$\Rightarrow$ nächste Woche: keine Vorlesung

Wdh $\mathcal{L}_\tau^{CG}$ $\quad \dot\rho_S = \mathcal{L}_\tau^{CG}\rho_S$ ist gut für $t=\tau$

noch besser $\rho_S(t) = e^{\mathcal{L}_t^{CG}\cdot t}\rho_S^0 \longrightarrow \frac{d}{dt}\rho_S \neq \mathcal{L}_t^{CG}\rho_S$

$$\hookrightarrow \frac{d}{dt}\rho_S = \underbrace{\left(\frac{d}{dt} e^{\mathcal{L}_t^{CG}\cdot t}\right) e^{-\mathcal{L}_t^{CG}\cdot t}}_{\mathcal{L}^{DCG}(t)} \rho_S(t)$$

zeigten $\lim_{\tau\to\infty}\mathcal{L}_\tau^{CG} = \mathcal{L}_{BMS}$

- Bsp. Spin-Boson-Modell

$$H = \underbrace{\Omega\sigma^z + T\sigma^x}_{H_S} + \underbrace{\sigma^z}_{A}\otimes\underbrace{\sum_k\left(h_k b_k + h_k^* b_k^\dagger\right)}_{B} + \underbrace{\sum_k \overset{\geq 0}{\omega_k} b_k^\dagger b_k}_{H_B}$$

$$C(\tau) = \mathrm{Tr}_B\{\tilde B(\tau) B \bar\rho_B\} = \frac{1}{2\pi}\int \underbrace{\Gamma(\omega)[1+n_B(\omega)]}_{\gamma(\omega)}\cdot e^{-i\omega\tau} d\omega$$

$$= \mathrm{Tr}_B\left\{\left[\sum_k h_k b_k\cdot e^{-i\omega_k\tau} + h_k^* b_k^\dagger e^{+i\omega_k\tau}\right]\left[\sum_q h_q b_q + h_q^* b_q^\dagger\right]\rho_B\right\}$$

$$\mathrm{Tr}_B\{b_k b_q^\dagger \rho_B\} = \delta_{kq}\,\mathrm{Tr}_B\{(1+b_q^\dagger b_q)\rho_B\} = \delta_{kq}\cdot(1+n_B(\omega_q))$$

$$= \sum_k |h_k|^2\left[e^{-i\omega_k\tau}(1+n_B(\omega_k)) + e^{+i\omega_k\tau}\cdot n_B(\omega_k)\right] \qquad \Gamma(\omega) = 2\pi\sum_k |h_k|^2\delta(\omega-\omega_k)$$

$$= \frac{1}{2\pi}\int_0^\infty \Gamma(\omega)\left[[1+n_B(\omega)]e^{-i\omega\tau} + n_B(\omega)e^{+i\omega\tau}\right]d\omega \qquad \Gamma(\omega<0) = 0$$

$$= \frac{1}{2\pi}\int_{-\infty}^{+\infty}\tilde\Gamma(\omega)\,[1+n_B(\omega)]\,e^{-i\omega\tau}d\omega \;\Bigg\|\; \begin{array}{l} n_B(-\omega) = -[1+n_B(\omega)] \\ \tilde\Gamma(-\omega) = -\tilde\Gamma(\omega)\end{array} \qquad \tilde\Gamma(|\omega|) \stackrel{!}{=} \Gamma(|\omega|)$$

a.) exakte Lösung (nur $T=0$)

Entkopplung durch Polaron-Trafo $\quad U_P = \exp\left\{-\sigma^z \sum_k \left(\frac{h_k}{\omega_k} b_k^\dagger - \frac{h_k^*}{\omega_k} b_k\right)\right\}$

$\rightsquigarrow$ Populationen konstant $\quad \tilde\rho_{00}(t) = \rho_{00}(0)$

Kohärenzen zerfallen $\quad \tilde\rho_{01}(t) = e^{-\gamma(t)} \rho_{01}(0)$

b.) BMS-Mastergleichung

$\gamma_{+-,+-}$: Rate von $|-\rangle \to |+\rangle$ $\quad$ verschwindet für $T=0$

