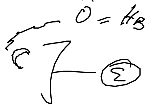
Wdh

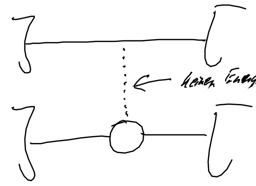
- SRL $\hat{O} = H_B = \sum_k \varepsilon_k c_k^\dagger c_k$



$$\rightsquigarrow \mathcal{L}(\xi) = \Gamma \begin{pmatrix} -f & +(1-f) e^{+i\xi\cdot\varepsilon} \\ f e^{-i\xi\cdot\varepsilon} & -(1-f) \end{pmatrix}$$

$\rightsquigarrow$ Methode ist konsistent

- SET



QPC

4-Terminal System

kein Energie-Austausch

SET

$$A = \mathbb{1} - \alpha d^\dagger d \quad \rightsquigarrow \quad \mathcal{L}_{QPC}(\xi) = \begin{pmatrix} 1 & 0 \\ 0 & |1-\alpha|^2 \end{pmatrix} \left[\gamma^{\xi}(0) - \gamma^{0}(0)\right]$$

$\mathbb{1}$: Unterdrückungsfaktor

ZF; $\gamma^{\xi}(0)$: Energie-Übertrag

$$\gamma^{\xi}(0) = \frac{e^{-i\xi}}{2\pi} \int T(\omega,\omega) \left[1 - f_L(\omega)\right] \cdot f_R(\omega)\, d\omega + \frac{e^{+i\xi}}{2\pi} \int T(\omega,\omega) \cdot f_L(\omega) \left[1 - f_R(\omega)\right] d\omega$$

betr. $\alpha = 0$

$$C(\xi,t) = \ln \mathrm{Tr}\left\{ e^{\mathcal{L}_{QPC}(\xi)\cdot t} \rho_0 \right\} = \underbrace{\left[\gamma^{\xi}(0) - \gamma^{0}(0)\right]}_{C_I(\xi)} \cdot t$$

$$I = (-i\partial_\xi)\, C_I(\xi)\Big|_{\xi=0} = \frac{1}{2\pi} \int T(\omega,\omega) \left\{ f_L(\omega)\left[1 - \cancel{f_R(\omega)}\right] - \left[1 - \cancel{f_L(\omega)}\right] \cdot f_R(\omega) \right\} d\omega$$

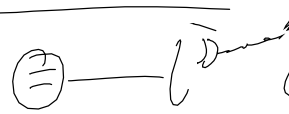
$$= \frac{1}{2\pi} \int T(\omega,\omega) \left[f_L(\omega) - f_R(\omega) \right] d\omega \qquad \text{"Landauer Formel"}$$

$T(\omega,\omega)$: Transmission (hier nur perturbativ in t_{kq})

allg
$$\mathcal{L}_{tot}(\xi) = \sum_{\alpha \in \{L,R\}} \Gamma_\alpha \begin{pmatrix} -f_\alpha & +(1-f_\alpha) \\ +f_\alpha & -(1-f_\alpha) \end{pmatrix} + \mathcal{L}_{QPC}(\xi)$$

$\rightsquigarrow$ Interpretation als Ladungsdetektor

3.4.3. Pure-Dephasing



Welche Energie wird in das Bad abgestrahlt

$\hat{O} = H_B$

$$H = \underbrace{\Omega \sigma^z}_{H_S} + \underbrace{\sigma^z}_{A} \otimes \underbrace{\sum_k \left(h_k b_k + h_k^* b_k^\dagger\right)}_{B} + \underbrace{\sum_k \omega_k b_k^\dagger b_k}_{H_B}$$

ex. Lsg: · Populationen konstant
· Kohärenzen zerfallen

(CG liefert die ex. Lösung für $\tau = t$
$$\rho(t) = e^{\mathcal{L}_t \cdot t} \rho_0 = \rho_{ex}(t))$$

a.) Mastergl. mit ZF

$$C^{\chi}(t) = \mathrm{Tr}\left\{ e^{-iH_B\cdot\chi} \left(\sum_k h_k b_k e^{-i\omega_k\cdot t} + h_k^* b_k^\dagger e^{+i\omega_k\cdot t}\right) e^{+iH_B\chi} \left(\sum_q h_q b_q + h_q^* b_q^\dagger\right) \bar\rho_B \right\}$$

$$= \sum_k |h_k|^2 \left[e^{+i\omega_k\cdot\chi} e^{-i\omega_k\cdot t} (1+n_B(\omega_k)) + e^{-i\omega_k\cdot\chi} e^{+i\omega_k\cdot t} n_B(\omega_k) \right]$$

$$= \frac{1}{2\pi}\int_0^\infty \Gamma(\omega) \left[e^{+i\omega\chi - i\omega t} [1+n_B(\omega)] + e^{-i\omega\chi + i\omega t} \underset{\omega\to-\omega}{n_B(\omega)} \right] d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^\infty \Gamma(\omega) [1+n_B(\omega)] \Theta(\omega) e^{+i\omega\chi} e^{-i\omega t} d\omega$$

$$+ \frac{1}{2\pi}\int_{-\infty}^\infty \Gamma(-\omega)\cdot n_B(-\omega) \Theta(-\omega) e^{+i\omega\chi} e^{-i\omega t} d\omega$$

$$\rightsquigarrow \gamma^{\chi}(\omega) = \Gamma(\omega)[1+n_B(\omega)]\cdot\Theta(\omega) e^{+i\omega\chi} + \Gamma(-\omega) n_B(-\omega)\Theta(-\omega) e^{+i\omega\chi}$$

