WdH

- $\mathrm{Tr}\{[A,B]\} = 0$ für endlich-dim. HR

$[a, a^\dagger] = \mathbb{1}$

$$\tilde{a} = \begin{pmatrix} 0 & \sqrt{1} & 0 & \cdots & 0 \\ \vdots & & \sqrt{2} & \ddots & 0 \\ \vdots & & & \ddots & \sqrt{N} \\ 0 & \cdots & & \cdots & 0 \end{pmatrix} \qquad \tilde{a}^\dagger = \begin{pmatrix} 0 & - & \cdots & - & 0 \\ \sqrt{1} & \ddots & & & \vdots \\ \vdots & \ddots & \ddots & & \vdots \\ 0 & \cdots & 0 & \sqrt{N} & 0 \end{pmatrix}$$

$$[\tilde{a}, \tilde{a}^\dagger] = \begin{pmatrix} 1 & & (0) \\ & \ddots & \\ (0) & & 1-N \end{pmatrix}$$

- Kraus-Abb.: allg. Abb., welche die DM-Eigenschaften erhält

$$\rho' = \sum_\alpha K_\alpha \rho K_\alpha^\dagger \qquad \sum_\alpha K_\alpha^\dagger K_\alpha = \mathbb{1}$$

- Lindblad-ME

Superoperator: $N^2 \times N^2$ Matrix

Vektor: N^2 Einträge

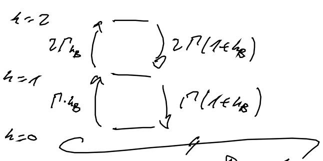
$$\dot{\rho} = -i[H, \rho] + \sum_\alpha \left[L_\alpha \rho L_\alpha^\dagger - \frac{1}{2}\{L_\alpha^\dagger L_\alpha, \rho\}\right] = \mathcal{L}\rho \rightsquigarrow \rho(t) = e^{\mathcal{L}t}\rho_0$$

$H = H^\dagger$ • alle Eigenschaften einer DM bleiben erhalten

- Beispiel: HO in therm. Bad

$$\dot{\rho} = -i[\Omega a^\dagger a, \rho] + \Gamma(1+n_B)\left[a\rho a^\dagger - \frac{1}{2}\{a^\dagger a, \rho\}\right] + \Gamma \cdot n_B\left[a^\dagger \rho a - \frac{1}{2}\{a a^\dagger, \rho\}\right] \qquad n_B = \frac{1}{e^{\beta\Omega} - 1}$$

$$\bar{P}_n = \frac{e^{-\beta E_n}}{\sum_n e^{-\beta E_n}} \qquad \bar{\rho} = \lim_{t\to\infty} \rho(t) = \frac{e^{-\beta\Omega a^\dagger a}}{\mathrm{Tr}\{\;''\;\}} \qquad \mathcal{L}\bar{\rho} = 0$$



b.) transiente Dynamik

$$\frac{d}{dt}\langle n\rangle = \frac{d}{dt}\langle a^\dagger a\rangle = \mathrm{Tr}\{a^\dagger a \dot{\rho}\} =$$

- $\mathrm{Tr}\{a^\dagger a(\Omega a^\dagger a \rho - \rho \Omega \underline{a^\dagger a})\} = \Omega\, \mathrm{Tr}\{[(a^\dagger a)^2 - (a^\dagger a)^2]\rho\} = 0$
- $\mathrm{Tr}\{a^\dagger a(a\rho a^\dagger - \frac{1}{2}a^\dagger a \rho - \frac{1}{2}\rho a^\dagger a)\} = \mathrm{Tr}\{[a^\dagger \underbrace{a^\dagger a}_{aa^\dagger - \mathbb{1}} a - \frac{1}{2}(a^\dagger a)^2 \cdot 2]\rho\}$

$$= -\mathrm{Tr}\{a^\dagger a\, \rho\} = \langle a^\dagger a\rangle_t$$

- analog der letzte Term

$$\frac{d}{dt}\langle n\rangle = -\Gamma(1+n_B)\langle n\rangle + \Gamma \cdot n_B \langle n\rangle \qquad \rightsquigarrow \frac{\bar{n}}{1+\bar{n}} = \frac{n_B}{1+n_B} = e^{-\beta\cdot\Omega}$$

$$0 = (1+n_B)\bar{n} + \Gamma \cdot n_B \bar{n} \qquad \bar{n} = \lim_{t\to\infty}\langle n\rangle_t$$

$$\frac{d}{dt}\langle a\rangle = \left[-i\Omega - \frac{\Gamma(1+n_B) + \Gamma \cdot n_B}{2}\right]\langle a\rangle \qquad \langle x\rangle = \frac{1}{\sqrt{2m\Omega}}(a + a^\dagger)$$

$$\frac{d}{dt}\langle a^\dagger\rangle = \left[+i\Omega - \frac{\Gamma(1+n_B) + \Gamma n_B}{2}\right]\langle a^\dagger\rangle \qquad \langle p\rangle = i\sqrt{\frac{m\Omega}{2}}(a^\dagger - a)$$

$$\Rightarrow \frac{d}{dt}\langle x\rangle = \frac{1}{m}\langle p\rangle - \frac{\Gamma(1+2n_B)}{2}\langle x\rangle$$

$$\frac{d}{dt}\langle p\rangle = -m\Omega^2\langle x\rangle - \frac{\Gamma(1+2n_B)}{2}\langle p\rangle$$

