$\Rightarrow$ nächste Woche: keine Vorlesung

Wdh $\mathcal{L}_\tau^{CG}$ $\quad \dot{\rho}_S = \mathcal{L}_\tau^{CG}\rho_S$ ist gut für $t=\tau$

noch besser $\quad \rho_S(t) = e^{\mathcal{L}_t^{CG}\cdot t}\rho_S^0 \longrightarrow \frac{d}{dt}\rho_S \neq \mathcal{L}_t^{CG}\rho_S$

$$\hookrightarrow \frac{d}{dt}\rho_S = \underbrace{\left(\frac{d}{dt}e^{\mathcal{L}_t^{CG}\cdot t}\right)e^{-\mathcal{L}_t^{CG}\cdot t}}_{\mathcal{L}^{DCG}(t)}\rho_S(t)$$

zeigten $\quad \lim_{\tau\to\infty}\mathcal{L}_\tau^{CG} = \mathcal{L}_{BMS}$

• Bsp. Spin-Boson-Modell

$$H = \underbrace{\Omega\sigma^z + T\sigma^x}_{H_S} + \underbrace{\sigma^z}_{A}\otimes\underbrace{\sum_k\left(h_k b_k + h_k^* b_k^\dagger\right)}_{B} + \underbrace{\sum_k \overbrace{\omega_k}^{\geq 0} b_k^\dagger b_k}_{H_B}$$

$$C(\tau) = \mathrm{Tr}_B\{\tilde{B}(\tau) B \bar{\rho}_B\} = \frac{1}{2\pi}\int\underbrace{\tilde{\Gamma}(\omega)[1+n_B(\omega)]}_{\gamma(\omega)}\cdot e^{-i\omega\tau}d\omega$$

$$= \mathrm{Tr}_B\left\{\left[\sum_k h_k b_k\cdot e^{-i\omega_k\tau} + h_k^* b_k^\dagger e^{+i\omega_k\tau}\right]\left[\sum_q h_q b_q + h_q^* b_q^\dagger\right]\bar{\rho}_B\right\}$$

$$\mathrm{Tr}_B\{b_k b_q^\dagger \bar{\rho}_B\} = \delta_{kq}\,\mathrm{Tr}_B\{(1+b_q^\dagger b_q)\bar{\rho}_B\} = \delta_{kq}\cdot(1+n_B(\omega_q))$$

$$= \sum_k |h_k|^2\left[e^{-i\omega_k\tau}(1+n_B(\omega_k)) + e^{+i\omega_k\tau}\cdot n_B(\omega_k)\right] \qquad \Gamma(\omega) = 2\pi\sum_k|h_k|^2\delta(\omega-\omega_k)$$

$$= \frac{1}{2\pi}\int_0^\infty\Gamma(\omega)\left[[1+n_B(\omega)]e^{-i\omega\tau} + n_B(\omega)e^{+i\omega\tau}\right]d\omega \qquad \Gamma(\omega<0) = 0$$

$$= \frac{1}{2\pi}\int_{-\infty}^{+\infty}\tilde{\Gamma}(\omega)[1+n_B(\omega)]e^{-i\omega\tau}d\omega \quad\Bigg\|\quad n_B(-\omega) = -[1+n_B(\omega)] \qquad \tilde{\Gamma}(-\omega) = -\tilde{\Gamma}(\omega) \qquad \tilde{\Gamma}(|\omega|) \overset{!}{=} \Gamma(|\omega|)$$

a.) exakte Lösung (nur $T=0$)

Entkopplung durch Polaron-Trafo $\quad U_P = \exp\left\{-\sigma^z \sum_k \left(\frac{h_k}{\omega_k} b_k^\dagger - \frac{h_k^*}{\omega_k} b_k\right)\right\}$

$\rightsquigarrow$ Populationen konstant $\quad \tilde{\rho}_{00}(t) = \rho_{00}(0)$

Kohärenzen zerfallen $\quad \tilde{\rho}_{01}(t) = e^{-\gamma(t)} \rho_{01}(0)$

b.) BMS-Mastergleichung

$\gamma_{+-,+-}$: Rate von $|-\rangle \to |+\rangle$ $\quad$ verschwindet für $T=0$

