $[N_S, H_S] = 0$

im Langzeitlimes:

$$\bar{I}_{M/E}^{(L)} + \bar{I}_{M/E}^{(R)} = 0$$

$$\boxed{I_M^{(\nu)} = \mathrm{Tr}\{N_S(\mathcal{L}^{(\nu)}\rho)\}}$$

Teilchenstrom aus Reservoir ν

Bsp: SET

$$I_E^{(L)}(t) = (1, 1)\begin{pmatrix} 0 & 0 \\ 0 & \varepsilon \end{pmatrix}\begin{pmatrix} -\Gamma_L \cdot f_L & +\Gamma_L(1-f_L) \\ +\Gamma_L \cdot f_L & -\Gamma_L(1-f_L) \end{pmatrix}\begin{pmatrix} \rho_{00}(t) \\ \rho_{11}(t) \end{pmatrix}$$

$\rightsquigarrow t\to\infty$: $\rho_{00} \to 1-\bar{f} \qquad \rho_{11} \to \bar{f}$

$$I_E^{(L)} = \varepsilon \cdot \frac{\Gamma_L \cdot \Gamma_R}{\Gamma_L + \Gamma_R}\left(f_L - f_R\right) = \varepsilon \cdot I_M^{(L)}$$

$\rightsquigarrow$ betrachte Ratengl.: $\quad \dot{\rho}_{aa} = \sum_\nu \sum_b \gamma_{ab,ab}^{(\nu)} \rho_{bb} - \sum_\nu \sum_b \gamma_{ba,ba}^{(\nu)} \rho_{aa}$

Rate von $b \to a$

$$\rho = \sum_a \rho_{aa} |a\rangle\langle a| + \sum_{ab:\, a\neq b} \rho_{ab} |a\rangle\langle b|$$

$$N_S = \sum_a N_a |a\rangle\langle a| \qquad \left([H_S, N_S] = 0 \right)$$

$$H_S = \sum_a E_a |a\rangle\langle a|$$

$$I_N^{(\nu)} = \sum_{a,b} \frac{(N_a - N_b)}{\Delta t} \underbrace{\Delta t \cdot \gamma_{ab,ab}^{(\nu)} \rho_{bb}}_{\text{WS für Sprung von } b \to a \text{ in } \Delta t} = \sum_{ab} (N_a - N_b) \gamma_{ab,ab}^{(\nu)} \rho_{bb}$$