1.3. Mikroskopische Lindblad Ableitung

1.3.1. Voraussetzungen

a) Tensor Produkt

seien V & W Hilberträume. Dann ist $V\otimes W$ ein HR mit einer Basis $\{|v_i\rangle\otimes|w_j\rangle\}$

- Bilinear: $(z_1|v_1\rangle + z_2|v_2\rangle)\otimes|w\rangle = z_1|v_1\rangle\otimes|w\rangle + z_2|v_2\rangle\otimes|w\rangle$

  $z_i\in\mathbb{C}$ $\quad |v\rangle\otimes(z_1|w_1\rangle + z_2|w_2\rangle) = z_1|v\rangle\otimes|w_1\rangle + z_2|v\rangle\otimes|w_2\rangle$

- komb. Operatoren

  $$(A\otimes B)(|v\rangle\otimes|w\rangle) \stackrel{!}{=} (A|v\rangle)\otimes(B|w\rangle)$$

- jeder Operator auf $V\otimes W$ kann zerlegt werden

  $$C = \sum_\alpha c_i A_\alpha\otimes B_\alpha$$

- Das Skalarprodukt wird vererbt

  $$|a\rangle = \sum_{ij} a_{ij}|v_i\rangle\otimes|w_j\rangle \qquad |b\rangle = \sum_{kl} b_{kl}|v_k\rangle\otimes|w_l\rangle$$

  $$\langle a|b\rangle = \sum_{ijkl} a_{ij}^* b_{kl}\underbrace{\langle v_i|v_k\rangle}_{\delta_{ik}}\cdot\underbrace{\langle w_j|w_l\rangle}_{\delta_{jl}} = \sum_{ij} a_{ij}^* b_{ij}$$

Bsp: $\hat{\Sigma} = a\,\mathbb{1}\otimes\mathbb{1} + \sum_{i=1}^{3}\alpha_i\sigma^i\otimes\mathbb{1} + \sum_{i=1}^{3}\beta_i\mathbb{1}\otimes\sigma^i + \sum_{ij=1}^{3}a_{ij}\sigma^i\otimes\sigma^j$

$$\mathrm{Tr}_{AB}\{A\otimes B\} = \mathrm{Tr}_A\{A\}\cdot\mathrm{Tr}_B\{B\} \qquad \mathrm{Tr}\{\hat{\Sigma}\} = a\cdot 2\cdot 2 = 4a$$

b.) Die partielle Spur

$$\mathrm{Tr}_B\{|a_1\rangle\langle a_2|\otimes|b_1\rangle\langle b_2|\} \stackrel{!}{=} |a_1\rangle\langle a_2|\cdot\mathrm{Tr}\{|b_1\rangle\langle b_2|\}$$

Bsp: $\rho_{AB} = |\Psi\rangle\langle\Psi| = \rho_{AB}^2 \qquad |\Psi\rangle = \frac{1}{\sqrt{2}}[|01\rangle + |10\rangle]$

$$= \frac{1}{\sqrt{2}}[|0\rangle\otimes|1\rangle + |1\rangle\otimes|0\rangle]$$

$$\rho_A = \mathrm{Tr}_B\{\rho_{AB}\} = \mathrm{Tr}_B\{\tfrac{1}{2}[|01\rangle + |10\rangle][\langle 01| + \langle 10|]\}$$

$$= \mathrm{Tr}_B\{\tfrac{1}{2}[\underbrace{|01\rangle\langle 01|}_{|0\rangle\langle 0|\otimes|1\rangle\langle 1|} + \cancel{\underbrace{|01\rangle\langle 10|}_{|0\rangle\langle 1|\otimes|1\rangle\langle 0|}} + \cancel{|10\rangle\langle 01|} + |10\rangle\langle 10|]\}$$

$$= \tfrac{1}{2}|0\rangle\langle 0| + \tfrac{1}{2}|1\rangle\langle 1| \neq \rho_A^2$$

allg: $C = \sum_\alpha A_\alpha\otimes B_\alpha \qquad \mathrm{Tr}_B\{C\} = \sum_\alpha A_\alpha\cdot\mathrm{Tr}\{B_\alpha\}$

c.) red. Dichtematrix

Sei ρ_{AB} eine DM auf $A \otimes B$ mit Dim $N_A \cdot N_B$

$$\rho_{AB} = \sum_{nm} \rho^{AB}_{nm} |n\rangle\langle m| = \sum_{n_A m_A=1}^{N_A} \sum_{n_B m_B=1}^{N_B} \rho_{(n_A n_B),(m_A m_B)} |n_A\rangle\langle m_A| \otimes |n_B\rangle\langle m_B|$$

