

English Summary:

$$\text{Magnetization } \underline{M}(\underline{r}, t) = \frac{1}{\Delta V} \int_{\Delta V} d^3s \underline{M}_m(\underline{r} + \underline{s}, t)$$

$$\text{microscopic magnetic dipole density } \underline{M}_m(\underline{r}, t) = \sum_i m_i(t) \delta(\underline{r} - \underline{r}_i)$$

$$\underline{\nabla}_x \underline{M} = \underline{j}_M \quad \text{magnetization current density}$$

$$\text{magnetic field } \underline{H}(\underline{r}, t) = \frac{1}{\mu_0} \underline{B}' - \underline{M}, \quad \underline{\nabla}_x \underline{H} = \underline{j} \quad \text{free current density}$$

$$\frac{\partial \underline{P}}{\partial t} = \underline{j}_P \quad \text{polarization current density (contributes to displacement current)}$$

$$\left. \begin{aligned} \square \underline{A} &= -\mu_0 (\underline{j} + \underline{j}_P + \underline{j}_M) \\ \square \phi &= -\frac{1}{\epsilon_0} (\rho + \rho_p) \end{aligned} \right\} \text{Lorenz gauge}$$

⇒ Maxwell's eqs.

$$\left. \begin{aligned} \underline{\nabla}_x \underline{E} + \frac{\partial \underline{B}}{\partial t} &= 0 \\ \underline{\nabla} \cdot \underline{B} &= 0 \\ \underline{\nabla} \cdot \underline{D} &= \rho \\ \underline{\nabla}_x \underline{H} - \frac{\partial \underline{D}}{\partial t} &= \underline{j} \end{aligned} \right\}$$

$$\left. \begin{aligned} \underline{D} &= \epsilon_0 \underline{E} + \underline{P} \\ \underline{H} &= \frac{1}{\mu_0} \underline{B} - \underline{M} \end{aligned} \right\}$$

Die Feldgl. (1) - (6) sind nicht vollständig!  
Ergänzung durch Materialgleichungen notwendig.  
(Zus.hang  $\underline{P} \leftrightarrow \underline{E}$ ,  $\underline{M} \leftrightarrow \underline{B}$ )

Einfachster Fall:

$$(i) \text{ isotrope Materie } \Rightarrow \left. \begin{aligned} \underline{P} &\uparrow\uparrow \underline{E} \\ \underline{M} &\uparrow\uparrow \underline{B} \text{ oder } \underline{M} \uparrow\downarrow \underline{B} \end{aligned} \right\} \begin{array}{l} \text{skalarer} \\ \text{Zus.hang} \end{array}$$

(paramagn.)      (diamagn.)

$$(ii) \text{ nicht zu hohe Felder } \Rightarrow \left. \begin{aligned} \underline{P} &\sim \underline{E} \\ \underline{M} &\sim \underline{B} \end{aligned} \right\} \begin{array}{l} \text{linearer} \\ \text{Zus.hang} \end{array}$$

(iii) kein Gedächtniseffekt }  $\Rightarrow \underline{P}(\underline{r}, t) \sim \underline{E}(\underline{r}, t)$  } instanter, lokaler, Zus.hang  
 keine nichtlokale WW }  $\underline{M}(\underline{r}, t) \sim \underline{B}(\underline{r}, t)$  }

$\underline{P} = \epsilon_0 \chi_e \underline{E}$       elektr. Suszeptibilität  $\chi_e$

$\underline{M} = \chi_M \underline{H}$       magn. Suszeptibilität  $\chi_M$   
 (Materialkonstanten)

Die Materialkonstanten müssen aus mikroskop. Theorien (z.B. QM) abgeleitet werden.

$\underline{D} = \epsilon_0 \underline{E} + \underline{P} = \epsilon_0 (1 + \chi_e) \underline{E} = \epsilon_0 \epsilon \underline{E}$

mit  $\epsilon := 1 + \chi_e$  (relative Dielektrizitätskonstante,  
 relat. permittivity)

$\underline{B} = \mu_0 (\underline{H} + \underline{M}) = \mu_0 (1 + \chi_M) \underline{H} = \mu_0 \mu \underline{H}$

mit  $\mu := 1 + \chi_M$  (relative Permeabilität)

$\Rightarrow \underline{M} = \chi_M \underline{H} = \frac{1}{\mu_0} \frac{\chi_M}{\mu} \underline{B} = \frac{1}{\mu_0} \frac{\chi_M}{1 + \chi_M} \underline{B}$

> 0 paramagnet.

