__Wdh__

- $H_I = \sum_\alpha \underset{=A_\alpha^\dagger}{A_\alpha} \otimes \underset{=B_\alpha^\dagger}{B_\alpha}$ (System: A_α, Bad: B_α)

- WW-Bild bzgl. $H_S \otimes H_B$ $\qquad \dot{\tilde\rho} = -i \left[\underbrace{e^{+i(H_S+H_B)t} H_I e^{-i(H_S+H_B)t}}_{\tilde H_I}, \tilde\rho \right]$

- Bad-korrelations-Fkt.:

$$C_{\alpha\beta}(\tau) = \mathrm{Tr}\left\{ e^{+iH_B\tau} B_\alpha e^{-iH_B\tau} B_\beta \cdot \bar\rho_B \right\} \qquad \hat\rho_{tot}(t) \approx \tilde\rho_S(t) \otimes \bar\rho_B$$

- Nicht-Markovsche MGL

$$\dot{\tilde\rho}_S = -\sum_{\alpha\beta} \int_0^t dt' \left\{ C_{\alpha\beta}(t-t') \left[\tilde A_\alpha(t), \tilde A_\beta(t') \tilde\rho_S(t') \right] + h.c. \right\} \qquad \mathrm{Tr}\{\rho_S(t)\} = 1, \quad \rho_S = \rho_S^\dagger$$

$$= \int_0^t \underset{\text{superoperator}}{W(t,t')}\, \rho(t')\, dt'$$

- Markov-Näherungen $\quad C_{\alpha\beta}(\tau)$ fallen schnell ab

$$\longrightarrow \rho_S(t') \to \rho_S(t)$$

$$\int_0^t [\dots] dt' \longrightarrow \int_0^\infty [\dots] dt'$$

$$\Rightarrow \dot\rho_S = \underset{\text{Rücktrafo ins Schröd.-Bild}}{-i[H_S, \rho_S]} - \left\{ \sum_{\alpha\beta} \int_0^\infty C_{\alpha\beta}(\tau) \left[A_\alpha, e^{-iH_S\tau} A_\beta e^{+iH_S\tau} \rho_S(t) \right] + h.c. \right\}$$

$$= W \rho_S(t) \qquad \leadsto \rho_S(t) = e^{W\cdot t} \rho_S^0$$

$\Rightarrow \mathrm{Tr}\{\rho_S\} = 1 \quad \forall t \qquad \rho_S = \rho_S^\dagger \quad \forall t$

__aber__ keine Lindblad-Form (i.A. Allg.) $\qquad$ Bsp.: $H = \frac{\omega_1}{2}\sigma_1^z + \frac{\omega_2}{2}\sigma_2^z + \lambda\,\sigma_1^x \otimes \sigma_2^x$

c.) Säkular-Näherung

$$\dot{\tilde\rho}_S = \sum_{\omega\omega'} \underset{\text{Super-Operatoren}}{\mathcal{L}_{\omega\omega'}} e^{i(\omega-\omega')\cdot t} \tilde\rho_S(t) = \sum_\omega \mathcal{L}_{\omega\omega} \tilde\rho_S(t) + \underbrace{\sum_{\omega\neq\omega'} \mathcal{L}_{\omega\omega'} e^{+i(\omega-\omega')t} \tilde\rho_S(t)}_{\approx 0}$$

Bohr-Frequenzen des Systems (Übergänge) $\qquad$ ("Säkular-Näherung")

$$\dot{\tilde\rho}_S = -\sum_{\alpha\beta} \int_0^\infty \left\{ C_{\alpha\beta}(\tau) \left[\tilde A_\alpha(t), \tilde A_\beta(t-\tau) \tilde\rho_S(t) \right] + h.c. \right\}$$

$$e^{+iH_S t} A_\alpha e^{-iH_S t} = \sum_{a,b} e^{+iE_a t} |a\rangle\langle a| A_\alpha |b\rangle\langle b| e^{-iE_b t}$$

$$H_S = \sum_a E_a |a\rangle\langle a| \qquad = \sum_{a,b} e^{+i\underbrace{(E_a-E_b)}_{\omega}\cdot t} \langle a|A_\alpha|b\rangle\, |a\rangle\langle b|$$

$\longrightarrow$ Gleichung mit $\delta_{E_b-E_a, E_d-E_c}$ $\longrightarrow$ führt zurück auf __Lindblad Form__

Halbseitige FT: $\quad \Gamma_{\alpha\beta}(\omega) = \int_0^\infty C_{\alpha\beta}(\tau) e^{+i\omega\tau} d\tau = \frac{1}{2}\gamma_{\alpha\beta}(\omega) + \frac{1}{2}\sigma_{\alpha\beta}(\omega)$

$\gamma_{\alpha\beta}(\omega) = \Gamma_{\alpha\beta}(\omega) + \Gamma_{\beta\alpha}^*(\omega) = \int_{-\infty}^\infty C_{\alpha\beta}(\tau) e^{+i\omega\tau} d\tau \quad$ volle FT (gerade)

