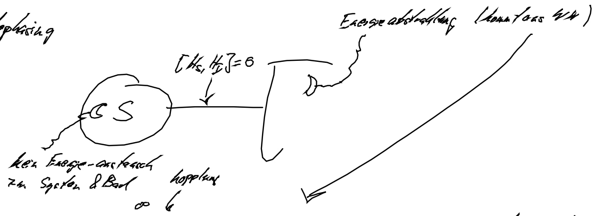
Wdh

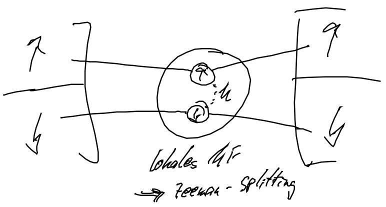
- pure dephasing



$$\langle \Delta E\rangle = \frac{2}{\pi}\int_0^\infty \frac{\Gamma(\omega)}{\omega}\sin^2\left(\frac{\omega t}{2}\right)d\omega$$

dyn. coarse-graining + ZF $\rightsquigarrow \rho(\chi,t) = e^{\mathcal{L}_t^{\chi}\cdot t}\rho_0$ (dynamisch) $\rightsquigarrow$ reprod. die exakte Lösung (nur für pure-dephasing)

- Spin-aufgelöstes Zählen $\hat{O} = S = \sum_k \left(C_{k\uparrow}^\dagger C_{k\uparrow} - C_{k\downarrow}^\dagger C_{k\downarrow}\right)$



$\rightsquigarrow \mathcal{L}(\chi) \rightsquigarrow \bar{I} = -i\,\mathrm{Tr}\{\mathcal{L}'(0)\bar{\rho}\}$

$\rightsquigarrow$ Spinventil

3.5. Symmetrien in der FCS

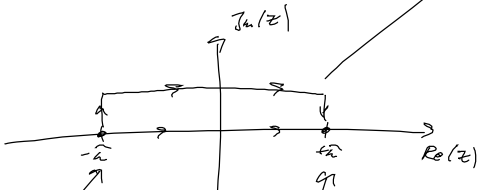
3.5.1. Mathem. Einführung

$$P_n(t) = \frac{1}{2\pi}\int_{-\pi}^{+\pi} M(\chi,t)\,e^{-in\chi}d\chi = \frac{1}{2\pi}\int_{-\pi}^{+\pi} e^{C(\chi,t)-in\chi}d\chi$$

sei $C(-\chi,t) = C(+\chi+i\alpha,t)$

$$\frac{P_{+n}(t)}{P_{-n}(t)} = \frac{\int M(\chi,t)\,e^{-in\chi}d\chi}{\int M(\chi,t)\,e^{+in\chi}d\chi} = \frac{\int_{-\pi}^{+\pi} e^{C(\chi,t)-in\chi}d\chi}{\int_{-\pi}^{+\pi} e^{C(-\chi,t)-in\chi}d\chi}$$

$$= \frac{\int_{-\pi}^{+\pi} e^{C(\chi,t)-in\chi}d\chi}{\int_{-\pi+i\alpha}^{+\pi+i\alpha} e^{C(\tilde{\chi},t)-in[\underbrace{\tilde{\chi}-i\alpha}_{\chi}]}d\tilde{\chi}} = \frac{\cancel{\int_{-\pi}^{+\pi} e^{C(\chi,t)-in\chi}d\chi}}{e^{-n\alpha}\cancel{\int_{-\pi}^{+\pi} e^{C(\chi,t)-in\chi}d\chi}} = e^{+n\alpha}$$



$e^{+i(-\pi+i\alpha)} = -e^{-\alpha}$ $\qquad$ $e^{+i(+\pi+i\alpha)} = -e^{-\alpha}$

$$C(-\chi,t) = C(+\chi+i\alpha,t) \implies \frac{P_n(t)}{P_{-n}(t)} = e^{+n\alpha}$$

