English Summary:

<u>Magnetization</u> $\underline{M}(\underline{r},t) = \frac{1}{\Delta V}\int_{\Delta V} d^3s\, \underline{M}_m(\underline{r}+\underline{s},t)$

microscopic magnetic dipole density $\underline{M}_m(\underline{r},t) = \sum_i \underline{m}_i(t)\,\delta(\underline{r}-\underline{r}_i)$

$\boxed{\underline{\nabla}\times\underline{M} = \underline{j}_M}$ magnetization current density

magnetic field $\boxed{\underline{H}(\underline{r},t) = \frac{1}{\mu_0}\underline{B}' - \underline{M}}$, $\boxed{\underline{\nabla}\times\underline{H} = \underline{j}}$ free current density

$\boxed{\frac{\partial}{\partial t}\underline{P} = \underline{j}_P}$ polarization current density (contributes to displacement current)

$$\left.\begin{aligned} \Box\underline{A} &= -\mu_0(\underline{j}+\underline{j}_P+\underline{j}_M) \\ \Box\phi &= -\frac{1}{\epsilon_0}(\rho+\rho_P) \end{aligned}\right\}\ \text{Lorenz gauge}$$

$\Rightarrow$ Maxwell's eqs.

$$\boxed{\begin{aligned} \underline{\nabla}\times\underline{E} + \frac{\partial}{\partial t}\underline{B} &= 0 \\ \underline{\nabla}\cdot\underline{B} &= 0 \\ \underline{\nabla}\cdot\underline{D} &= \rho \\ \underline{\nabla}\times\underline{H} - \frac{\partial}{\partial t}\underline{D} &= \underline{j} \end{aligned}} \qquad \boxed{\begin{aligned} \underline{D} &= \epsilon_0\underline{E} + \underline{P} \\ \underline{H} &= \frac{1}{\mu_0}\underline{B} - \underline{M} \end{aligned}}$$

Die Feldgln. (1) – (6) sind nicht vollständig!
Ergänzung durch <u>Materialgleichungen</u> notwendig.
(Zus.hang $\underline{P} \leftrightarrow \underline{E}$, $\underline{M} \leftrightarrow \underline{B}$)

Einfachster Fall:

(i) isotrope Materie $\Rightarrow$ $\underline{P} \uparrow\uparrow \underline{E}$

$$\left.\begin{aligned} &\underline{P} \uparrow\uparrow \underline{E} \\ &\underline{M} \uparrow\uparrow \underline{B} \text{ (paramagn.)} \quad \text{oder} \quad \underline{M} \uparrow\downarrow \underline{B} \text{ (diamagn.)} \end{aligned}\right\}\ \underline{\text{skalarer}}\ \text{Zus.hang}$$

(ii) nicht zu hohe Felder $\Rightarrow$

$$\left.\begin{aligned} \underline{P} &\sim \underline{E} \\ \underline{M} &\sim \underline{B} \end{aligned}\right\}\ \underline{\text{linearer}}\ \text{Zus.hang}$$

(iii) kein Gedächtniseffekt, keine nichtlokale WW $\left.\right\} \Rightarrow \left.\begin{matrix} \underline{P}(\underline{r},t) \sim \underline{E}(\underline{r},t) \\ \underline{M}(\underline{r},t) \sim \underline{B}(\underline{r},t) \end{matrix}\right\}$ instantaner, lokaler Zus.hang

$\underline{P} = \epsilon_0 \chi_e \underline{E}$ elektr. Suszeptibilität χ_e

$\underline{M} = \chi_M \underline{H}$ magn. Suszeptibilität χ_M

(Materialkonstanten)

Die Materialkonstanten müssen aus mikroskop. Theorien (z.B. QM) abgeleitet werden.

$$\underline{D} = \epsilon_0 \underline{E} + \underline{P} = \epsilon_0 (1 + \chi_e) \underline{E} = \epsilon_0 \epsilon \underline{E}$$

mit $\epsilon := 1 + \chi_e$ (relative Dielektrizitätskonstante, relat. permittivity)

$$\underline{B} = \mu_0 (\underline{H} + \underline{M}) = \mu_0 (1 + \chi_M) \underline{H} = \mu_0 \mu \underline{H}$$

mit $\mu := 1 + \chi_M$ (relative Permeabilität)

$$\Rightarrow \underline{M} = \chi_M \underline{H} = \frac{1}{\mu_0} \frac{\chi_M}{\mu} \underline{B} = \frac{1}{\mu_0} \frac{\chi_M}{\underbrace{1 + \chi_M}} \underline{B}$$

> 0 paramagnet.