$\gamma_{-+,-+}$: " $\quad |+\rangle \to |-\rangle$

$$\frac{d}{dt}\begin{pmatrix}\rho_{--}\\ \rho_{++}\\ \rho_{-+}\\ \rho_{+-}\end{pmatrix} = \underbrace{\begin{pmatrix}-\gamma_{+-,+-} & +\gamma_{-+,-+} & 0 & 0\\ +\gamma_{+-,+-} & -\gamma_{-+,-+} & 0 & 0\\ 0 & 0 & \xi & 0\\ 0 & 0 & 0 & \xi^*\end{pmatrix}}_{\mathcal{L}_{BMS}}\begin{pmatrix}\rho_{--}\\ \rho_{++}\\ \rho_{-+}\\ \rho_{+-}\end{pmatrix} \qquad \mathrm{Re}(\xi) \le 0$$

$$\frac{\gamma_{+-,+-}}{\gamma_{-+,-+}} = e^{-\beta\cdot 2\sqrt{\varepsilon^2+T^2}} \qquad \text{"Dekohärenz"}$$

c.) CG-Mastergleichung : $(T=0)$

$\rightsquigarrow \tilde A(t) = \sigma^z \longrightarrow$ Lamb shift trägt nicht bei

$$\rightsquigarrow \dot\rho = \frac{1}{\tau}\int_0^\tau dt_1 \int_0^\tau dt_2\, C(t_1-t_2)\left[\sigma^z\rho\sigma^z - \rho\right]$$

$$= \frac{1}{2\pi\tau}\int d\omega\, \Gamma(\omega)\left[1+n_B(\omega)\right] \underbrace{\int_0^\tau dt_1 \int_0^\tau dt_2\, e^{-i\omega(t_1-t_2)}}_{\tau^2 \operatorname{sinc}^2\left(\frac{\omega\tau}{2}\right)} \left[\quad\right]$$

$$= \Sigma(\tau)\cdot\left[\sigma^z\rho\sigma^z - \rho\right] \longrightarrow \left(\frac{d}{dt}\langle\sigma_z\rangle = \mathrm{Tr}\left\{\Sigma(\tau)\,\sigma^z\left(\sigma^z\rho\sigma^z - \rho\right)\right\} = 0\right)$$

$$\frac{d}{dt}\langle\sigma^\pm\rangle = -2\Sigma(\tau)\langle\sigma^\pm\rangle \rightsquigarrow \langle\sigma^\pm\rangle_t = e^{-2\Sigma(\tau)\cdot t}\langle\sigma^\pm\rangle_0$$

bei $t=\tau$ $\rightsquigarrow$ exakte Lösung wird reproduziert (nur für pure dephasing)

$$\frac{d}{dt}\langle\sigma^z\rangle = \mathrm{Tr}\{\sigma^z\dot\rho\} = \mathrm{Tr}\{\sigma^z(\mathcal{L}\rho)\}$$

1.3.7 Fermionen

bisher: $H_I = \sum_\alpha A_\alpha \otimes B_\alpha \qquad [A_\alpha\otimes\mathbb{1}, \mathbb{1}\otimes B_\alpha] = 0$

generischer Tunnel-Hamiltonian

$$H_T = \sum_k t_k \hat{d}^\dagger \hat{c}_k + \sum_k t_k^* \hat{c}_k^\dagger \hat{d} = d^\dagger \sum_k t_k c_k - d \sum_k t_k^* c_k^\dagger$$

fermionische Operatoren : sind nicht-lokal

$$\boxed{\begin{array}{l}\text{Jordan-Wigner-Trafo : Fermionen} \longrightarrow \text{Spin-}\tfrac{1}{2}\text{ Operatoren}\\ \text{Betrachte Fermionen auf } N \text{ Plätzen}\\ \left.\begin{array}{l} c_i = \sigma^z\otimes\dots\otimes\sigma^z\otimes\underset{i}{\sigma^-}\otimes 1\otimes\dots\otimes\underset{N}{1}\\ c_i^\dagger = \sigma^z\otimes\dots\otimes\sigma^z\otimes\sigma^+\otimes 1\otimes\dots\otimes \underset{N}{1}\end{array}\right\} \begin{array}{l}\{c_i, c_j\} = 0\\ \{c_i, c_j^\dagger\} = \delta_{ij}1\end{array}\end{array}}$$

