WdH

- Floquet: $U_\varphi(t) = U_{\text{mich}}(t) \cdot e^{-i\bar{H}\cdot t} = \sum_r e^{-i\varepsilon_r \cdot t} \, |r(t)\rangle\langle r(0)|$

$$U_{\text{ad}}(t) = \sum_n e^{+i\gamma_n(t)} \cdot \exp\left\{-i\int_0^t E_n(t')dt'\right\} |n(t)\rangle\langle n(0)|$$

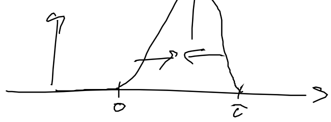
Berry-Phase $\qquad H(t)|n(t)\rangle = E_n(t)|n(t)\rangle$

- POVM $\qquad \{M_m\}$ Messoperatoren $\; : \; \sum_m M_m^\dagger M_m = \mathbb{1}$

$m \hat{=}$ Messergebnis

$$\rho \to \rho^{(m)} = \frac{M_m \rho M_m^\dagger}{\mathrm{Tr}\{M_m^\dagger M_m \rho\}} = \frac{M_m \rho M_m^\dagger}{p(m)}$$

+ falls $M_m = |m\rangle\langle m|$ → projektive Messung
+ alle DM-Eigenschaften bleiben erhalten
+ ergibt sich aus projektiver (lokaler) Messung auf Kompositsystem aus S & D
+ Bsp:



$$H(t) = H_S + H_D + g(t) H_{SD}$$

$$U(\tau) = \hat{\tau} \exp\left\{-i\int_0^\tau H(t)dt\right\}$$

$$\Rightarrow (\tau \to 0) \quad U_c = e^{-i\lambda H_{SD}} \qquad \lambda = \int_0^\tau g(t)dt$$

$$p(0) = \langle 1|\rho_S^0|1\rangle + \langle 0|\rho_S^0|0\rangle \cdot e^{-\lambda^2}$$

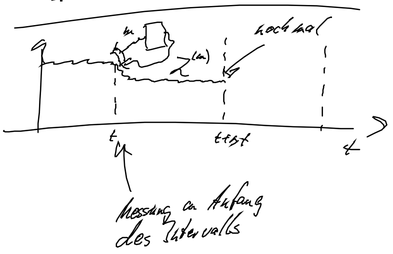
$$p(n \geq 1) = \frac{\lambda^{2n} e^{-\lambda^2}}{n!} \langle 0|\rho_S^0|0\rangle \qquad \sum_{n=1}^{\infty} p(n) = (1 - e^{-\lambda^2})\langle 0|\rho_S^0|0\rangle$$

falls $\rho_S^0 = |\psi\rangle\langle\psi| \qquad |\psi\rangle = \frac{1}{\sqrt{2}}[|1\rangle + |0\rangle]$

6.2. Externe stückweise konstante Kontrollschleifen

- kein Delay zw. Messung & Kontrolle
- Kontrolloperationen seien unendlich schnell

6.2.1. Wiederholte FB-Operationen



$$\rho^{(m)} = \frac{M_m \rho(t) M_m^\dagger}{p(m)} \hat{=} \frac{\mathcal{M}_m \rho}{p(m)} \quad \Bigg| \quad p(m) = \mathrm{Tr}\{M_m^\dagger M_m \rho(t)\}$$

$$\rho^{(m)}(t+\Delta t) = e^{\mathcal{L}^{(m)}\cdot\Delta t} \mathcal{M}_m \rho(t) \cdot \frac{1}{p(m)}$$

$$\rho(t+\Delta t) = \sum_m p(m)\rho^{(m)}(t+\Delta t) = \boxed{\sum_m e^{\mathcal{L}^{(m)}\cdot\Delta t} \mathcal{M}_m \rho(t) = \rho(t+\Delta t)}$$

(noch allgemeiner
$$\rho(t+\Delta t) = \sum_m \mathcal{K}^{(m)}(\Delta t, t) \mathcal{M}_m \rho(t)$$
)