$\rightarrow$ CG-Mastergl. τ τ $\qquad e^{+iH_S t}\sigma^z e^{-iH_S t} = \sigma^z$

$$\dot{\tilde\rho}_S = \underset{\text{CG-Zeitskala}}{\frac{1}{2\pi\tau}} \int_0^\tau dt_1 \int_0^\tau dt_2 \int d\omega \Big\{ \Theta(\omega)\Gamma(\omega)[1+n_B(\omega)] e^{-i\omega(t_1-t_2)} \left[e^{+i\omega\chi} \sigma^z \tilde\rho \sigma^z - \tilde\rho_S \right]$$

$$\Theta(-\omega)\Gamma(-\omega) n_B(-\omega) e^{-i\omega(t_1-t_2)} \left[e^{+i\omega\chi} \sigma^z \tilde\rho_S \sigma^z - \tilde\rho_S \right]$$

$$\dot{\tilde\rho}_S = [\gamma_-(\chi,\tau)\sigma^z\tilde\rho_S\sigma^z - \gamma_-(0,\tau)\tilde\rho_S] + [\gamma_+(\chi,\tau)\sigma^z\tilde\rho_S\sigma^z - \gamma_+(0,\tau)\tilde\rho_S]$$

$$\dot\rho_{00} = [\gamma_-(\chi,\tau) - \gamma_-(0,\tau) + \gamma_+(\chi,\tau) - \gamma_+(0,\tau)]\rho_{00}$$

$$\dot\rho_{11} = [\quad - \;\text{''}\; - \quad]\rho_{11} \qquad \dot\rho_{01} = [\ldots]\rho_{01}$$

$$C(\chi,t) = \ln \mathrm{Tr}\{ e^{Z(\chi)\cdot t} \rho_0 \}$$

$$= \ln (1,1,0,0) \left(e^{Z(\chi)\cdot t} \right) \begin{pmatrix} \rho_{00}^0 \\ \rho_{11}^0 \\ \rho_{01}^0 \\ \rho_{10}^0 \end{pmatrix}$$

$$= \ln e^{[\ldots]\cdot t} \underbrace{(1,1)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} \rho_{00}^0 \\ \rho_{11}^0 \end{pmatrix}} = \underline{[\ldots]\cdot t}$$

$$\Delta E(t) = -i\partial_\chi C(\chi,t)|_{\chi=0} = \int d\omega\, \Theta(\omega)\cdot\omega\, \Gamma(\omega)[1+n_B(\omega)] \frac{\tau\cdot t}{2\pi}\operatorname{sinc}^2\left[\frac{\omega\tau}{2}\right] d\omega$$

$$+ \int d\omega\, \Theta(-\omega)\cdot\omega\, \Gamma(-\omega) n_B(-\omega) \frac{\tau\cdot t}{2\pi}\operatorname{sinc}^2\left[\frac{\omega\tau}{2}\right] d\omega$$

$$\Delta E(\tau) \overset{\tau=t}{=} \boxed{\frac{2}{\pi}\int_0^\infty \frac{\Gamma(\omega)}{\omega}\sin^2\left(\frac{\omega\cdot\tau}{2}\right) d\omega = \Delta E(\tau)}$$

b) exakte Lösung

Heisenberg-Bild

$$\rho(t) = \rho_S^0 \otimes \bar{\rho}_B$$

$$\tilde{\sigma}^z = e^{+iH_{tot}t}\,\sigma^z\,e^{-iH_{tot}t}$$

$$\frac{d}{dt}\tilde{\sigma}^z = i\,e^{+iH_{tot}t}[H_{tot},\sigma^z]\,e^{-iH_{tot}t} = 0 \;\rightarrow\; \tilde{\sigma}^z = \sigma^z$$

$$\left.\begin{aligned}\frac{d}{dt}\tilde{b}_k &= -i\omega_k\tilde{b}_k - i h_k^*\sigma^z\\ \frac{d}{dt}\tilde{b}_k^\dagger &= +i\omega_k\tilde{b}_k^\dagger + i h_k\sigma^z\end{aligned}\right\}\quad \begin{aligned}&\tilde{b}_k(t) = b_k\,e^{-i\omega_k t} + \frac{h_k^*}{\omega_k}\sigma^z\left(e^{-i\omega_k t}-1\right)\\ &\text{analog } \tilde{b}_k^\dagger\end{aligned}$$

$$\langle E\rangle_t = \sum_k \omega_k \,\mathrm{Tr}\left\{\left[b_k^\dagger e^{+i\omega_k t} + \frac{h_k}{\omega_k}\sigma^z\left(e^{+i\omega_k t}-1\right)\right]\left[b_k e^{-i\omega_k t} + \frac{h_k^*}{\omega_k}\sigma^z\left(e^{-i\omega_k t}-1\right)\right]\rho_S^0\otimes\bar{\rho}_B\right\}$$

