W14

- Polaron-Mastergleichung

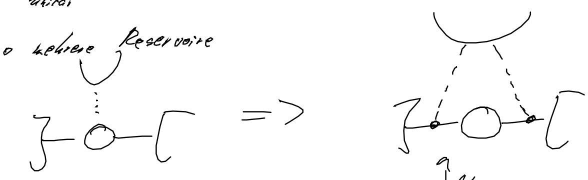
$$H = H_S + \underbrace{S}_{\text{Kopplung}} \sum_k \left(h_k b_k + h_k^* b_k^\dagger\right) + \underbrace{\sum_k \omega_k b_k^\dagger b_k}_{\text{Bad}} + \sum_k \frac{|h_k|^2}{\omega_k} S^2$$

$$U_p = \exp\left[S \sum_k \frac{h_k^*}{\omega_k}\left(b_k^\dagger - b_k\right)\right] \quad \text{vermischt System- \& Bad-Observablen}$$

$$H' = U_p H U_p^\dagger = \underbrace{U_p H_S U_p^\dagger}_{\text{beinhaltet Kopplung}} + \sum_k \omega_k b_k^\dagger b_k \quad \rightsquigarrow \text{Ableitung einer Standard-Mastergl.}$$

unitär

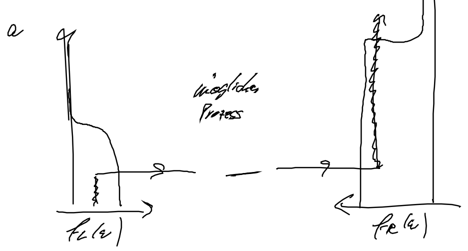
- mehrere Reservoire



Dissipatoren sind nicht additiv in den Bädern

$$C_{ij}(\tau) = C_{ij,el}(\tau) \cdot C_{ph}(\tau)$$

$\rightsquigarrow \gamma_{ij}(\omega)$ ist i.A. eine Faltung

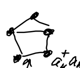


- Bogoliubov-Trafo ~~bo~~ neue bosonische Erz./Vern. ("Quasiteilchen")

$$a_k = \sum_q \left(u_{kq} b_q + v_{kq} b_q^\dagger\right) \qquad U = (u_{kq})$$
$$a_k^\dagger = \sum_q \left(u_{kq}^* b_q^\dagger + v_{kq}^* b_q\right) \qquad V = (v_{kq})$$

$$\boxed{\begin{aligned} U U^\dagger - V V^\dagger &= \mathbb{1} \\ U V^T - V U^T &= 0 \end{aligned}}$$

Wahlfreiheit $\rightsquigarrow$ vereinfache Hamiltonian



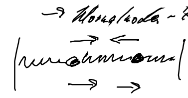
$a_k^\dagger a_k,\ a_k^2,\ (a_k^\dagger)^2 \Rightarrow$ $\Omega_1 b_1^\dagger b_1$, ..., $\Omega_5 b_5^\dagger b_5$

$a_k^\dagger a_q,\ a_k a_q$ nur quadratische Terme

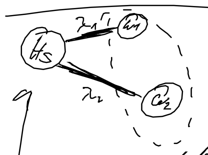
entkoppelte Moden

Analogon Mechanik: gek. Oszillatoren

$\rightarrow$ Normalmoden-Zerlegung



4.3.2. 2 Moden Beispiel



kann WW enthalten

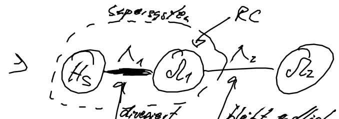
2 Moden Bog-Trafo

$$H = H_S + \omega_1\left(a_1^\dagger + \frac{\lambda_1}{\omega_1} S\right)\left(a_1 + \frac{\lambda_1}{\omega_1} S\right) + \omega_2\left(a_2^\dagger + \frac{\lambda_2}{\omega_2} S\right)\left(a_2 + \frac{\lambda_2}{\omega_2} S\right)$$