6.2.2. kontinuierliches feedback ($\Delta t \to 0$)

$$\rho(t+\Delta t) = \left[\sum_m M_m + \Delta t \sum_m \mathcal{L}^{(m)} M_m\right] \rho(t) + \mathcal{O}(\Delta t^2) \qquad \left(\sum_m M_m\right)$$

wissen: $\sum_m M_m^\dagger M_m = \mathbb{1}$ aber $\sum_m M_m \neq \mathbb{1}$ $\qquad \sum_m M_m \rho \stackrel{!}{=} \sum_m M_m \rho M_m^\dagger \neq \rho$

$$M_\uparrow = |\uparrow\rangle\langle\uparrow| \qquad M_\downarrow = |\downarrow\rangle\langle\downarrow|$$

$$\rho = \begin{pmatrix} \rho_{\uparrow\uparrow} & \rho_{\uparrow\downarrow} \\ \rho_{\downarrow\uparrow} & \rho_{\downarrow\downarrow} \end{pmatrix} \to M_\uparrow \rho M_\uparrow^\dagger + M_\downarrow \rho M_\downarrow^\dagger = \begin{pmatrix} \rho_{\uparrow\uparrow} & 0 \\ 0 & \rho_{\downarrow\downarrow} \end{pmatrix}$$

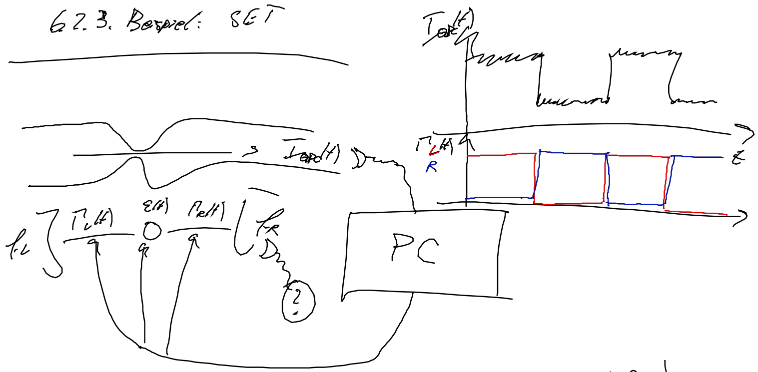
für projektive Messungen: $M_m \cdot M_n = M_n \cdot \delta_{nm}$

$$\underbrace{\sum_m M_m \rho(t+\Delta t)}_{\tilde\rho(t+\Delta t)} - \underbrace{\sum_m M_m \sum_n M_n \rho(t)}_{\tilde\rho(t)} = \Delta t \sum_n M_n \sum_m \mathcal{L}^{(m)} M_m \underbrace{\sum_k M_k \rho(t)}_{\tilde\rho(t)}$$

für $\Delta t \to 0$

$$\boxed{\dot{\tilde\rho} = \mathcal{L}_{fb} \tilde\rho \qquad \mathcal{L}_{fb} = \sum_n M_n \sum_m \mathcal{L}^{(m)} M_m}$$

6.2.3. Beispiel: SET



$$\mathrm{vec}(M_0 \rho M_0^\dagger) = \mathcal{M}_0 \,\mathrm{vec}(\rho) \qquad \mathrm{vec}(\rho) = \begin{pmatrix} \rho_{00} \\ \rho_{11} \\ \rho_{01} \\ \rho_{10} \end{pmatrix}$$

$$\rho = \begin{pmatrix} \rho_{00} & \rho_{01} \\ \rho_{10} & \rho_{11} \end{pmatrix} \qquad M_0 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \rightsquigarrow M_0 \rho M_0^\dagger = \begin{pmatrix} \rho_{00} & 0 \\ 0 & 0 \end{pmatrix} \Rightarrow \mathcal{M}_0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\mathcal{L}^{(0)} = \left(\begin{array}{c|c} \mathcal{L}^{(0)}_{pop} & 0 \\ \hline 0 & \begin{matrix} \xi^{(0)} & 0 \\ 0 & \xi^{(0)*} \end{matrix} \end{array}\right) \qquad \mathcal{L}^{(1)} = \left(\begin{array}{c|c} \mathcal{L}^{(1)}_{pop} & 0 \\ \hline 0 & \begin{matrix} \xi^{(1)} & 0 \\ 0 & \xi^{(1)*} \end{matrix} \end{array}\right)$$