Bsp.: SET

$$\mathcal{L}(\chi) = \Gamma_L \begin{pmatrix} -f_L & +(1-f_L)e^{-i\chi} \\ f_L \cdot e^{+i\chi} & -(1-f_L) \end{pmatrix} + \Gamma_R \begin{pmatrix} -f_R & +(1-f_R) \\ +f_R & -(1-f_R) \end{pmatrix}$$

betrachte EW $\quad \underbrace{\left| \mathcal{L}(\chi) - \lambda(\chi)\cdot \mathbf{1} \right|}_{D(\chi)} = 0$

$$\rightsquigarrow D(-\chi) = D\left(+\chi + i \cdot \ln\left[\frac{f_L(1-f_R)}{(1-f_L)\cdot f_R}\right]\right) = D\left(+\chi + i \underbrace{\left[(\beta_R-\beta_L)\,\varepsilon + (\beta_L \mu_L - \beta_R \mu_R)\right]}_{\alpha}\right)$$

$$\rightsquigarrow \lambda_{dom}(-\chi) = \lambda_{dom}(+\chi + i\alpha) = \lim_{t\to\infty} \frac{C(\chi,t)}{t} \qquad \lambda_{dom}(0) = 0$$

$$\rightsquigarrow \boxed{\lim_{t\to\infty} \frac{P_{+n}(t)}{P_{-n}(t)} = e^{n\left[(\beta_R-\beta_L)\varepsilon + \beta_L\mu_L - \beta_R\mu_R\right]} \xrightarrow{\beta_L=\beta_R} e^{n\cdot\beta\cdot V}}$$

integrierte Entropieproduktionsrate

FT-Theorem

"universell" gültig!

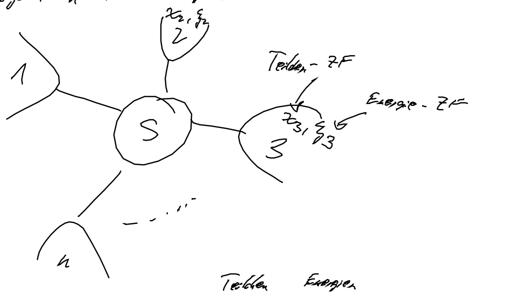
von links nach rechts

wir hatten $\lim_{t\to\infty} \dot{S}_i = (\beta_R-\beta_L)\cdot I_E + (\beta_L\mu_L - \beta_R\mu_R)\cdot I_M$

$$\text{allg.:} \quad \frac{P(+\Delta_i S)}{P(-\Delta_i S)} = e^{+\Delta_i S}$$

$P(\Delta_i S) \hat{=}$ WS für Trajektorie mit EP $\Delta_i S$

- für SET: "tight coupling": $I_E = \varepsilon \cdot I_M$
- allg.: n-terminal-System



$\Rightarrow$ 2n Zählfelder

Gesamtenergieerhaltung

+ TZ-Erhaltung

2n−2 ZF sind nötig

$$\hookrightarrow \dot{\underline{P}} = W(\underbrace{\underline{\chi}}_{\text{Teilchen}}, \underbrace{\underline{\xi}}_{\text{Energien}})\,\underline{P}$$

$$\boxed{W^T(-\underline{\chi} - i\underline{A}, -\underline{\xi} - i\underline{B}) = W(\underline{\chi}, \underline{\xi})}$$

$$\underline{B} = \begin{pmatrix} \beta_1 \\ \vdots \\ \beta_n \end{pmatrix} \qquad \underline{A} = -\begin{pmatrix} \mu_1\beta_1 \\ \vdots \\ \mu_n\beta_n \end{pmatrix} \qquad \text{"Affinitäten"}$$

$$\left| W(\underline{\chi},\underline{\xi}) - \lambda(\underline{\chi},\underline{\xi})\cdot\mathbf{1} \right| = 0$$

$$= \left| W(-\underline{\chi}-i\underline{A}, -\underline{\xi}-i\underline{B}) - \lambda(-\underline{\chi}-i\underline{A}, -\underline{\xi}-i\underline{B})\cdot\mathbf{1} \right|$$

$$= \left| W^T(-\underline{\chi}-i\underline{A}, -\underline{\xi}-i\underline{B}) - \lambda(-\underline{\chi}-i\underline{A}, -\underline{\xi}-i\underline{B})\cdot\mathbf{1} \right| \qquad \text{(EW sind inv. unter Transponierung)}$$