$$= H_S + \Omega_1\left(b_1^\dagger + \frac{\Lambda_1}{\Omega_1} S\right)\left(b_1 + \frac{\Lambda_1}{\Omega_1} S\right) + \Omega_2\left(b_2^\dagger + \frac{\Lambda_2}{\Omega_2}(b_1 + b_1^\dagger)\right)\left(b_2 + \frac{\Lambda_2}{\Omega_2}(b_1 + b_1^\dagger)\right)$$

finde Bog.-Trafo so, dass

$$a_k = \sum_{q=1}^{2}\left(u_{kq} b_q + v_{kq} b_q^\dagger\right)$$



$\lambda_i \to \infty$ (starke Kopplung)

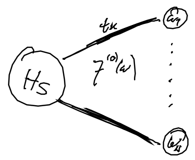
$$\mathrm{Tr}_{RC}\{\rho_{S+RC}(t)\} = \rho_S(t)$$

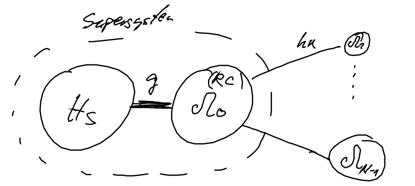
4.3.3. Ableitung der RC-Abbildung

$$H = H_S + \sum_k \omega_k \left(a_k^\dagger + \frac{t_k}{\omega_k} S\right)\left(a_k + \frac{t_k}{\omega_k} S\right) = H_S + \Omega_0 \left(b^\dagger + \frac{g}{\Omega_0} S\right)\left(b + \frac{g}{\Omega_0} S\right) + \sum_k \Omega_k \left(b_k^\dagger + \frac{h_k}{\Omega_k}(b+b^\dagger)\right) \times$$
$$\times \left(b_k + \frac{h_k}{\Omega_k}(b+b^\dagger)\right)$$

Kopplung: $\mathcal{J}^{(0)}(\omega) = 2\pi \sum_k |t_k|^2 \delta(\omega - \omega_k)$

ursprüngliche SD





$\mathcal{J}^{(1)}(\omega) = 2\pi \sum_k |h_k|^2 \delta(\omega - \Omega_k)$

residuale SD

Res.-Kopplung

Wo liegen die Residual-Bad-Energ.

$$\mathcal{J}^{(0)}(\omega) \quad \Rightarrow \quad g, \Omega_0, \mathcal{J}^{(1)}(\omega) \; ?$$

Trafo: $a_k = u_{k0} \cdot b + \sum_{q \geq 1} u_{kq} \cdot b_q$
$\quad + v_{k0} b^\dagger + \sum_{q \geq 1} v_{kq} b_q^\dagger$

Koeffizientenvergleich
- Terme linear in S: $\sum_k t_k (a_k + a_k^\dagger) \overset{!}{=} g(b + b^\dagger)$
- Terme quadratisch in S: $\sum_k \frac{t_k^2}{\omega_k} = \frac{g^2}{\Omega_0}$

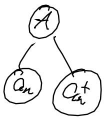
$$\boxed{\Omega_0^2 = \frac{\int_0^\infty \omega \cdot \mathcal{J}^{(0)}(\omega)\, d\omega}{\int_0^\infty \frac{\mathcal{J}^{(0)}(\omega)}{\omega}\, d\omega} \qquad g^2 = \frac{1}{2\pi \Omega_0} \int_0^\infty \omega \cdot \mathcal{J}^{(0)}(\omega)\, d\omega}$$

Abb. der spektralen Dichte: finde BWGL für Operator A im System

Heisenberg-BWGL $\tilde{A} = e^{+iHt} A e^{-iHt}$

① $\dot{\tilde{A}} = e^{+iHt} i[H, A] e^{-iHt}$

$= i \cdot \tilde{S}_1 + i \tilde{S}_2 \sum_k t_k (\tilde{a}_k + \tilde{a}_k^\dagger)$