$$\mathcal{M}_1 = \begin{pmatrix} 0 & & 0 & \\ & 1 & & \\ 0 & & 0 & \\ & & & 0 \end{pmatrix}$$

$$\mathcal{M}_0 + \mathcal{M}_1 \neq \mathbb{1}_4$$

$$\mathcal{L}_{fb} = \left(\begin{array}{cc|cc} -\Gamma_L^0 f_L^0 - \Gamma_R^0 f_R^0 & +\Gamma_L^1(1-f_L^1) + \Gamma_R^1(1-f_R^1) & 0 & 0 \\ +\Gamma_L^0 f_L^0 + \Gamma_R^0 f_R^0 & -\Gamma_L^1(1-f_L^1) - \Gamma_R^1(1-f_R^1) & 0 & 0 \\ \hline 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$= \left(\begin{array}{c|c} \mathcal{L}_{fb}^{pop} & 0 \\ \hline 0 & 0 \end{array}\right)$$

$$\mathcal{M}_0 + \mathcal{M}_1 = \left(\begin{array}{c|c} \mathbb{1}_{2\times 2} & 0 \\ \hline 0 & 0 \end{array}\right)$$

Lokales det. GG wird verletzt

$\rightarrow$ scheinbare Verletzung des 2. HS

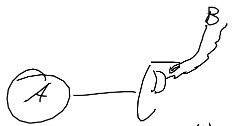
6.3. Wiseman-Milburn-Kontrolle

6.3.1. Dissipative Kontrollschleifen

System (A) + Detektor (B)

Operator in A, Basis in B

Komposit-DM: $\mathfrak{S}(t) = \sum_{nm} \rho^{(nm)}(t) \otimes |n\rangle\langle m|$



FCS $\quad \rho^{(n)}(t) \equiv \rho^{(nn)}(t)$

Sprünge

$$\dot{\rho}^{(n)}(t) = \mathcal{L}_0 \rho^{(n)}(t) + \mathcal{L}_+ \rho^{(n-1)}(t) + \mathcal{L}_- \rho^{(n+1)}(t)$$

kein Sprung

$$\rho^{(n)}(t) = \frac{1}{2\pi}\int_{-\pi}^{+\pi} \rho(\chi,t)\, e^{-in\chi} d\chi$$

$$\rho(\chi,t) = \sum_n \rho^{(n)}(t)\, e^{+in\chi}$$

$$\rightsquigarrow \mathcal{L}(\chi) = \mathcal{L}_0 + e^{+i\chi}\mathcal{L}_+ + e^{-i\chi}\mathcal{L}_-$$

$$\mathfrak{S}(t+\Delta t) = \mathfrak{S}(t) + \Delta t \cdot \sum_{nm} \dot{\rho}^{(nm)}(t) \otimes |n\rangle\langle m| + \mathcal{O}(\Delta t^2)$$

$$= \mathfrak{S}(t) + \Delta t \sum_n \left[\mathcal{L}_0 \rho^{(n)}(t) + \mathcal{L}_+ \rho^{(n-1)}(t) + \mathcal{L}_- \rho^{(n+1)}(t)\right] \otimes |n\rangle\langle n| + \underbrace{\Delta t \sum_{n\neq m} \dot{\rho}^{(nm)} \otimes |n\rangle\langle m|}_{\text{fliegt raus wg. } n\neq m} + \ldots$$

jetzt: bei t sei der Detektor in ZS $|0\rangle\langle 0|$ $\qquad \mathfrak{S}(t) = \rho(t) \otimes |0\rangle\langle 0|$

$$\mathcal{M}_0(\Delta t)\, \rho(t) = \mathrm{Tr}_B\left\{|0\rangle\langle 0|\, \mathfrak{S}(t+\Delta t)\, |0\rangle\langle 0|\right\} = \langle 0|\, \mathfrak{S}(t+\Delta t)\, |0\rangle = \left(\mathbb{1} + \mathcal{L}_0 \Delta t\right) \cdot \rho(t)$$

$$\mathfrak{S}(t+\Delta t) = \rho(t) \otimes |0\rangle\langle 0| + \Delta t\left[\mathcal{L}_0 \rho(t) \otimes |0\rangle\langle 0| + \mathcal{L}_+ \rho(t) \otimes |{+1}\rangle\langle{+1}| + \mathcal{L}_- \rho(t) \otimes |{-1}\rangle\langle{-1}|\right]$$
$$+ \text{off.} + \mathcal{O}(\Delta t^2)$$