$\gamma_{-+,-+}$: " $\quad |+\rangle \to |-\rangle$

$$\frac{d}{dt}\begin{pmatrix}\rho_{--}\\ \rho_{++}\\ \rho_{-+}\\ \rho_{+-}\end{pmatrix} = \underbrace{\begin{pmatrix}-\gamma_{+-,+-} & +\gamma_{-+,-+} & 0 & 0\\ +\gamma_{+-,+-} & -\gamma_{-+,-+} & 0 & 0\\ 0 & 0 & \xi & 0\\ 0 & 0 & 0 & \xi^*\end{pmatrix}}_{\mathcal{L}_{BMS}}\begin{pmatrix}\rho_{--}\\ \rho_{++}\\ \rho_{-+}\\ \rho_{+-}\end{pmatrix} \qquad Re(\xi) \le 0$$

$$\underbrace{\frac{\gamma_{+-,+-}}{\gamma_{-+,-+}} = e^{-\beta \cdot 2\sqrt{\varepsilon^2+T^2}}}_{\#} \qquad \text{"Dekohärenz"}$$

c.) CG-Mastergleichung : $(T=0)$

$\rightsquigarrow$ $\tilde{A}(t) = \sigma^z$ $\rightsquigarrow$ Lamb shift trägt nicht bei

$$\rightsquigarrow \dot{\rho} = \frac{1}{\tau}\int_0^\tau dt_1 \int_0^\tau dt_2\, C(t_1-t_2)\left[\sigma^z\rho\sigma^z - \rho\right]$$

$$= \frac{1}{2\pi\tau}\int d\omega\, \tilde{\Gamma}(\omega)\left[1+n_B(\omega)\right] \underbrace{\int_0^\tau dt_1 \int_0^\tau dt_2\, e^{-i\omega(t_1-t_2)}}_{\tau^2 \operatorname{sinc}^2\left(\frac{\omega\tau}{2}\right)} \left[\quad\right]$$

$$= \Sigma(\tau)\cdot\left[\sigma^z\rho\sigma^z - \rho\right] \rightsquigarrow \left(\frac{d}{dt}\langle\sigma_z\rangle = Tr\left\{\Sigma(\tau)\,\sigma^z\left(\sigma^z\rho\sigma^z-\rho\right)\right\} = 0\right)$$

$$\frac{d}{dt}\langle\sigma^\pm\rangle = -2\,\Sigma(\tau)\langle\sigma^\pm\rangle \rightsquigarrow \langle\sigma^\pm\rangle_t = e^{-2\Sigma(\tau)\cdot t}\langle\sigma^\pm\rangle_0$$

bei $t=\tau$ $\rightsquigarrow$ exakte Lösung wird reproduziert (nur für pure dephasing)

$$\frac{d}{dt}\langle\sigma^z\rangle = Tr\{\sigma^z\dot{\rho}\} = Tr\{\sigma^z(\mathcal{L}\rho)\}$$

1.3.7 Fermionen

bisher: $H_I = \sum_\alpha A_\alpha \otimes B_\alpha \qquad [A_\alpha \otimes \mathbb{1}, \mathbb{1} \otimes B_\alpha] = 0$

generischer Tunnel-Hamiltonian

$$H_T = \sum_k t_k \hat{d}^+ \hat{c}_k + \sum_k t_k^* \hat{c}_k^+ \hat{d} = d^+ \sum_k t_k c_k - d \sum_k t_k^* c_k^+$$

fermionische Operatoren : sind nicht-lokal

Jordan-Wigner-Trafo : Fermionen $\longrightarrow$ Spin-1/2 Operatoren

Betrachte Fermionen auf N Plätzen

$$C_i = \sigma^z \otimes \dots \otimes \sigma^z \otimes \underset{i}{\sigma^-} \otimes 1 \otimes \dots \otimes \underset{N}{1} \qquad \{c_i, c_j\} = 0$$

$$C_i^+ = \sigma^z \otimes \dots \otimes \sigma^z \otimes \underset{i}{\sigma^+} \otimes 1 \otimes \dots \otimes \underset{N}{1} \qquad \{c_i, c_j^+\} = \delta_{ij} 1$$