$\sigma_{\alpha\beta}(\omega) = \Gamma_{\alpha\beta}(\omega) - \Gamma_{\beta\alpha}^*(\omega) = \int C_{\alpha\beta}(\tau) \cdot \mathrm{sgn}(\tau) e^{+i\omega\tau} d\tau \quad$ volle ungerade FT

$$= \frac{i}{\pi} \mathcal{P} \int \frac{\gamma_{\alpha\beta}(\omega')}{\omega - \omega'} d\omega'$$

Cauchy Hauptwert

Vorr.: $H_I = \sum_\alpha A_\alpha \otimes B_\alpha \qquad A_\alpha = A_\alpha^\dagger \quad B_\alpha = B_\alpha^\dagger$

$[H_B, \bar{\rho}_B] = 0 \qquad Tr\{B_\alpha \bar{\rho}_B\} = 0 \qquad C_{\alpha\beta}(\tau) = Tr\{e^{+iH_B\tau} B_\alpha e^{-iH_B\tau} B_\beta \cdot \bar{\rho}_B\}$

berechne $\gamma_{\alpha\beta}(\omega) = \int C_{\alpha\beta}(\tau) e^{+i\omega\tau} d\tau \qquad \sigma_{\alpha\beta}(\omega)$ analog

$$\dot{\rho}_S = -i[\underbrace{H_S + \sum_{ab} \sigma_{ab} \hat{L}_{ab}}_{H_{LS}}, \rho_S] + \sum_{ab,cd} \gamma_{ab,cd} [\hat{L}_{ab} \rho \hat{L}_{cd}^\dagger - \frac{1}{2}\{\hat{L}_{cd}^\dagger \hat{L}_{ab}, \rho\}]$$

$\hat{L}_{ab} = |a\rangle\langle b| \qquad H_S|a\rangle = E_a|a\rangle$

$$\sigma_{ab} = \sum_{\alpha\beta} \sum_c \frac{1}{2i} \sigma_{\alpha\beta}(E_b - E_c) \delta_{E_b, E_a} \langle c|A_\beta|b\rangle\langle c|A_\alpha|a\rangle^*$$

$$\gamma_{ab,cd} = \sum_{\alpha\beta} \gamma_{\alpha\beta}(E_b - E_a) \delta_{E_b - E_a, E_d - E_c} \langle a|A_\beta|b\rangle\langle c|A_\alpha|d\rangle^*$$

BMS Mastergleichung

+ zeige: $\cdot\ H_{LS} = \sum_{ab} \sigma_{ab} \hat{L}_{ab} = H_{LS}^\dagger$

$\cdot \sum_{ab,cd} x_{ab}^* \gamma_{ab,cd} x_{cd} \geq 0$

folgt aus $\gamma_{\alpha\beta}(\omega)$ ist pos. definit

+ L_{ab} sind nicht lokal

+ oft wird H_{LS} vernachlässigt

+ falls $\rho: N \times N \longrightarrow \mathcal{L}\rho = \dot{\rho} \qquad \mathcal{L}: N^2 \times N^2$

+ falls H_S keine Entartungen hat

$\rightarrow \delta_{E_a, E_b} = \delta_{a,b}$

$$\rightarrow \dot{\rho}_{aa} = +\sum_b \gamma_{ab,ab} \rho_{bb} - \left(\sum_b \gamma_{ba,ba}\right) \rho_{aa} \qquad \text{Ratengleichung}$$

$$\gamma_{ab,ab} = \sum_{\alpha\beta} \underbrace{\gamma_{\alpha\beta}(E_b - E_a)}_{\text{pos. definit}} \langle a|A_\beta|b\rangle\langle a|A_\alpha|b\rangle^* \geq 0$$

$$\mathcal{L}_{BMS} = \begin{pmatrix} \mathcal{L}_{pop} & 0 \\ 0 & \mathcal{L}_{coh} \end{pmatrix} \qquad \rho = \begin{pmatrix} \rho_{pop} \\ \rho_{coh} \\ \vdots \end{pmatrix}$$

+ Lindblad Davis - Abb.: $\mathcal{L}_{BMS}\, e^{-\beta H_S} = 0$

für $\bar{\rho}_B = \frac{e^{-\beta H_B}}{Z_B}$

1.3.3. Harmon. Oszillator im therm. Bad

$$H_{tot} = \underbrace{\Omega a^\dagger a}_{H_S} + \underbrace{\sum_k \omega_k b_k^\dagger b_k}_{H_B} + \underbrace{\overbrace{(a + a^\dagger)}^{A} \otimes \overbrace{\sum_k (h_k b_k + h_k^* b_k^\dagger)}^{B}}_{H_I} \qquad \begin{matrix} A = A^\dagger \\ B = B^\dagger \end{matrix}$$