$$\mathcal{M}_{\pm 1}(\Delta t)\, \rho(t) = \Delta t \cdot \mathcal{L}_\pm\, \rho(t)$$

$$\mathcal{M}_0(\Delta t) + \mathcal{M}_{-1}(\Delta t) + \mathcal{M}_{+1}(\Delta t) = \mathbb{1} + \mathcal{L}_0 \cdot \Delta t + \mathcal{L}_{-1} \cdot \Delta t + \mathcal{L}_{+1} \cdot \Delta t$$

$$\mathcal{U}_\pm \, vec(\rho) = vec(u_\pm \rho u_\pm^\dagger) \qquad u = e^{-iV}$$

$$\rho(t+\Delta t) = \left[\mathcal{M}_0(\Delta t) + \mathcal{U}_- \mathcal{M}_-(\Delta t) + \mathcal{U}_+ \mathcal{M}_+(\Delta t)\right]\rho(t)$$
$$= \left\{\mathbb{1} + \Delta t \underbrace{\left[\mathcal{L}_0 + \mathcal{U}_+\mathcal{L}_+ + \mathcal{U}_-\mathcal{L}_-\right]}_{\mathcal{L}_{fb}}\right\}\rho(t)$$

$$\boxed{\begin{aligned}\mathcal{L}_{fb} &= \mathcal{L}_0 + \mathcal{U}_+\mathcal{L}_+ + \mathcal{U}_-\mathcal{L}_-\\ \mathcal{L} &= \mathcal{L}_0 + \mathcal{L}_+ + \mathcal{L}_-\end{aligned}}$$

Sprungterm

$$\dot\rho = -i[H,\rho] + \sum_\alpha \overset{\geq 0}{\gamma_\alpha}\left[L_\alpha \rho L_\alpha^\dagger - \frac{1}{2}\{L_\alpha^\dagger L_\alpha, \rho\}\right] = \mathcal{L}\rho$$

$$\mathcal{L}_{fb}\,\rho = -i[H,\rho] + \sum_\alpha \gamma_\alpha\left[\underbrace{U_\alpha L_\alpha}_{\tilde L_\alpha} \rho L_\alpha^\dagger U_\alpha^\dagger - \frac{1}{2}\{L_\alpha^\dagger L_\alpha, \rho\}\right]$$

Anwendung: dissipative fb-Präparation reiner Zustände durch geeignete Kontrolle U_α

Umschreibung

$$\dot\rho = -i\underbrace{\left[H - i\sum_\alpha \frac{\gamma_\alpha}{2} L_\alpha^\dagger L_\alpha\right]}_{H_{eff}}\rho + i\rho\underbrace{\left[H + i\sum_\alpha \frac{\gamma_\alpha}{2} L_\alpha^\dagger L_\alpha\right]}_{H_{eff}^\dagger} + \sum_\alpha \gamma_\alpha U_\alpha L_\alpha \rho L_\alpha^\dagger U_\alpha^\dagger$$

"nicht-hermitescher Ham."

$$H_{eff}|\Psi\rangle = \underset{\in\mathbb{C}}{\varepsilon}|\Psi\rangle \qquad \text{möchten} \quad \mathcal{L}_{fb}\left(|\Psi\rangle\langle\Psi|\right) = 0$$
$$\langle\Psi| H_{eff}^\dagger = \langle\Psi|\varepsilon^*$$

$$0 = -i\varepsilon|\Psi\rangle\langle\Psi| + i\varepsilon^*|\Psi\rangle\langle\Psi| + \sum_\alpha \gamma_\alpha U_\alpha L_\alpha|\Psi\rangle\langle\Psi|L_\alpha^\dagger U_\alpha^\dagger$$

$$2\,\mathrm{Im}(\varepsilon) + \sum_\alpha \gamma_\alpha |b_\alpha|^2 = 0 \qquad U_\alpha L_\alpha|\Psi\rangle = \underset{\in\mathbb{C}}{b_\alpha}|\Psi\rangle$$

ermöglicht Stabilisierung von $|\Psi\rangle\langle\Psi|$