- $c_i^2 = 0$ ✓
- $i<j$:

$$c_i = \sigma^z\otimes\dots\otimes\sigma^z\otimes\underset{i}{\sigma^-}\otimes 1\otimes\dots\otimes 1\otimes\underset{j}{1}\otimes 1\otimes\dots\otimes 1$$
$$c_j = \sigma^z\otimes\dots\otimes\sigma^z\otimes\sigma^z\otimes\sigma^z\otimes\dots\otimes\sigma^z\otimes\sigma^-\otimes 1\otimes\dots\otimes 1$$

i: $\sigma^-\sigma^z + \sigma^z\sigma^- = 0 \rightsquigarrow \{c_i, c_j\} = 0$

jetzt $d = \sigma^-\otimes\underbrace{1\otimes\dots\otimes 1}_{N\times} = \sigma^- \otimes 1$, $\tilde{c}_k$

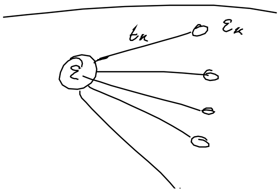
$$c_k = \underset{0}{\sigma^z}\otimes\underbrace{\sigma^z\otimes\dots\otimes\underset{k}{\sigma^-}\otimes 1\otimes\dots\otimes 1}_{N\times}$$

$$H_T = \underbrace{\sigma^+\sigma^z}_{-\tilde{d}^\dagger}\otimes\sum_k t_k\tilde{c}_k - \underbrace{\sigma^-\sigma^z}_{\tilde{d}}\otimes\sum_k t_k^*\tilde{c}_k^\dagger \qquad \sigma^-\sigma^z = \sigma^-$$

jetzt: $[\tilde{d}, \tilde{c}_k] = 0$ $\qquad \tilde{d}^2 = 0$, $\{\tilde{c}_k, \tilde{c}_q^\dagger\} = \delta_{kq}$

- beachte $c_k^\dagger c_k = 1\otimes\tilde{c}_k^\dagger\tilde{c}_k$

1.3.8. Single resonant level



$$H = \varepsilon d^\dagger d + d^\dagger\sum_k t_k c_k - d\sum_k t_k^* c_k^\dagger + \sum_k \varepsilon_k c_k^\dagger c_k \qquad (1)$$
$$= \varepsilon\tilde{d}^\dagger\tilde{d} - \tilde{d}^\dagger\otimes\sum_k t_k\tilde{c}_k - \tilde{d}\otimes\sum_k t_k^*\tilde{c}_k^\dagger + \sum_k \varepsilon_k\tilde{c}_k^\dagger\tilde{c}_k$$

a.) Exakte Lösung : $t=0$ $\rho(0) = \rho_S^0 \otimes \bar{\rho}_B$ aus (1)

Heisenberg-Bild $\quad \frac{d}{dt}\tilde{d} = i[H, \tilde{d}]$

$$= i\varepsilon\left(\cancel{\tilde{d}^\dagger\tilde{d}\tilde{d}} - \tilde{d}\underbrace{\tilde{d}^\dagger\tilde{d}}_{1-\tilde{d}\tilde{d}^\dagger}\right) + i\sum_k t_k\left(\tilde{d}^\dagger\tilde{c}_k\tilde{d} - \tilde{d}\tilde{d}^\dagger\tilde{c}_k\right)$$

$$= -i\varepsilon\tilde{d} - i\sum_k t_k\tilde{c}_k$$

$$\frac{d}{dt}\tilde{c}_k = -i\varepsilon_k\tilde{c}_k - it_k^*\tilde{d}$$

$$D(s) = \int_0^\infty \tilde{d}(t)e^{-st}dt \qquad C_k(s) = \int_0^\infty \tilde{c}_k(t)e^{-st}dt$$

$$s\cdot D(s) - \hat{d} = -i\varepsilon D(s) - i\sum_k t_k C_k(s) \xleftarrow{\text{einsetzen}}$$

$$s\cdot C_k(s) - \hat{c}_k = -i\varepsilon_k C_k(s) - it_k D(s) \longrightarrow \text{löse nach } C_k(s)$$

$$n_d(t) = Tr\{\tilde{d}^\dagger(t)\tilde{d}(t)\rho_0\} = \qquad d(t) = \frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty} D(s)e^{+s\cdot t}ds$$

$Tr\{d^\dagger d\rho_0\}$ $\qquad$ $Tr\{c_k^\dagger c_k\rho_0\} = f(\varepsilon_k)$

$$= \ldots = n_0\cdot e^{-\Gamma\cdot t} + \frac{1}{2\pi}\int d\omega\left[1 + e^{-\Gamma t} - 2\cos[(\omega-\varepsilon)t]e^{-\Gamma t/2}\right]\frac{4\Gamma f(\omega)}{\Gamma^2 + 4(\omega-\varepsilon)^2}$$

$$\Gamma(\omega) = 2\pi\sum_k |t_k|^2\delta(\omega-\varepsilon_k) \longrightarrow \Gamma$$