- $C_i^2 = 0$ ✓
- $i < j$:

$$c_i = \sigma^z \otimes \dots \otimes \sigma^z \otimes \underset{i}{\sigma^-} \otimes 1 \otimes \dots \otimes 1 \otimes 1 \otimes 1 \otimes \dots \otimes 1$$

$$c_j = \sigma^z \otimes \dots \otimes \sigma^z \otimes \sigma^z \otimes \sigma^z \otimes \dots \otimes \sigma^z \otimes \underset{j}{\sigma^-} \otimes 1 \otimes \dots \otimes 1$$

i : $\sigma^- \sigma^z + \sigma^z \sigma^- = 0 \rightsquigarrow \{c_i, c_j\} = 0$

jetzt $d = \sigma^- \otimes \overbrace{1 \otimes \dots \otimes 1}^{N\times}$

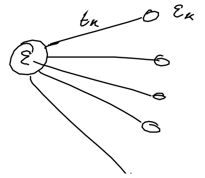
$$C_k = \underset{0}{\sigma^z} \otimes \underbrace{\sigma^z \otimes \dots \otimes \underset{k}{\sigma^-} \otimes 1 \otimes \dots \otimes 1}_{N\times,\ \tilde{c}_k}$$

$$H_T = \underbrace{\sigma^+ \sigma^z}_{-\tilde{d}^+} \otimes \sum_k t_k \tilde{c}_k - \underbrace{\sigma^- \sigma^z}_{\tilde{d}} \otimes \sum_k t_k^* \tilde{c}_k^+ \qquad \sigma^- \sigma^z = \sigma^-$$

jetzt: $[\tilde{d}, \tilde{c}_k] = 0$ $\qquad \tilde{d}^2 = 0$, $\{\tilde{c}_k, \tilde{c}_q^+\} = \delta_{kq}$

∘ beachte $c_k^+ c_k = 1 \otimes \tilde{c}_k^+ \tilde{c}_k$

1.3.8. Single resonant level



$$H = \varepsilon d^+ d + d^+ \sum_k t_k c_k - d \sum_k t_k^* c_k^+ + \sum_k \varepsilon_k c_k^+ c_k \qquad (1)$$

$$= \varepsilon \tilde{d}^+ \tilde{d} - \tilde{d}^+ \otimes \sum_k t_k \tilde{c}_k - \tilde{d} \otimes \sum_k t_k^* \tilde{c}_k^+ + \sum_k \varepsilon_k \tilde{c}_k^+ \tilde{c}_k$$

a.) Exakte Lösung: $t=0 \quad \rho(0) = \rho_S^0 \otimes \bar{\rho}_B$ aus (1)

Heisenberg-Bild $\quad \frac{d}{dt}\tilde{d} = i[H, \tilde{d}]$

$$= i\varepsilon\left(\cancel{\tilde{d}^\dagger\tilde{d}\tilde{d}} - \tilde{d}\underbrace{\tilde{d}^\dagger\tilde{d}}_{1-\tilde{d}\tilde{d}^\dagger}\right) + i\sum_k t_k\left(\tilde{d}^\dagger\tilde{c}_k\tilde{d} - \tilde{d}\tilde{d}^\dagger\tilde{c}_k\right)$$

$$= -i\varepsilon\tilde{d} - i\sum_k t_k\tilde{c}_k$$

$$\frac{d}{dt}\tilde{c}_k = -i\varepsilon_k\tilde{c}_k - i t_k^*\tilde{d}$$

$$D(s) = \int_0^\infty \tilde{d}(t)\, e^{-st}\, dt \qquad C_k(s) = \int_0^\infty \tilde{c}_k(t)\, e^{-st}\, dt$$

$$\to s\cdot D(s) - \hat{d} = -i\varepsilon D(s) - i\sum_k t_k C_k(s) \longleftarrow \text{einsetzen}$$

$$s\cdot C_k(s) - \hat{c}_k = -i\varepsilon_k C_k(s) - i t_k D(s) \longrightarrow \text{löse nach } C_k(s)$$

$$n_d(t) = \mathrm{Tr}\{\tilde{d}^\dagger(t)\tilde{d}(t)\rho_0\} = \qquad d(t) = \frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty} D(s)\, e^{+s\cdot t}\, ds$$

$\mathrm{Tr}\{d^\dagger d\,\rho_0\}$ $\qquad$ $\mathrm{Tr}\{c_k^\dagger c_k \rho_0\} = f(\varepsilon_k)$

$$= \ldots = n_0\cdot e^{-\Gamma t} + \frac{1}{2\pi}\int d\omega\left[1 + e^{-\Gamma t} - 2\cos((\omega-\varepsilon)t)\, e^{-\Gamma t/2}\right]\frac{4\Gamma f(\omega)}{\Gamma^2 + 4(\omega-\varepsilon)^2}$$

$$\Gamma(\omega) = 2\pi\sum_k |t_k|^2\delta(\omega-\varepsilon_k) \longrightarrow \Gamma$$