$$= \left| W(\underline{\chi},\underline{\xi}) - \lambda(-\underline{\chi}-i\underline{A}, -\underline{\xi}-i\underline{B})\cdot\mathbf{1} \right|$$

$$\Rightarrow \lim_{t\to\infty} C(-\underline{\chi}-i\underline{A}, -\underline{\xi}-i\underline{B}, t) = \lim_{t\to\infty} C(\underline{\chi}, \underline{\xi}, t)$$

integrierte EP-Rate

$$\rightarrow \lim_{t\to\infty} \frac{P_{+\Delta N, +\Delta E}(t)}{P_{-\Delta N, -\Delta E}(t)} = e^{-(\underline{\Delta E}\cdot\underline{B} + \underline{\Delta N}\cdot\underline{A})} = e^{-\sum_\nu \beta_\nu (\Delta E_\nu - \mu_\nu \Delta N_\nu)}$$

$$\boxed{\frac{P_{+\Delta_i S}}{P_{-\Delta_i S}} = e^{\Delta_i S}}$$

Crooks Fluktuations theorem

4. Transport bei starker Kopplung

bisher $H_I \to 0$ (ME wird gut)

$$\rightsquigarrow \bar\rho_S = \frac{e^{-\beta(H_S - \mu N_S)}}{Z_S}$$

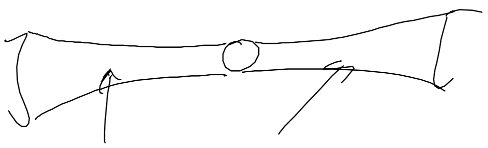
Thermalisierung

$$\rightarrow \dot S_i = \frac{d}{dt} S_{vN}(\rho) - \sum_\nu \beta_\nu \left[\bar I_E^{(\nu)} - \bar I_M^{(\nu)} \mu_\nu \right] \geq 0$$

$$\rightarrow \frac{P_{+\Delta_i S}}{P_{-\Delta_i S}} = e^{+\Delta_i S}$$ stoch. Formulierung des 2. HS

jetzt H_I endlich

4.1. Exakt lösbare Modelle: Das Fano-Anderson-Modell (SET, exakt)



starke Kopplung

$$H_S = \varepsilon d^\dagger d \qquad H_B = \sum_k \varepsilon_{kL} c_{kL}^\dagger c_{kL} + \sum_k \varepsilon_{kR} c_{kR}^\dagger c_{kR}$$

$$H_I = \sum_k \left(t_{kL} d c_{kL}^\dagger + h.c.\right) + \sum_k \left(t_{kR} d c_{kR}^\dagger + h.c.\right)$$

$$H = H_S + H_I + H_B$$

H ist eine quadrat. Form von Erzeugern & Vernichtern

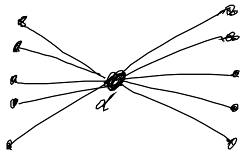
$\Rightarrow$ exakte Lösung ist möglich

analog zum Single res. Level (nur hier 2 Bäder)

⇝ Heisenberg-Gleichungen

$$\dot{d} = e^{+iHt}\, i[H, d]\, e^{-iHt} = e^{+iHt}\, i\left(-\varepsilon d + \sum_k t_{kL}^* c_{kL} + t_{kR}^* c_{kR}\right) e^{-iHt}$$

$$\boxed{\begin{aligned} \dot{\tilde{d}} &= -i\varepsilon \tilde{d} + i\sum_k \left(t_{kL}^* \tilde{c}_{kL} + t_{kR}^* \tilde{c}_{kR}\right) \\ \dot{\tilde{c}}_{k\nu} &= -i\,\varepsilon_{k\nu} \tilde{c}_{k\nu} + i\, t_{k\nu} \cdot \tilde{d} \end{aligned}}$$ ist geschlossen



⋮ [Skript]

⋮

wideband-limes $\Gamma_\nu(\omega) = 2\pi \sum_k |t_{k\nu}|^2 \delta(\omega - \varepsilon_{k\nu}) \approx \Gamma_\nu$

