Wdh

- Stromformel $I = \frac{d}{dt}\langle n\rangle = -i\,\mathrm{Tr}\{\mathcal{L}'(0)\,\rho(t)\}$ $\qquad \rho(t) = e^{\mathcal{L}(0)\cdot t}\rho_0$

z.B. $\mathcal{L} = \mathcal{L}_0 + \sum_\nu \mathcal{L}^{(\nu)} \longrightarrow \mathcal{L}(\vec{\chi}) = \mathcal{L}_0 + \sum_\nu \mathcal{L}^{(\nu)}(\chi^\nu)$

$\mathcal{L}_0 = -i[H_S,\rho]$, $\mathcal{L}^{(\nu)}$: Bad ν

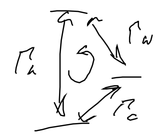
$I^{(\nu)}_{n}(t) = -i\,\mathrm{Tr}\{\partial_{\chi_\nu}\mathcal{L}(\vec{\chi})\,\rho(t)\}$

im SS gilt $\sum_\nu \bar{I}^{(\nu)}_{n} = 0$

- Bsp. SET

$$P_n(t) = \frac{1}{2\pi}\int_{-\pi}^{+\pi} \underbrace{M(\chi,t)}\, e^{-in\chi}\,d\chi = \frac{1}{2\pi}\int \mathrm{Tr}\{\rho(\chi,t)\}\cdot e^{-in\chi}\,d\chi$$
$$= e^{C(\chi,t)} \overset{t\to\infty}{\approx} e^{\lambda_{\max}(\chi)\cdot t} \qquad \rho(\chi,t) = e^{\mathcal{L}(\chi)\cdot t}\rho_0$$

- Absorbtionskühler



- Energie-aufgelöste FCS: $\mathcal{L}_0 + \sum_n \mathcal{L}_n \longrightarrow \mathcal{L}_0 + \sum_n \mathcal{L}_n \cdot e^{i\xi\Delta E_n} = \mathcal{L}(\xi)$

$\mathcal{L}_n$: Sprung mit ΔE_n; ξ: nur 1 Zählfeld nötig

3.3. Wartezeiten vs. FCS

a) • nur ein Sprungtyp:

+ WS für keinen Sprung $[0,t]$ $\qquad P_0(t) = \mathrm{Tr}\{e^{\mathcal{L}_0\cdot t}\rho_0\}$

+ " $[0,t+dt]$ $\qquad P_0(t+dt)$

+ WS für einen Sprung in $[t,t+dt]$ $\qquad W(t)\cdot dt = P_0(t) - P_0(t+dt) \rightsquigarrow W(t) = -\frac{dP_0(t)}{dt} = -\mathrm{Tr}\{\mathcal{L}_0 e^{\mathcal{L}_0 t}\rho_0\}$ (WZ-Verteilung)

$$\langle\tau\rangle = \int_0^\infty W(\tau)\cdot\tau\,d\tau \quad \text{mittl. Wartezeit}$$

b.) Was ist die WZ-Verteilung zwischen zwei versch. Sprüngen $\mathcal{L}_i$ & $\mathcal{L}_j$

betr. Entw. des Propagators Anteil für 1 Sprung $\qquad \mathcal{L} = \mathcal{L}_0 + \sum_{i=1}^N \mathcal{L}_i$

$$\mathcal{P}^{(1)}(\Delta t) = \int_0^{\Delta t} e^{\mathcal{L}_0(\Delta t - t_1)}\mathcal{L}_1 e^{\mathcal{L}_0 t_1}\,dt_1 \xrightarrow{\Delta t\to 0} \Delta t\cdot\mathcal{L}_1 + \ldots$$

DM direkt nach einem Sprung: Typ i: $\qquad \rho_0^{(i)} = \frac{\cancel{\Delta t}\,\mathcal{L}_i\rho_0}{\mathrm{Tr}\{\cancel{\Delta t}\,\mathcal{L}_i\rho_0\}} = \frac{\mathcal{L}_i\rho_0}{\mathrm{Tr}\{\mathcal{L}_i\rho_0\}} \longrightarrow$ wird AZ jetzt: WZ-Verteilung zwischen Sprung 1 des Typs (i) & Sprung 2 des Typs (j)

$$\underset{\uparrow}{W^i(\tau)} = -\mathrm{Tr}\{Z_0 e^{Z_0\cdot\tau}\rho_0^{(i)}\} = \mathrm{Tr}\{\underset{\uparrow}{(Z-Z_0)} e^{Z_0\cdot\tau}\rho_0^{(i)}\} = \sum_{j=1}^{N} \underbrace{\mathrm{Tr}\{Z_j e^{Z_0\cdot\tau}\rho_0^{(i)}\}}_{W^{ij}(\tau)}$$