- $t=0: \; n_d(0) = n_0$ ✓
- $t\to\infty: \; \bar{n}_d \to \frac{1}{2\pi}\int d\omega \frac{4\Gamma f(\omega)}{\Gamma^2+4(\omega-\varepsilon)^2}$

$$\lim_{\Gamma\to 0}\frac{4\Gamma}{\Gamma^2+4(\omega-\varepsilon)^2} = 2\pi\cdot\delta(\omega-\varepsilon)$$

$$\lim_{\Gamma\to 0}\bar{n}_d = f(\varepsilon) \qquad \text{System thermalisiert}$$

Energie des System-Quantenpunktes

b.) Coarse-graining ME: $A_1 = \tilde{d}^\dagger \quad A_2 = \tilde{d}$

$B_1 = -\sum_k t_k\tilde{c}_k \quad B_2 = -\sum_k t_k^*\tilde{c}_k^\dagger$

$$C_{11}(\tau) = 0 = C_{22}(\tau)$$

$$C_{12}(\tau) = \mathrm{Tr}\left\{\left(\sum_k t_k\tilde{c}_k e^{-i\varepsilon_k\tau}\right)\left(\sum_q t_q^*\tilde{c}_q^\dagger\right)\bar{\rho}_B\right\} = \sum_k |t_k|^2 e^{-i\varepsilon_k\tau}(1-f(\varepsilon_k))$$

$$C_{21}(\tau) = \sum_k |t_k|^2 e^{+i\varepsilon_k\tau} f(\varepsilon_k) \qquad \Gamma(\omega) = 2\pi\sum_k |t_k|^2\delta(\omega-\varepsilon_k) \quad \text{spektr. Dichte}$$

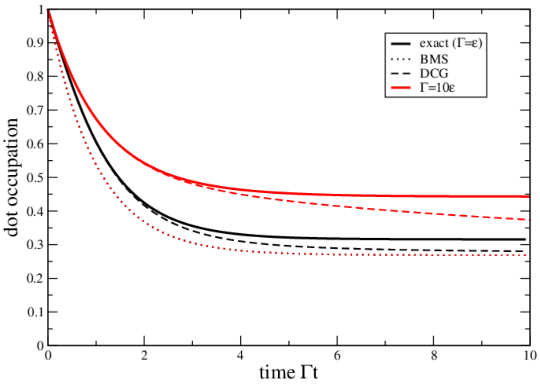
$$\gamma_{12}(\omega) = \Gamma(\omega)[1-f(\omega)] \qquad \gamma_{21}(\omega) = \Gamma(-\omega)f(-\omega)$$

$\rightsquigarrow \sigma_{12}(\omega) \qquad \rightsquigarrow \sigma_{21}(\omega)$

$$\longrightarrow \dot{\tilde{\rho}}_S = -i\left[(\ldots)d^\dagger d + (\ldots)dd^\dagger, \tilde{\rho}_S\right]$$
$$+ \frac{1}{2\pi}\int\gamma_{12}(\omega)\,\tau\cdot\mathrm{sinc}^2\left[\frac{(\varepsilon-\omega)\tau}{2}\right]\left[d\rho d^\dagger - \frac{1}{2}d^\dagger d\rho - \frac{1}{2}\rho d^\dagger d\right]$$
$$+ \frac{1}{2\pi}\int\gamma_{21}(\omega)\,\tau\,\mathrm{sinc}^2\left[\frac{(\varepsilon+\omega)\tau}{2}\right]\left[d^\dagger\rho d - \frac{1}{2}dd^\dagger\rho - \frac{1}{2}\rho dd^\dagger\right]$$

$\langle 0|\dot{\rho}_S|0\rangle = R_{1\to 0}(\tau)\langle 1|\rho_S|1\rangle - R_{0\to 1}(\tau)\langle 0|\rho_S|0\rangle$ DCG: $t=\tau$

analog $\langle 1|\dot{\rho}_S|1\rangle$ BMS: $\tau \to \infty$



$\gamma_{12}(\omega) = \Gamma(\omega)\,[1 - f(\omega)]$

$\gamma_{21}(\omega) = \Gamma(\omega)\cdot f(\omega)$