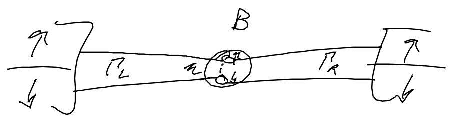
$$= \underbrace{\sum_k \omega_k\,\mathrm{Tr}_B\left\{b_k^\dagger b_k\,\bar{\rho}_B\right\}}_{\langle E\rangle_0} + \underbrace{\sum_k \frac{|h_k|^2}{\omega_k^{\cancel{2}}}\cdot\cancel{\omega_k}\left(2-2\cos(\omega_k t)\right)}_{\langle \Delta E\rangle_t}$$

$$\langle \Delta E\rangle_t = \frac{2}{\pi}\int_0^\infty \frac{\Gamma(\omega)}{\omega}\sin^2\left(\frac{\omega_k t}{2}\right)d\omega$$

konstant für $t \gg \tau$ Energie kommt aus der Kopplung

3.4.4 Beispiel Spin-Ströme

lokales Magnetfeld



$$H_S = (\varepsilon + b)\,d_\uparrow^\dagger d_\uparrow + (\varepsilon - b)\,d_\downarrow^\dagger d_\downarrow + U\,d_\uparrow^\dagger d_\uparrow d_\downarrow^\dagger d_\downarrow$$

Onsite-Energie Zeeman-Splitting Coulomb-WW

$$H_I = \sum_k \sum_{\nu\in\{L,R\}} \sum_{\sigma\in\{\uparrow,\downarrow\}} \left[t_{k\nu}\,d_\sigma^\dagger c_{k\nu\sigma} + h.c.\right]$$

$$H_B = \sum_k \sum_\nu \sum_\sigma \varepsilon_{k\nu}\,c_{k\nu\sigma}^\dagger c_{k\nu\sigma}$$

$$\hat{O} = S_R = \sum_k \left(c_{kR\uparrow}^\dagger c_{kR\uparrow} - c_{kR\downarrow}^\dagger c_{kR\downarrow}\right)$$

$$e^{+i\alpha c^\dagger c}\,c\,e^{-i\alpha c^\dagger c} = c\,e^{-i\alpha}$$

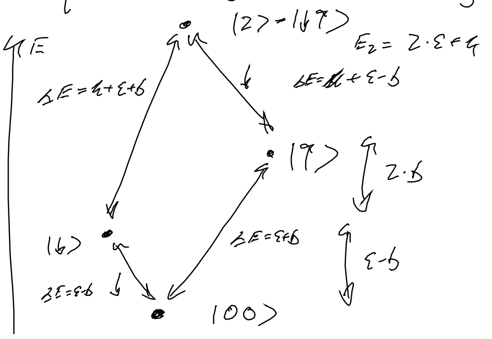
$$\circ\; e^{-iS\chi} e^{+iH_B t}\, c_{kR\uparrow}\, e^{-iH_B t} e^{+iS\chi} = c_{kR\uparrow} \cdot e^{-i\varepsilon_{kR} t} e^{+i\chi}$$

$$[\;\;]\, c_{kR\downarrow}\, [\;\;] = c_{kR\downarrow}\, e^{-i\varepsilon_{kR} t} e^{-i\chi}$$

$$[\;\;]\, c_{kL\uparrow}\, [\;\;] = c_{kL\uparrow} \cdot e^{-i\varepsilon_{kL} t}$$

$$[\;\;]\, c_{kL\downarrow}\, [\;\;] = c_{kL\downarrow}\, e^{-i\varepsilon_{kL} t}$$

Erzeuger analog

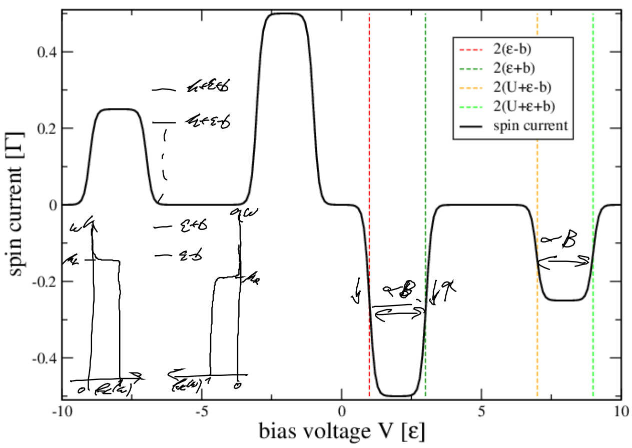


$$Z(\chi)$$

$$Z(0)\,\bar\rho = 0 \rightsquigarrow \bar\rho$$

$$I = -i\,\mathrm{Tr}\{Z'(0)\,\bar\rho\}$$

z.B. $I\left(\mu_L = +\frac{V}{2}, \mu_R = -\frac{V}{2}\right)$



Spinventil