English Summary:

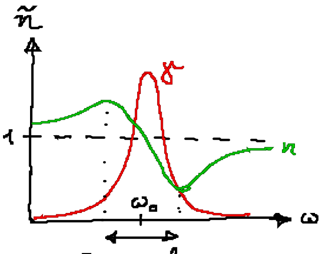
Dielectric dispersion

$$\boxed{\hat{\underline{P}}(\omega) = \epsilon_0 \hat{\chi}(\omega) \hat{\underline{E}}(\omega)} \qquad \hat{\chi}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dt\, \chi(t) e^{i\omega t}$$

$$\underline{P}(t) = \frac{\epsilon_0}{\sqrt{2\pi}} \int_{-\infty}^{t} dt' \chi(t-t') \underline{E}(\underline{r}, t')$$

dielectric fct. $\epsilon(\omega) = \underbrace{\epsilon'(\omega)}_{\text{dispersion}} + i \underbrace{\epsilon''(\omega)}_{\text{absorption}}$

(group velocity $v_g = \frac{d\omega}{dk'} = \frac{c}{n + \omega \frac{dn}{d\omega}}$) $\quad (\kappa > 0)$



anomalous dispersion

Kramers-Kronig:

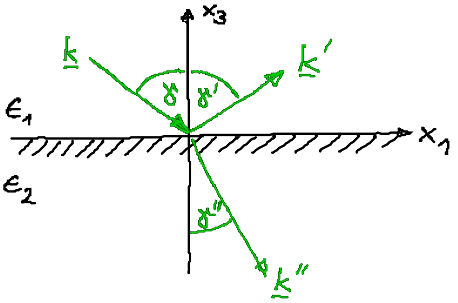
(follows from causality: $\chi(t) = 0$ for $t < 0$)

$$\boxed{\begin{aligned} \epsilon'(\omega) - 1 &= \frac{1}{\pi} \mathcal{P} \int_{-\infty}^{\infty} d\omega' \frac{\epsilon''(\omega')}{\omega' - \omega} \\ \epsilon''(\omega) &= -\frac{1}{\pi} \mathcal{P} \int_{-\infty}^{\infty} d\omega' \frac{\epsilon'(\omega') - 1}{\omega' - \omega} \end{aligned}}$$

5.7 Brechung und Reflexion

Wellenausbreitung in geschichteten Medien:

(transparent $\Rightarrow$ $\epsilon_i \in \mathbb{R}$, $i = 1, 2$)



$$\boxed{\begin{aligned} \frac{\omega}{c_1} &= |\underline{k}| = |\underline{k}'| = \frac{\omega'}{c_1} \\ |\underline{k}''| &= \frac{\omega''}{c_2} \\ c_i &= \frac{c}{n_i} = \frac{c}{\sqrt{\epsilon_i}} \end{aligned}} \quad i = 1, 2$$

Einfallende Welle $\underline{E} = \underline{E}_0 e^{i(\underline{k} \cdot \underline{r} - \omega t)}$

Reflektierte Welle $\underline{E}' = \underline{E}_0' e^{i(\underline{k}' \cdot \underline{r} - \omega' t)}$

Transmittierte Welle $\underline{E}'' = \underline{E}_0'' e^{i(\underline{k}'' \cdot \underline{r} - \omega'' t)}$

Grenzbedingungen für Felder

$\underline{n}\times(\underline{E}^{(1)}-\underline{E}^{(2)})=0$ Tang. komp. v. $\underline{E}$ stetig

$\underline{n}\cdot(\underline{D}^{(1)}-\underline{D}^{(2)})=\cancel{\sigma}$ Norm. komp. v. $\underline{D}=\epsilon_0\epsilon\underline{E}$ stetig

$\underline{n}\times(\underline{H}^{(1)}-\underline{H}^{(2)})=\cancel{\underline{g}}$ Tang. komp. v. $\underline{H}$ stetig

$\underline{n}\cdot(\underline{B}^{(1)}-\underline{B}^{(2)})=0$ Normalkomp. v. $\underline{B}=\mu_0\mu\underline{H}$ stetig

Grenzbed. für $\underline{E}$ (linear polarisiert)

$$E_1+E_1'\Big|_{x_3=0} = E_1''\Big|_{x_3=0}$$ Tang. komp. stetig

$$\underline{r}=0:\ E_{01}e^{-i\omega t}+E_{01}'e^{-i\omega' t}=E_{01}''e^{-i\omega'' t}\ \Rightarrow\ \boxed{\begin{array}{c}\omega=\omega'=\omega''\\ E_{01}+E_{01}'=E_{01}''\end{array}}$$

$$t=0:\ E_{01}e^{ik_1x_1}+E_{01}'e^{ik_1'x_1}=E_{01}''e^{ik_1''x_1}\ \Rightarrow\ \boxed{k_1=k_1'=k_1''}$$