< 0 diamagnet.

paramagn.  $\chi_M > 0 \Rightarrow \mu > 1$

diamagn.  $-1 < \chi_M < 0 \Rightarrow 0 < \mu < 1$

Bem.: (i)  $\underline{E} = 0 \Rightarrow \underline{P} = 0$  beschreibt kein Ferroelektrikum!

$\underline{B} = 0 \Rightarrow \underline{M} = 0$  " keinen Ferromagnet!

(ii) stets  $\chi_e > 0$

aber  $\chi_M \geq 0$  Para- magnet  
 Dia- magnet

(iii) Ein Term  $\sim \underline{B}$  in  $\underline{P}$  oder  $\sim \underline{E}$  in  $\underline{M}$  kann nicht auftreten wegen des falschen Raumspiegelungsverhaltens (vgl. §3.3):

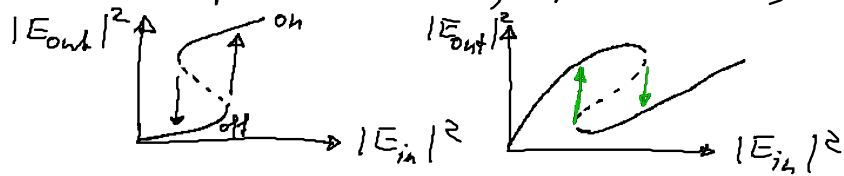
$\underline{E}, \underline{P}$  polarer Vektor,  $\underline{B}, \underline{M}$  axialer Vektor

$\rho_p = -\text{div } \underline{P}$  skalar,  $\underline{j}_M = \nabla \times \underline{M}$  polarer Vektor

(iv) Abweichungen

- für anisotrope Kristalle:  $\underline{P} = \epsilon_0 \underline{\chi}_e \underline{E}$   
(symm. Tensor  $\underline{\chi}_e$ .)

- für starke Felder:  $\underline{P} = \epsilon_0 (\chi_e^{(1)} \underline{E} + \chi_e^{(2)} |\underline{E}|^2 \underline{E} + \dots)$   
(Anwendung: opt. Nichtlinearität,  
z.B. opt. Bistabilität, opt. Schalter)



SEED (self-electrooptic effect) device

Merbach, Schöll, Gutowski,  
J. Appl. Phys. 85, 7051 (1999)

- für hochfrequente Felder:

$\underline{P}(\underline{r}, t) = \epsilon_0 \int d\underline{r}' dt' \chi_e(\underline{r}, \underline{r}', t, t') \underline{E}(\underline{r}', t')$

(räumliche bzw. zeitliche Dispersion:)  
 $\hat{\underline{P}}(\underline{k}, \omega) = \epsilon_0 \hat{\chi}_e(\underline{k}, \omega) \hat{\underline{E}}(\underline{k}, \omega)$

5.4 Grenzbedingungen für Felder

Frage:  $\underline{E}, \underline{D}, \underline{H}, \underline{B}$  an Grenzflächen, die verschiedene el. u. magn. Materialien (oder Vakuum/Materie) trennen?

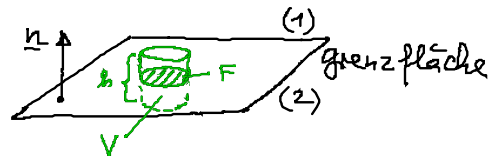
Integration der Maxwell-Ges. über Volumen  $V$ :

(1)  $\int_V d\underline{r}^3 \nabla \cdot \underline{E} = - \int_V d\underline{r}^3 \dot{\underline{B}}$

(2)  $\int_V d\underline{r}^3 \nabla \times \underline{H} = \int_V d\underline{r}^3 (\underline{j} + \dot{\underline{D}})$

(3)  $\int_V d\underline{r}^3 \nabla \cdot \underline{B} = 0 = \oint_{\partial V} d\underline{f} \cdot \underline{B}$

(4)  $\int_V d\underline{r}^3 \nabla \cdot \underline{D} = \int_V d\underline{r}^3 \rho = \oint_{\partial V} d\underline{f} \cdot \underline{D}$



Normalkomp.

$$\underline{h \rightarrow 0} : (3) \lim_{h \rightarrow 0} \oint_{\partial V} d\underline{f} \cdot \underline{B} = \int_F d\underline{f} \cdot (\underline{B}^{(1)} - \underline{B}^{(2)}) = \int_F d\underline{f} \cdot \underline{n} \cdot (\underline{B}^{(1)} - \underline{B}^{(2)})$$

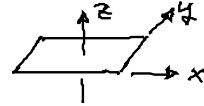
↑ n
Deckel
Boden
F

des Dose

$$(4) \lim_{h \rightarrow 0} \oint_{\partial V} d\underline{f} \cdot \underline{D} = \int_F d\underline{f} \cdot (\underline{D}^{(1)} - \underline{D}^{(2)}) = \int_F d\underline{f} \cdot \underline{n} \cdot (\underline{D}^{(1)} - \underline{D}^{(2)})$$