Dann ist $\rho_A = \mathrm{Tr}_B\{\rho_{AB}\} = \sum_{n_B=1}^{N_B} \langle n_B|\rho_{AB}|n_B\rangle$

$$= \sum_{n_A, m_A=1}^{N_A} \Big[\underbrace{\sum_{n_B=1}^{N_B} \rho_{(n_A n_B),(m_A n_B)}}_{\rho^{(A)}_{n_A m_A}}\Big] |n_A\rangle\langle m_A| \qquad \text{eine DM in } A$$

- $\mathrm{Tr}_A\{\rho_A\} = 1 \qquad \rho_A = \rho_A^\dagger \qquad \langle\psi_A|\rho_A|\psi_A\rangle \geq 0$
- $\boxed{\mathrm{Tr}\{\hat{A} \otimes \mathbb{1} \cdot \rho_{AB}\} = \mathrm{Tr}_A\{\hat{A} \cdot \rho_A\}}$

1.3.2. Allg. quanten-opt. Ableitung

System $\leftrightarrow$ Bad

$H_{tot} = H_S \otimes \mathbb{1} + H_I + \mathbb{1} \otimes H_B = H_S + \underset{\uparrow\ \text{Störungstheorie}}{H_I} + H_B$

$\dot{\rho}_{tot} = -i[H, \rho_{tot}]$

$\rightsquigarrow \mathrm{Tr}_B\{e^{-iH_{tot} \cdot t} \rho_S^0 \otimes \rho_B^0 e^{+iH_{tot} \cdot t}\} = \rho_S(t)$

$= \sum_\alpha K_\alpha(t) \rho_S^0 K_\alpha^\dagger(t)$

$: \sum_\alpha K_\alpha^\dagger K_\alpha = \mathbb{1}$

$H_I = \sum_\alpha A_\alpha \otimes B_\alpha = H_I^\dagger$

z.B. dA $\boxed{\begin{matrix} A_\alpha = A_\alpha^\dagger \\ B_\alpha = B_\alpha^\dagger \end{matrix}}$ $\Big\}$ kann durch Umschreiben erreicht werden

$a b^\dagger + b a^\dagger$?

$$H_I = \frac{1}{2} \sum_\alpha \left(A_\alpha \otimes B_\alpha + A_\alpha^\dagger \otimes B_\alpha^\dagger\right)$$
$$= \frac{1}{2} \sum_\alpha \left[(A_\alpha^S + A_\alpha^A) \otimes (B_\alpha^S + B_\alpha^A) + (A_\alpha^S - A_\alpha^A) \otimes (B_\alpha^S - B_\alpha^A)\right]$$
$$= \sum_\alpha \left[A_\alpha^S \otimes B_\alpha^S - (iA_\alpha^A) \otimes (iB_\alpha^A)\right]$$

$H_{tot} = H_0 + H_1$

$\tilde{\rho}_{tot}(t) = e^{+i(H_S + H_B) \cdot t} \rho_{tot}(t) e^{-i(H_S + H_B) \cdot t}$

$\rightsquigarrow \dot{\tilde{\rho}}_{tot} = -i[\tilde{H}_I(t), \tilde{\rho}_{tot}(t)]$

$$\tilde{H}_I(t) = e^{+i(H_S + H_B) \cdot t} H_I e^{-i(H_S + H_B) \cdot t}$$
$$= \sum_\alpha \underbrace{e^{+iH_S \cdot t} A_\alpha e^{-iH_S \cdot t}}_{\tilde{A}_\alpha(t)} \otimes \underbrace{e^{+iH_B \cdot t} B_\alpha e^{-iH_B \cdot t}}_{\tilde{B}_\alpha(t)}$$

z.B.: $H = \underbrace{\Omega a^\dagger a}_{H_S} + \underbrace{\sum_k \omega_k b_k^\dagger b_k}_{H_B} + \underbrace{(a + a^\dagger) \otimes \sum_k \lambda_k (b_k + b_k^\dagger)}_{H_I}$

$$\tilde{H}(t) = \left(e^{+i\Omega t a^\dagger a} (a + a^\dagger) e^{-i\Omega t a^\dagger a}\right) \otimes \left(\sum_k \lambda_k e^{+i\omega_k t b_k^\dagger b_k} (b_k + b_k^\dagger) e^{-i\omega_k t b_k^\dagger b_k}\right)$$

$$\frac{d}{dt} \underbrace{e^{+i\Omega t a^\dagger a} a e^{-i\Omega t a^\dagger a}}_{\tilde{a}(t)} = i\Omega e^{+i\Omega t a^\dagger a} \underbrace{[a^\dagger a, a]}_{-a} e^{-i\Omega t a^\dagger a} = -i\Omega \cdot \tilde{a}(t)$$

$\rightsquigarrow \tilde{a}(t) = a \cdot e^{-i\Omega t}$

$$\tilde{H}_V(t) = \left(a \cdot e^{-i\Omega t} + a^\dagger e^{+i\Omega t}\right) \otimes \left(\sum_k \lambda_k \left(b_k \cdot e^{-i\omega_k t} + b_k^\dagger e^{+i\omega_k t}\right)\right)$$

klein