WZ-Verteilung für bel. Sprung nach Sprung i ; $\mathrm{Tr}\{Z\hat{O}\} = 0$

jetzt: $\rho_0 = \bar{\rho}$

$Z\bar{\rho} = 0$

$$\Rightarrow \boxed{\begin{array}{l} Z = Z_0 + \sum_{i=1}^{N} Z_i \\ \text{WZ-Verteilung zwischen Sprung (i)} \to \text{(j)} \\ W^{ij}(\tau) = \dfrac{\mathrm{Tr}\{Z_j e^{Z_0\cdot\tau} Z_i\bar{\rho}\}}{\mathrm{Tr}\{Z_i\bar{\rho}\}} \end{array}}$$

Konsistent bei nur einem Sprung: $W^{11}(\tau) = \dfrac{\mathrm{Tr}\{Z_1 e^{Z_0\tau} Z_1\bar{\rho}\}}{\mathrm{Tr}\{Z_1\bar{\rho}\}} = \mathrm{Tr}\{(Z-Z_0)e^{Z_0\tau}\rho_0\} = -\mathrm{Tr}\{Z_0 e^{Z_0\tau}\rho_0\}$

$$\rho_0 = \frac{Z_1\bar{\rho}}{\mathrm{Tr}\{Z_1\bar{\rho}\}}$$

<u>Bsp</u>

a) SRL $\quad f \,\big\}\xrightarrow{\Gamma} \bigcirc$

$Z_0 = \Gamma\begin{pmatrix} -f & 0 \\ 0 & -(1-f) \end{pmatrix}$ $\quad Z_1 = \Gamma\begin{pmatrix} 0 & 0 \\ f & 0 \end{pmatrix}$ (rein) $\quad Z_2 = \Gamma\begin{pmatrix} 0 & 1-f \\ 0 & 0 \end{pmatrix}$ (raus)

$\bar{\rho} = \begin{pmatrix} 1-f \\ f \end{pmatrix}$ $\quad$ rein → rein $\quad$ raus → rein (dot ist leer)

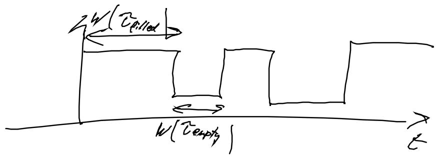
$$W(\tau) = \begin{pmatrix} 0 & \Gamma f\cdot e^{-\Gamma f\cdot\tau} \\ \Gamma(1-f)e^{-\Gamma(1-f)\tau} & 0 \end{pmatrix}$$

raus → raus

b) SET $\quad f_L\to 1 \,\big\}\xrightarrow{\Gamma_L} \bigcirc \xrightarrow{\Gamma_R}\big\{\, f_R \to 0$

$Z_0 = \begin{pmatrix} -\Gamma_L & 0 \\ 0 & -\Gamma_R \end{pmatrix}$ $\quad Z_1 \overset{\text{(rein)}}{=} \begin{pmatrix} 0 & 0 \\ \Gamma_L & 0 \end{pmatrix}$ $\quad Z_2 \overset{\text{(raus)}}{=} \begin{pmatrix} 0 & \Gamma_R \\ 0 & 0 \end{pmatrix}$

$$\to W(\tau) = \begin{pmatrix} 0 & \Gamma_L\cdot e^{-\Gamma_L\cdot\tau} \\ \Gamma_R\cdot e^{-\Gamma_R\cdot\tau} & 0 \end{pmatrix}$$

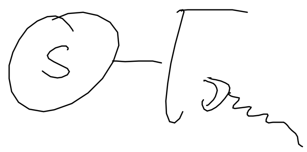


3.4. Mikroskop. Ableitung von Zählfeldern (zum Nikolaus)

zähle mal in Reservoir

Bad-Observable $\hat O = \sum_\ell \overset{\text{EWs}}{O_\ell} |\ell\rangle\langle\ell|$

Annahme: $[\hat O, H_B] = 0$ $\quad \left(H_B = \sum_\ell E_B^\ell |\ell\rangle\langle\ell|\right)$