Annahme: Grenzfläche trägt Flächenladungsdichte  $\sigma$

$$\rho(\underline{r}, t) = \sigma(x, y, t) \delta(z)$$



$$\Rightarrow \lim_{h \rightarrow 0} \int_V d^3r \rho = \int_F d\underline{f} \sigma$$

Also für bel. Fläche F:

$$\int_F d\underline{f} \cdot \underline{n} \cdot (\underline{B}^{(1)} - \underline{B}^{(2)}) = 0 \Rightarrow$$

$$\underline{n} \cdot (\underline{B}^{(1)} - \underline{B}^{(2)}) = 0$$

$$\underline{n} \cdot (\underline{D}^{(1)} - \underline{D}^{(2)}) = \sigma$$

$$\int_F d\underline{f} \cdot \underline{n} \cdot (\underline{D}^{(1)} - \underline{D}^{(2)}) = \int_F d\underline{f} \sigma \Rightarrow$$

Tangenzialkomp.

Verallg. Gauß'scher Satz:

$$(1) \int_V d^3r \nabla \times \underline{E} = \oint_{\partial V} d\underline{f} \times \underline{E} = - \int_V d^3r \dot{\underline{B}}$$

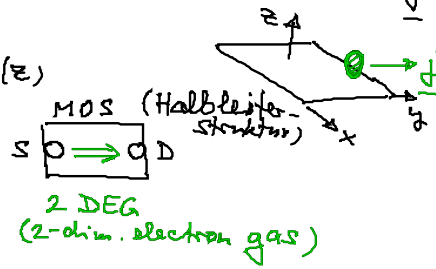
$$(2) \int_V d^3r \nabla \times \underline{H} = \oint_{\partial V} d\underline{f} \times \underline{H} = \int_V d^3r (\underline{j} + \dot{\underline{D}})$$

h → 0: (1)  $\Rightarrow \lim_{h \rightarrow 0} \oint_{\partial V} d\underline{f} \times \underline{E} = \int_F d\underline{f} \cdot \underline{n} \times (\underline{E}^{(1)} - \underline{E}^{(2)})$   
(Tangenzialkomponente)

(2)  $\Rightarrow \lim_{h \rightarrow 0} \oint_{\partial V} d\underline{f} \times \underline{H} = \int_F d\underline{f} \cdot \underline{n} \times (\underline{H}^{(1)} - \underline{H}^{(2)})$

Annahme: Grenzfläche trägt freie Flächenstromdichte  $\underline{j}$

$$\underline{j}(\underline{r}, t) = \underline{j}(x, y, t) \delta(z)$$



$$\Rightarrow \lim_{h \rightarrow 0} \int_V d^3r \underline{j} = \int_F d\underline{f} \underline{j}$$

$\underline{B}$ ,  $\underline{D}$  und  $\underline{\dot{B}}$ ,  $\underline{\dot{D}}$  sind beschränkt:

$$\Rightarrow \lim_{h \rightarrow 0} \int_V d^3r \underline{\dot{B}} = \lim_{h \rightarrow 0} \int_V d^3r \underline{\dot{D}} = 0$$

Also für bel. Fläche  $F$ :

$$\int_F d\underline{f} \underline{n} \times (\underline{E}^{(1)} - \underline{E}^{(2)}) = 0$$

$$\int_F d\underline{f} \underline{n} \times (\underline{H}^{(1)} - \underline{H}^{(2)}) = \int_F d\underline{f} \underline{j}$$

Zus. fassung:  $\delta \underline{E} := \underline{E}^{(1)} - \underline{E}^{(2)}$  usw.

Maxwell-gln.	Grenzbed.	
$\nabla \times \underline{E} = -\underline{\dot{B}}$	$\underline{n} \times \delta \underline{E} = 0$	Tang. Komp. v. $\underline{E}$ stetig
$\nabla \cdot \underline{D} = \rho$	$\underline{n} \cdot \delta \underline{D} = \sigma$	Normalkomp. v. $\underline{D}$ springt (Flächendivergenz)
$\nabla \times \underline{H} = \underline{j} + \underline{\dot{D}}$	$\underline{n} \times \delta \underline{H} = \underline{j}$	Tang. Komp. v. $\underline{H}$ springt (Flächenrotation)
$\nabla \cdot \underline{B} = 0$	$\underline{n} \cdot \delta \underline{B} = 0$	Normalkomp. v. $\underline{B}$ stetig



frohe Weihnachten!