< 0 diamagnet.

paramagn. $\chi_M > 0 \Rightarrow \mu > 1$

diamagn. $-1 < \chi_M < 0 \Rightarrow 0 < \mu < 1$

Bem.: (i) $\underline{E} = 0 \Rightarrow \underline{P} = 0$ beschreibt kein Ferroelektrikum!

$\underline{B} = 0 \Rightarrow \underline{M} = 0$ " keinen Ferromagneten!

(ii) stets $\chi_e > 0$

aber $\chi_M \gtrless 0$ Para- / Dia- magnet

(iii) Ein Term $\sim \underline{B}$ in $\underline{P}$ oder $\sim \underline{E}$ in $\underline{M}$ kann nicht auftreten wegen des falschen Raumspiegelungsverhaltens (vgl. §3.3):

$\underline{E}, \underline{P}$ polarer Vektor, $\underline{B}, \underline{M}$ axialer Vektor

$\varrho_P = -\mathrm{div}\,\underline{P}$ skalar , $\underline{j}_M = \underline{\nabla} \times \underline{M}$ polarer Vektor

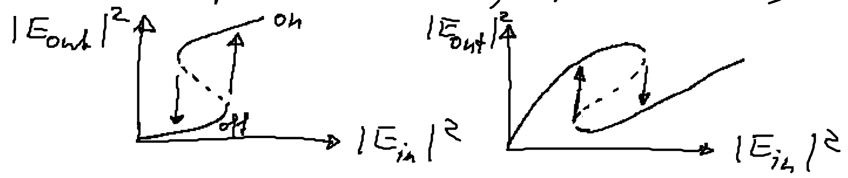
(iv) Abweichungen

– für anisotrope Kristalle: $\underline{P} = \epsilon_0 \underline{\underline{\chi}}_e \underline{E}$

(symm. Tensor $\underline{\underline{\chi}}_e$)

– für starke Felder: $\underline{P} = \epsilon_0 (\chi_e^{(1)} \underline{E} + \chi_e^{(3)} |\underline{E}|^2 \underline{E} + \ldots)$

(Anwendung: opt. Nichtlinearität,
z.B. opt. Bistabilität , opt. Schalter)



SEED (self-electrooptic effect device)

Merbach, Schöll, Gutowski,
J. Appl. Phys 85, 7051 (1999)

– für hochfrequente Felder:

$$\underline{P}(\underline{r},t) = \epsilon_0 \int d^3r'\,dt'\; \chi_e(\underline{r},\underline{r}',t,t')\, \underline{E}(\underline{r}',t')$$

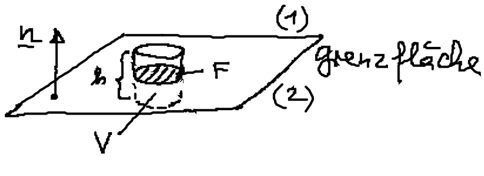
(räumliche bzw. zeitliche Dispersion:

$$\hat{\underline{P}}(\underline{k},\omega) = \epsilon_0 \hat{\chi}_e(\underline{k},\omega)\, \hat{\underline{E}}(\underline{k},\omega)\;)$$

5.4 Grenzbedingungen für Felder

Frage: $\underline{E}, \underline{D}, \underline{H}, \underline{B}$ an Grenzflächen, die verschiedene el. u. magn. Materialien (oder Vakuum/Materie) trennen?

Integration der Maxwell-Gln. über Volumen V:



(1) $\int_V d^3r\, \underline{\nabla} \times \underline{E} = -\int_V d^3r\, \dot{\underline{B}}$

(2) $\int_V d^3r\, \underline{\nabla} \times \underline{H} = \int_V d^3r\, (\underline{j} + \dot{\underline{D}})$

(3) $\int_V d^3r\, \underline{\nabla} \cdot \underline{B} = 0 \overset{\text{Gauß}}{=} \oint_{\partial V} d\underline{f} \cdot \underline{B}$

(4) $\int_V d^3r\, \underline{\nabla} \cdot \underline{D} = \int_V d^3r\, \varrho \overset{\text{Gauß}}{=} \oint_{\partial V} d\underline{f} \cdot \underline{D}$

Normalkomp.