- $t=0$: $n_d(0) = n_0$ ✓
- $t\to\infty$: $\bar{n}_d \to \frac{1}{2\pi}\int d\omega \frac{4\Gamma f(\omega)}{\Gamma^2 + 4(\omega-\varepsilon)^2}$

$$\lim_{\Gamma\to 0}\frac{4\Gamma}{\Gamma^2 + 4(\omega-\varepsilon)^2} = 2\pi\cdot\delta(\omega-\varepsilon)$$

$$\lim_{\Gamma\to 0}\bar{n}_d = f(\varepsilon) \qquad \text{System thermalisiert}$$

Energie des System-Quantenpunktes

b.) Coarse-graining ME : $A_1 = \tilde{d}^\dagger$ $\quad A_2 = \tilde{d}$

$B_1 = -\sum_k t_k\tilde{c}_k$ $\quad B_2 = -\sum_k t_k^*\tilde{c}_k^\dagger$

$$C_{11}(\tau) = 0 = C_{22}(\tau)$$

$$C_{12}(\tau) = Tr\left\{\left(\sum_k t_k\tilde{c}_k e^{-i\varepsilon_k\cdot\tau}\right)\left(\sum_q t_q^*\tilde{c}_q^\dagger\right)\bar{\rho}_B\right\} = \sum_k |t_k|^2 e^{-i\varepsilon_k\tau}(1-f(\varepsilon_k))$$

$$C_{21}(\tau) = \sum_k |t_k|^2 e^{+i\varepsilon_k\cdot\tau}\cdot f(\varepsilon_k) \qquad \Gamma(\omega) = 2\pi\sum_k |t_k|^2\delta(\omega-\varepsilon_k) \quad \text{spektr. Dichte}$$

$$\gamma_{12}(\omega) = \Gamma(\omega)[1-f(\omega)] \qquad \gamma_{21}(\omega) = \Gamma(-\omega)f(-\omega)$$

$\rightsquigarrow \sigma_{12}(\omega)$ $\qquad$ $\rightsquigarrow \sigma_{21}(\omega)$

$$\longrightarrow \dot{\tilde{\rho}}_S = -i[(\ldots)d^\dagger d + (\ldots)dd^\dagger, \tilde{\rho}_S]$$

$$+\frac{1}{2\pi}\int\gamma_{12}(\omega)\,\tau\cdot\operatorname{sinc}^2\left[\frac{(\omega-\varepsilon)\cdot\tau}{2}\right]\left[d\rho d^\dagger - \frac{1}{2}d^\dagger d\rho - \frac{1}{2}\rho d^\dagger d\right]$$

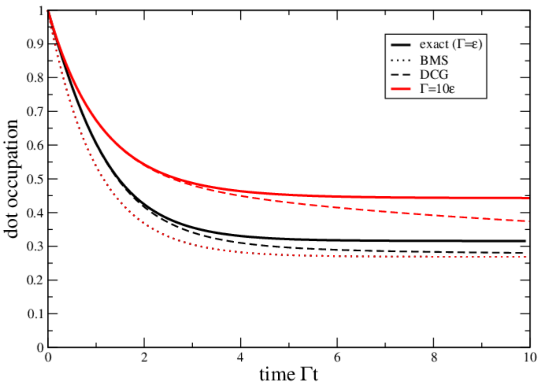
$$+\frac{1}{2\pi}\int\gamma_{21}(\omega)\,\tau\operatorname{sinc}^2\left[\frac{(\varepsilon+\omega)\cdot\tau}{2}\right]\left[d^\dagger\rho d - \frac{1}{2}dd^\dagger\rho - \frac{1}{2}\rho dd^\dagger\right]$$

$$\langle 0|\dot{\rho}_S|0\rangle = R_{1\to 0}(\tau)\langle 1|\rho_S|1\rangle - R_{0\to 1}(\tau)\langle 0|\rho_S|0\rangle$$

analog $\langle 1|\dot{\rho}_S|1\rangle$

DCG: $t=\tau$

BMS: $\tau\to\infty$



$$\gamma_{12}(\omega) = \Gamma(\omega)\,[1-f(\omega)]$$
$$\gamma_{21}(\omega) = \Gamma(\omega)\cdot f(\omega)$$