$\dot{\tilde{\rho}} = -i[\tilde{H}_I, \tilde{\rho}] \qquad \tilde{H}_I = (a \cdot e^{-i\Omega t} + a^\dagger e^{+i\Omega t}) \otimes \sum_k (h_k b_k e^{-i\omega_k t} + h_k^* b_k^\dagger e^{+i\omega_k t})$

$$\tilde{a}(t) = e^{+i\Omega t a^\dagger a}\, a\, e^{-i\Omega t a^\dagger a}$$

$$\rightsquigarrow \frac{d}{dt}\tilde{a}(t) = -i\Omega \tilde{a}(t)$$

$\alpha = \beta = 1$

$$\mathrm{Tr}\{B\,\bar\varrho_B\} = 0$$

$$C_{11}(\tau) = C(\tau) = \mathrm{Tr}\left\{\left(\sum_k h_k b_k e^{-i\omega_k \tau} + h_k^* b_k^\dagger e^{+i\omega_k\tau}\right)\left(\sum_q h_q b_q + h_q^* b_q^\dagger\right)\frac{e^{-\beta\sum_k \omega_k b_k^\dagger b_k}}{Z_B}\right\}$$

$$= \frac{1}{2\pi}\int_0^\infty \Gamma(\omega)\left[e^{-i\omega\tau}[1+n_B(\omega)] + e^{+i\omega\tau}\cdot n_B(\omega)\right]d\omega$$

$$\tilde\Gamma(\omega) = -\tilde\Gamma(-\omega)$$

$$\tilde\Gamma(|\omega|) = \Gamma(\omega)$$

$$= \frac{1}{2\pi}\int \underbrace{\tilde\Gamma(\omega)[1+n_B(\omega)]}_{\gamma_{11}(\omega)} e^{-i\omega\tau}d\omega$$

$$\Gamma(\omega) = \frac{1}{2\pi}\sum_k |h_k|^2\cdot\delta(\omega-\omega_k)$$

spektrale Dichte

$$\dot\varrho = -i[\Omega a^\dagger a, \varrho(t)] - \left[\int_0^\infty C(\tau)\left[(a+a^\dagger), e^{-i\Omega a^\dagger a\cdot\tau}(a+a^\dagger)e^{+i\Omega a^\dagger a\cdot\tau}\varrho(t)\right]d\tau + h.c.\right]$$

Markov-ME (keine Lindblad-Form)

vernachlässige $e^{\pm 2i\Omega t}$ im WW-Bild

$$\Gamma(\omega) = \int_0^\infty C(\tau)e^{+i\omega\tau}d\tau \Rightarrow \frac{1}{2}\gamma + \frac{i}{2}\sigma = \Gamma(+\Omega)$$

$$\frac{1}{2}\bar\gamma + \frac{i}{2}\bar\sigma = \Gamma(-\Omega)$$

$$\Rightarrow \dot\varrho = -i\left[\frac{\sigma}{2}a^\dagger a + \frac{\bar\sigma}{2}aa^\dagger, \varrho\right] + \gamma\left[a\varrho a^\dagger - \frac{1}{2}\{a^\dagger a,\varrho\}\right] + \bar\gamma\left[a^\dagger\varrho a - \frac{1}{2}\{aa^\dagger,\varrho\}\right]$$

$$\Rightarrow \dot\varrho = -i\left[\Omega a^\dagger a + \left(\frac{\sigma}{2}+\frac{\bar\sigma}{2}\right)a^\dagger a, \varrho\right] + \Gamma(\Omega)[1+n_B(\Omega)]\left[a\varrho a^\dagger - \frac{1}{2}\{a^\dagger a,\varrho\}\right]$$
$$+\Gamma(\Omega)\cdot n_B(\Omega)\left[a^\dagger\varrho a - \frac{1}{2}\{aa^\dagger,\varrho\}\right]$$

$$H_S|n\rangle = n\Omega|n\rangle = E_n|n\rangle$$

$$a^\dagger a = \sum_{n=0}^\infty n\cdot|n\rangle\langle n|$$

$$a = \sum_{n=1}^\infty \sqrt{n}\,|n-1\rangle\langle n|$$

1.3.4. Gleichgewichts-TD

Warum ist $\sum e^{-\beta H_S} = 0$

falls $\varrho_B = \dfrac{e^{-\beta H_B}}{Z_B}$

$$C_{\alpha\beta}(\tau) = C_{\beta\alpha}(-\tau - i\beta)$$

Kubo-Martin-Schwinger Relation

KMS

$$C_{\beta\alpha}(-\tau-i\beta) = \mathrm{Tr}\left\{e^{+iH_B(-\tau-i\beta)}B_\beta\, e^{-iH_B(-\tau-i\beta)}B_\alpha\frac{e^{-\beta H_B}}{Z_B}\right\}$$

$$= \mathrm{Tr}\left\{e^{-\beta H_B}e^{+iH_B\tau}B_\alpha\cancel{\frac{e^{-\beta H_B}}{Z_B}}\cancel{e^{+\beta H_B}}e^{-iH_B\tau}B_\beta\right\} = \mathrm{Tr}\left\{B_\alpha(\tau)B_\beta\frac{e^{-\beta H_B}}{Z_B}\right\}$$

$$= C_{\alpha\beta}(\tau)$$

$$\frac{n_B(\Omega)}{1+n_B(\Omega)} = e^{-\beta\Omega}$$

morgen: 2. Hauptsatz