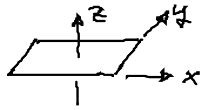
$h \to 0$: (3) $$\lim_{h\to 0} \oint_{\partial V} d\underline{f}\cdot\underline{B} = \int_F \overset{\underline{n}}{d\underline{f}}\cdot(\underset{\text{Deckel}}{\underline{B}^{(1)}} - \underset{\text{Boden}}{\underline{B}^{(2)}}) = \int_F df\,\underline{n}\cdot(\underline{B}^{(1)}-\underline{B}^{(2)})$$

der Dose

(4) $$\lim_{h\to 0} \oint_{\partial V} d\underline{f}\cdot\underline{D} = \int_F d\underline{f}\,(\underline{D}^{(1)}-\underline{D}^{(2)}) = \int_F df\,\underline{n}\cdot(\underline{D}^{(1)}-\underline{D}^{(2)})$$

Annahme: Grenzfläche trägt Flächenladungsdichte σ

$$\rho(\underline{r},t) = \sigma(x,y,t)\,\delta(z)$$



$$\Rightarrow \lim_{h\to 0}\int_V d^3r\,\rho = \int_F df\,\sigma$$

Also für bel. Fläche F:

$$\int_F df\,\underline{n}\cdot(\underline{B}^{(1)}-\underline{B}^{(2)}) = 0 \quad\Rightarrow\quad \boxed{\underline{n}\cdot(\underline{B}^{(1)}-\underline{B}^{(2)}) = 0}$$

$$\int_F df\,\underline{n}\cdot(\underline{D}^{(1)}-\underline{D}^{(2)}) = \int_F df\,\sigma \quad\Rightarrow\quad \boxed{\underline{n}\cdot(\underline{D}^{(1)}-\underline{D}^{(2)}) = \sigma}$$

Tangenzialkomp.

Verallg. gauß'scher Satz:

(1) $$\int_V d^3r\,\underline{\nabla}\times\underline{E} = \oint_{\partial V} d\underline{f}\times\underline{E} = -\int_V d^3r\,\dot{\underline{B}}$$

(2) $$\int_V d^3r\,\underline{\nabla}\times\underline{H} = \oint_{\partial V} d\underline{f}\times\underline{H} = \int_V d^3r\,(\underline{j}+\dot{\underline{D}})$$

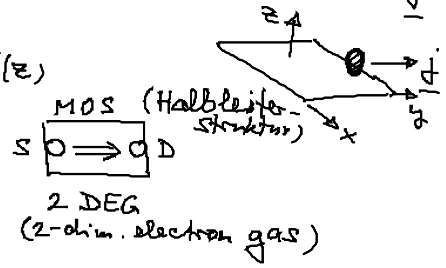
$h\to 0$: (1) $$\Rightarrow \lim_{h\to 0}\oint_{\partial V} d\underline{f}\times\underline{E} = \int_F df\,\underline{n}\times(\underline{E}^{(1)}-\underline{E}^{(2)})$$

(Tangenzialkomponente)

(2) $$\Rightarrow \lim_{h\to 0}\oint_{\partial V} d\underline{f}\times\underline{H} = \int_F df\,\underline{n}\times(\underline{H}^{(1)}-\underline{H}^{(2)})$$

Annahme: Grenzfläche trägt freie Flächenstromdichte $\underline{g}$

$$\underline{j}(\underline{r},t) = \underline{g}(x,y,t)\,\delta(z)$$



$$\Rightarrow \lim_{h\to 0} \int_V d^3r\, \underline{j} = \int_F df\, \underline{g}$$

$\underline{B}, \underline{D}$ und $\dot{\underline{B}}, \dot{\underline{D}}$ sind beschränkt:

$$\Rightarrow \lim_{h\to 0} \int_V d^3r\, \dot{\underline{B}} = \lim_{h\to 0} \int_V d^3r\, \dot{\underline{D}} = 0$$

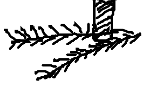
Also für bel. Fläche F:

$$\int_F df\, \underbrace{\underline{n}\times(\underline{E}^{(1)} - \underline{E}^{(2)})}_{0} = 0$$

$$\int_F df\, \underbrace{\underline{n}\times(\underline{H}^{(1)} - \underline{H}^{(2)}) = \int_F df\, \underline{g}}_{=}$$

Zus.fassung: $\delta\underline{E} := \underline{E}^{(1)} - \underline{E}^{(2)}$ usw.

Maxwell-Gln.	Grenzbed.	
$\nabla\times\underline{E} = -\dot{\underline{B}}$	$\underline{n}\times\delta\underline{E} = 0$	Tang. komp. v. $\underline{E}$ stetig
$\nabla\cdot\underline{D} = \varrho$	$\underline{n}\cdot\delta\underline{D} = \sigma$	Normalkomp. v. $\underline{D}$ springt (Flächendivergenz)
$\nabla\times\underline{H} = \underline{j} + \dot{\underline{D}}$	$\underline{n}\times\delta\underline{H} = \underline{g}$	Tang. komp. v. $\underline{H}$ springt (Flächenrotation)
$\nabla\cdot\underline{B} = 0$	$\underline{n}\cdot\delta\underline{B} = 0$	Normalkomp. v. $\underline{B}$ stetig



frohe Weihnachten!