Zweipunkt-Messung 1. Messung $t_0=0$, 2. Messung t } Differenz

1. Messung: Messpostulat $\quad \bar\rho_B \xrightarrow{\ell} \frac{|\ell\rangle\langle\ell|\bar\rho_B|\ell\rangle\langle\ell|}{P_\ell} = \frac{\bar\rho_B^{(\ell)}}{P_\ell}$

$$P_\ell = \mathrm{Tr}\{|\ell\rangle\langle\ell|\bar\rho_B\}$$

MGF für die Differenz der Messergebnisse

Zeitentwicklung vom Obs.

$$M_{tot}(\chi,t) = \sum_\ell \mathrm{Tr}\{ e^{+i\chi(\hat O - O_\ell)} U(t)\, \rho_S^0 \otimes \bar\rho_B^{(\ell)}\, U^\dagger(t)\}$$

Mittelung über Anfangs-Messergebnisse

z.B.: $-i\partial_\chi M(\chi,t)|_{\chi=0} = \sum_\ell \mathrm{Tr}\{(\hat O - O_\ell)\,\rho_{tot}^{(\ell)}(t)\}$

umschreiben
$$M_{tot}(\chi,t) = \sum_\ell \mathrm{Tr}\{ e^{+i\chi\frac{\hat O}{2}} U(t) \underbrace{e^{-i\frac{\chi}{2}O_\ell}\, \rho_S^0\otimes\bar\rho_B^{(\ell)}\, e^{-i\frac{\chi}{2}O_\ell}}_{e^{-i\frac{\chi}{2}\hat O}\rho_S^0\otimes\bar\rho_B^{(\ell)} e^{-i\frac{\chi}{2}\hat O}} U^\dagger(t)\, e^{+i\frac{\chi}{2}\hat O}\}$$

$$= \mathrm{Tr}\{ \underbrace{e^{+i\frac{\chi}{2}\hat O} U(t) e^{-i\frac{\chi}{2}\hat O}}_{U_{\chi/2}(t)} \left(\rho_S^0 \otimes \underbrace{\sum_\ell \bar\rho_B^{(\ell)}}\right) \underbrace{e^{-i\frac{\chi}{2}\hat O} U^\dagger(t) e^{+i\frac{\chi}{2}\hat O}}_{U^\dagger_{-\chi/2}(t)}\}$$

$$\bar{\bar\rho}_B = \sum_\ell |\ell\rangle\langle\ell|\bar\rho_B|\ell\rangle\langle\ell|$$

$$\boxed{U_{\chi/2}(t) = e^{+i\frac{\chi}{2}\hat O}\, U(t)\, e^{-i\frac{\chi}{2}\hat O}}$$
verallg. ZE-Operator

$[\hat O, H_B]=0$

$\rightsquigarrow e^{+iH_Bt}\hat O e^{-iH_Bt} = \tilde{\hat O} = \hat O$

U(t) entwickeln $\quad \tilde H_{\chi/2}(t) = e^{+i\frac{\chi}{2}\hat O}\tilde H_I(t) e^{-i\frac{\chi}{2}\hat O} = \sum_\alpha \tilde A_\alpha(t)\otimes e^{+i\frac{\chi}{2}\hat O} B_\alpha e^{-i\frac{\chi}{2}\hat O}$

$$U_{+\chi/2}(t) = 1 - i\int_0^t dt_1\, \tilde H_{\chi/2}(t_1) - \int_0^t dt_1\int_0^t dt_2\, \Theta(t_1-t_2)\,\tilde H_{\chi/2}(t_1)\,\tilde H_{\chi/2}(t_2) + \dots$$

$$U^\dagger_{-\chi/2}(t) = 1 + i\int_0^t dt_1\, \tilde H_{-\chi/2}(t_1) - \int_0^t dt_1\int_0^t dt_2\, \Theta(t_2-t_1)\,\tilde H_{-\chi/2}(t_1)\,\tilde H_{-\chi/2}(t_2) + \dots$$

vorher: $[1 + t\cdot\mathcal{L}_t + \dots]\rho_S^0 \stackrel{!}{=} \mathrm{Tr}_B\{U(t)\rho_S^0\otimes\bar\rho_B U^\dagger(t)\}$

jetzt $[1 + t\cdot\mathcal{L}_t(\chi)+\dots]\rho_S^0 \stackrel{!}{=} \mathrm{Tr}_B\{U_{\chi/2}(t)\rho_S^0\otimes\bar\rho_B U^\dagger_{-\chi/2}(t)\}$

analoges Vorgehen $\left(\mathrm{Tr}\{H_I \bar{\bar\rho}_B\} = 0\right)$

$$t \cdot \mathcal{L}_t(\chi)\rho_S^0 = + \int_0^t dt_1 dt_2 \, \mathrm{Tr}_B\{\tilde{H}_{\chi/2}(t_2)\, \rho_S^0 \otimes \bar{\bar{\rho}}_B \, \tilde{H}_{-\chi/2}(t_1)\}$$