⇝ stat. Besetzung des dots $\quad \bar{n} = \lim_{t\to\infty} \langle d^\dagger d \rangle \overset{\downarrow}{=} \sum_\nu \int_{-\infty}^{+\infty} d\omega \, \frac{\Gamma_\nu f_\nu(\omega)}{\Gamma_L + \Gamma_R} \, \frac{2}{\pi} \, \frac{\Gamma_L + \Gamma_R}{(\Gamma_L + \Gamma_R)^2 + 4(\omega - \varepsilon)^2}$

• $\delta(x) = \lim_{\epsilon \to 0} \frac{1}{\pi} \frac{\epsilon}{x^2 + \epsilon^2}$

für schwache Kopplung gilt $\quad \bar{n} \longrightarrow \dfrac{\Gamma_L \cdot f_L(\varepsilon) + \Gamma_R \cdot f_R(\varepsilon)}{\Gamma_L + \Gamma_R} \hat{=}$ Lösung von SET

∘ inf. bias $\quad f_L(\omega) \to 1, \; f_R(\omega) \to 0 \qquad \bar{n} \to \dfrac{\Gamma_L}{\Gamma_L + \Gamma_R}$ (Mastergleichung wird exakt für inf. bias)

stat. Strom

$$\bar{I}_M = \lim_{t\to\infty} \frac{d}{dt} \left\langle \sum_k c_{kR}^\dagger c_{kR} \right\rangle = -\lim_{t\to\infty} \frac{d}{dt} \left\langle \sum_k c_{kL}^\dagger c_{kL} \right\rangle$$

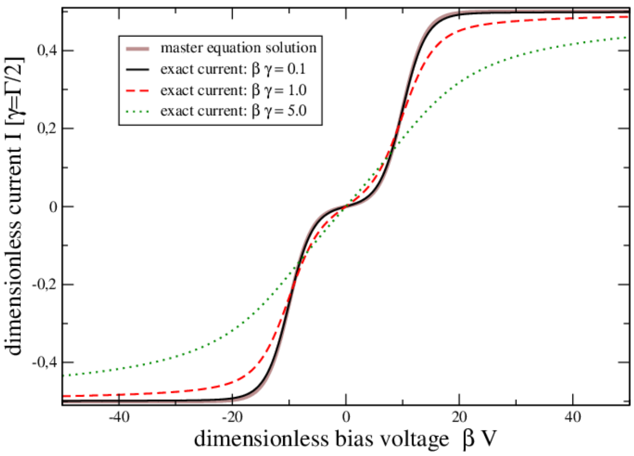
⋮ [Skript]

$$\bar{I}_M = \frac{\Gamma_L \Gamma_R}{\Gamma_L + \Gamma_R} \int d\omega \, [f_L(\omega) - f_R(\omega)] \underbrace{\frac{1}{\pi} \frac{(\Gamma_L + \Gamma_R)/2}{(\omega - \varepsilon)^2 + \left(\frac{\Gamma_L + \Gamma_R}{2}\right)^2}}_{\Gamma_L + \Gamma_R \to 0:\; \delta(\omega - \varepsilon)} \qquad I_{SET} = \frac{\Gamma_L \cdot \Gamma_R}{\Gamma_L + \Gamma_R} [f_L(\varepsilon) - f_R(\varepsilon)]$$

Landauer-Formel

$$\left.\begin{aligned} \bar{I}_M &= \frac{1}{2\pi} \int d\omega \, T(\omega) \, [f_L(\omega) - f_R(\omega)] \\ \bar{I}_E &= \frac{1}{2\pi} \int d\omega \, \omega \cdot T(\omega) \, [f_L(\omega) - f_R(\omega)] \end{aligned}\right\} \quad (\beta_R - \beta_L) \bar{I}_E + (\beta_L \cancel{\mu_L} - \beta_R \cancel{\mu_R}) \cdot \bar{I}_M \geq 0$$

2. HS gilt



master equation solution
exact current: β γ = 0.1
exact current: β γ = 1.0
exact current: β γ = 5.0
dimensionless current I [γ=Γ/2]
dimensionless bias voltage β V
0,4
0,2
0
-0,2
-0,4
-40
-20
0
20
40