$\tilde{S}_1 = e^{+iHt} \left[H_S + \sum_k \frac{t_k^2}{\omega_k} S^2, A\right] e^{-iHt}$

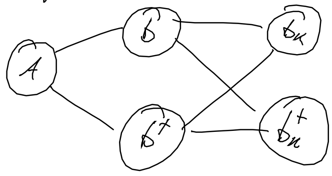
$\tilde{S}_2 = e^{+iHt} [S, A] e^{-iHt}$

$\dot{\tilde{a}}_k = -i\omega_k \tilde{a}_k - i t_k \tilde{S}$

$\dot{\tilde{a}}_k^\dagger = +i\omega_k \tilde{a}_k^\dagger + i t_k \tilde{S}$

$\rightsquigarrow$ eliminiere $\tilde{a}_k$ & $\tilde{a}_k^\dagger$ $\rightsquigarrow$ Gleichung für $\dot{\tilde{A}}$ $\longrightarrow$ Vergleich

② analog im Z. Bild mit $\tilde{A}, \tilde{b}, \tilde{b}^\dagger, \tilde{b}_k, \tilde{b}_k^\dagger$



eliminiere (nach FT) $b_k, b_k^\dagger$, dann $b, b^\dagger$ $\rightsquigarrow$ Gleichung für $\dot{\tilde{A}}$

Cauchy-Trafo von $\mathcal{J}(\omega)$ $\quad \mathcal{J}(-\omega) = -\mathcal{J}(\omega)$

$$W(z) = \frac{2}{\pi} \int_0^\infty \frac{\omega \cdot \mathcal{J}(\omega)}{\omega^2 - z^2} d\omega = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\mathcal{J}(\omega)}{\omega - z} d\omega$$

$$\lim_{\varepsilon \to 0} W(\omega + i\varepsilon) = \lim_{\varepsilon \to 0} \frac{1}{\pi} \int \frac{\mathcal{J}(\omega')}{\omega' - \omega - i\varepsilon} d\omega' = \lim_{\varepsilon \to 0} \frac{1}{\pi} \int \frac{\mathcal{J}(\omega')[\omega' - \omega + i\varepsilon]}{(\omega' - \omega)^2 + \varepsilon^2} d\omega'$$

$$= \frac{1}{\pi} \mathcal{P} \int \frac{\mathcal{J}(\omega')}{\omega' - \omega} d\omega' + i \mathcal{J}(\omega)$$

Trick

$\vdots$ [Skript]

$$\Rightarrow \boxed{\mathcal{J}^{(1)}(\omega) = \frac{4g^2 \cdot \mathcal{J}^{(0)}(\omega)}{\left[\frac{1}{\pi}\mathcal{P}\int\frac{\mathcal{J}^{(0)}(\omega')}{\omega-\omega'}d\omega'\right]^2 + \left[\mathcal{J}^{(0)}(\omega)\right]^2}}$$

- $\mathcal{J}^{(0)}(\omega) \to \alpha \cdot \mathcal{J}^{(0)}(\omega)$
  $g^2 \to \alpha \cdot g^2$
  $\mathcal{J}^{(1)}(\omega) \to \mathcal{J}^{(1)}(\omega)$ bleibt gleich
  $\rightsquigarrow$ Starke Kopplung $\mathcal{J}^{(0)}(\omega)$ kann mit Supersystem behandelt werden
- rekursive Anw. möglich
- für SDs mit komp. Support $\mathcal{J}^{(1)}(\omega) = 0$ $\forall\, \omega \notin [0, \omega_{max}]$

$$\mathcal{J}(\omega) = \omega \cdot \sqrt{1 - \frac{\omega^2}{\omega_{max}^2}}\ \Theta(\omega_{max}^2 - \omega^2)$$