$$- \int_0^t dt_1 dt_2 \, \Theta(t_1-t_2)\, \mathrm{Tr}_B\{\tilde{H}_{\chi/2}(t_1)\, \tilde{H}_{\chi/2}(t_2)\, \rho_S^0 \otimes \bar{\bar{\rho}}_B\}$$

$$- \int dt_1 dt_2 \, \Theta(t_2-t_1)\, \mathrm{Tr}_B\{\rho_S^0 \otimes \bar{\bar{\rho}}_B \, \tilde{H}_{-\chi/2}(t_1)\, \tilde{H}_{-\chi/2}(t_2)\}$$

$$\mathrm{Tr}_B\{e^{+i\frac{\chi}{2}\hat{O}} B_\alpha(t_1)\, \cancel{e^{-i\frac{\chi}{2}\hat{O}}\, e^{+i\frac{\chi}{2}\hat{O}}}\, B_\beta(t_2) \times e^{-i\frac{\chi}{2}\hat{O}}\, \bar{\bar{\rho}}_B\} = C_{\alpha\beta}(t_1,t_2)$$

$$= \sum_{\alpha\beta} \int dt_1 dt_2 \Big[C^\chi_{\alpha\beta}(t_1,t_2)\, A_\beta(t_2)\rho_S^0 A_\alpha(t_1)$$

$$- \Theta(t_1-t_2)\, C_{\alpha\beta}(t_1,t_2)\, A_\alpha(t_1) A_\beta(t_2)\rho_S^0 - \Theta(t_2-t_1)\, C_{\alpha\beta}(t_1,t_2)\, \rho_S^0 A_\alpha(t_1) A_\beta(t_2)\Big]$$

$$C^\chi_{\alpha\beta}(t_1,t_2) = \mathrm{Tr}_B\{e^{-i\hat{O}\chi/2} B_\alpha(t_1)\, e^{+i\hat{O}\frac{\chi}{2}}\, e^{+i\hat{O}\frac{\chi}{2}} B_\beta(t_2)\, e^{-i\hat{O}\frac{\chi}{2}}\, \bar{\bar{\rho}}_B\}$$

verallg. Korrelations-Fkt mit ZF

- falls $[H_B, \bar{\bar{\rho}}_B] = 0$, $[\hat{O}, \bar{\bar{\rho}}_B] = 0$ $\rightsquigarrow$ $C^\chi_{\alpha\beta}(t_1-t_2) = \mathrm{Tr}\{e^{-i\hat{O}\chi} \tilde{B}_\alpha(t_1-t_2)\, e^{+i\hat{O}\chi} B_\beta \bar{\bar{\rho}}_B\}$
- falls $[H_B, \bar{\rho}_B] = 0$ $\quad \bar{\bar{\rho}}_B = \sum_\ell |\ell\rangle\langle\ell| \bar{\rho}_B |\ell\rangle\langle\ell| = \bar{\rho}_B$
- falls $\hat{O} = H_B$ $\quad C^\chi_{\alpha\beta}(\tau) = C_{\alpha\beta}(\tau - \chi)$

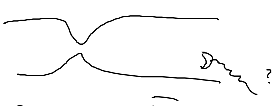
$\rightsquigarrow$ CG-Masterql. mit ZF:

$$\dot{\rho}_S = -i\Big[\frac{1}{2i\tau}\int_0^\tau dt_1 dt_2 \sum_{\alpha\beta} C_{\alpha\beta}(t_1-t_2)\,\mathrm{sgn}(t_1-t_2)\,\tilde{A}_\alpha(t_1)\tilde{A}_\beta(t_2), \rho_S\Big]$$

$$+ \frac{1}{\tau}\int_0^\tau dt_1 dt_2 \sum_{\alpha\beta}\Big[C^\chi_{\alpha\beta}(t_1-t_2)\hat{A}_\beta(t_2)\rho_S\hat{A}_\alpha(t_1) - \frac{1}{2} C_{\alpha\beta}(t_1-t_2)\{\hat{A}_\alpha(t_1)\hat{A}_\beta(t_2), \rho_S\}\Big]$$

$\rightsquigarrow$ ∘ $\rho(\chi,t)$ $\rightsquigarrow$ MGF $M(\chi,t) = \mathrm{Tr}\{\rho(\chi,t)\}$

- BMS-Limes $\rightsquigarrow$ $\tau \to \infty$
- Warum? QPC



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