Rubin-SD

4.3.4. Anw. Spin-Boson-Modell

$$H = \frac{\omega}{2}\sigma^z + \sigma^z \sum_k t_k (a_k + a_k^\dagger) + \sum_k \omega_k a_k^\dagger a_k + \sum_k \frac{t_k^2}{\omega_k} \cdot \mathbb{1}$$

$$= \underbrace{\underbrace{\frac{\omega}{2}\sigma^z}_{H_S} + \Omega_0 \left(b^\dagger + \frac{g}{\Omega_0}\sigma^z\right)\left(b + \frac{g}{\Omega_0}\sigma^z\right)}_{H_S' \rightsquigarrow \text{ME}} + \sum_k \Omega_k \left(b_k^\dagger + \frac{h_k}{\Omega_k}(b+b^\dagger)\right)\left(b_k + \frac{h_k}{\Omega_k}(b+b^\dagger)\right)$$

hier: Born-Markov-Näherungen

$$\dot\rho = -i[H_S', \rho] - \left\{\int_0^\infty C(\tau)\left[(b+b^\dagger), e^{-iH_S'\tau}(b+b^\dagger)e^{+iH_S'\tau}\rho\right]d\tau + \text{h.c.}\right\}$$

$H_S'|a\rangle = E_a|a\rangle$

$$\int_0^\infty C(\tau)e^{-iH_S'\tau}(b+b^\dagger)e^{+iH_S'\tau}d\tau = \sum_{ab}\langle a|(b+b^\dagger)|b\rangle \underbrace{\int_0^\infty C(\tau)\, e^{-i(E_a-E_b)\cdot\tau}d\tau}\, |a\rangle\langle b|$$

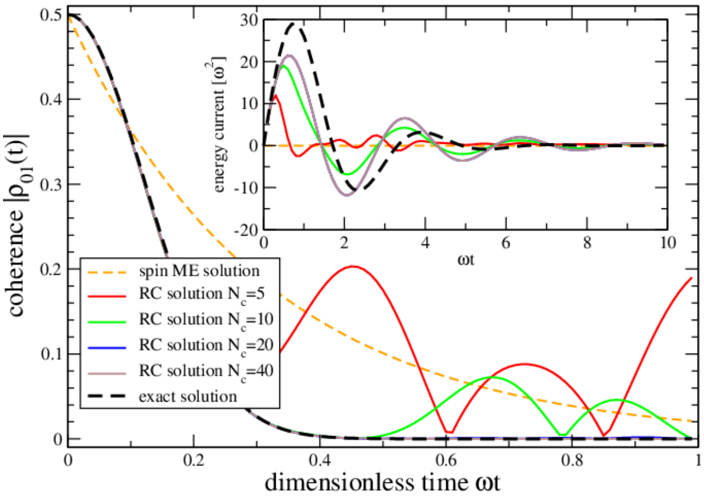
$$\to \frac{\gamma(E_b-E_a)}{2} + \underbrace{\frac{i}{2\pi}\mathcal{P}\int\frac{\gamma(\omega)}{E_b-E_a-\omega}d\omega}_{\text{Lamb-shift} \approx 0}$$

$\gamma(\omega) = \mathcal{J}^{(1)}(\omega)[1 + n_B(\omega)]$

berechne $\cdot\ |\rho_{01}| = \left|\frac{1}{2}\mathrm{Tr}\{(\sigma_x + i\sigma_y)\underbrace{\rho(t)}_{\text{Super-System}}\}\right|$ Kohärenz

$\cdot\ \frac{d}{dt}\left\langle \sum_k \omega_k a_k^\dagger a_k \right\rangle = -\frac{d}{dt}\left\langle \sigma^z \sum_k t_k (a_k + a_k^\dagger)\right\rangle = -\frac{d}{dt}\, g \left\langle \sigma^z (b+b^\dagger)\right\rangle$

für $\rho_0 \approx \rho_S^0 \otimes \frac{e^{-\beta\Omega_0[b^\dagger b + \Delta(b+b^\dagger)^2]}}{Z_{RC}}$


coherence $|\rho_{01}(t)|$
dimensionless time ωt
energy current [ω^2]
ωt
spin ME solution
RC solution N_c=5
RC solution N_c=10
RC solution N_c=20
RC solution N_c=40
exact solution