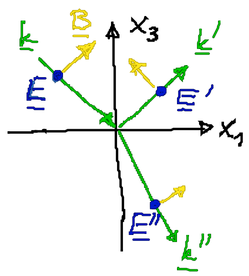
$$\Rightarrow\ \underbrace{|\underline{k}|}_{\omega/c_1}\sin\gamma=\underbrace{|\underline{k}'|}_{\omega/c_1}\sin\gamma'=\underbrace{|\underline{k}''|}_{\omega/c_2}\sin\gamma''$$

$$\Rightarrow\ \boxed{\begin{array}{l}\sin\gamma=\sin\gamma'\\[4pt] \dfrac{\sin\gamma''}{\sin\gamma}=\dfrac{c_2}{c_1}=\dfrac{n_1}{n_2}\end{array}}$$

Reflexionsgesetz

Brechungsgesetz (Snellius)

Bestimmung der Amplituden:



(a) Polarisation von $\underline{E}$ ⊥ Einfallsebene

$$E_{01}=E_{01}'=E_{01}''=0$$
$$E_{03}=E_{03}'=E_{03}''=0$$

(1) $\boxed{E_{02}+E_{02}'=E_{02}''}$ Tang. komp.

Mit $\underline{B}_0=\frac{c}{\omega}(\underline{k}\times\underline{E}_0)=\frac{c}{\omega}E_{02}\begin{pmatrix}-k_3\\0\\k_1\end{pmatrix}$ folgt für Tang. komp. v. $\underline{B}$:

$$B_{01}+B_{01}'=B_{01}''\ \Rightarrow\ k_3E_{02}+k_3'E_{02}'=k_3''E_{02}''$$

Reflexionsgesetz $\Rightarrow k_3=-k_3' \Rightarrow \boxed{k_3(E_{02}-E_{02}')=k_3''E_{02}''}$ (2)

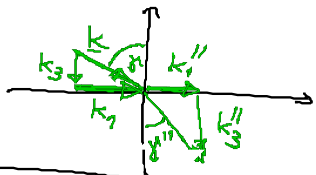
$$(1) \text{ in } (2) \implies k_3 (E_{02} - E'_{02}) = k''_3 (E_{02} + E'_{02}) \Rightarrow (k_3 - k''_3) E_{02} = (k_3 + k''_3) E'_{02}$$

$$\Rightarrow \boxed{\frac{E'_{02}}{E_{02}} = \frac{k_3 - k''_3}{k_3 + k''_3}, \quad \frac{E''_{02}}{E_{02}} = \frac{2k_3}{k_3 + k''_3}}$$

Drücke k''_3 durch Brechungswinkel γ'' aus!

$$\Rightarrow k''_3 = |\underline{k}''| \cos\gamma'' = |\underline{k}| \underbrace{\left(\frac{n_2}{n_1}\right)}_{\frac{\sin\gamma}{\sin\gamma''}} \cos\gamma''$$

$$k_3 = |\underline{k}| \cos\gamma$$



$$\Rightarrow \boxed{\begin{aligned} \frac{E'_{02}}{E_{02}} &= \frac{\cos\gamma \sin\gamma'' - \sin\gamma \cos\gamma''}{\cos\gamma \sin\gamma'' + \sin\gamma \cos\gamma''} = \frac{\sin(\gamma'' - \gamma)}{\sin(\gamma'' + \gamma)} \\ \frac{E''_{02}}{E_{02}} &= \frac{2 \sin\gamma'' \cos\gamma}{\sin(\gamma'' + \gamma)} \end{aligned}}$$

Fresnel'sche Formeln

Intensitätsverhältnisse

Zeitmittel des Poyntingvektors $\langle \underline{S} \rangle = \frac{1}{T} \int_0^T dt\, \underline{E} \times \underline{H} \sim |\underline{E}_0|^2$

Reflexionskoeff.:

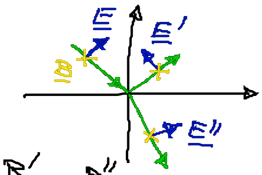
$$\boxed{R_\perp = \left| \frac{E'_{02}}{E_{02}} \right|^2 = \frac{\sin^2(\gamma'' - \gamma)}{\sin^2(\gamma'' + \gamma)}}$$

Transmissionskoeff.:

$$\boxed{T_\perp = \left| \frac{E''_{02}}{E_{02}} \right|^2 = \frac{4 \sin^2\gamma'' \cos^2\gamma}{\sin^2(\gamma'' + \gamma)} = 1 - R_\perp}$$

$\perp$ polarisiert

(b) Polarisation von $\underline{E}$ ∥ Einfallsebene



$\underline{B}$ ⊥ Einfallsebene

⇒ analoge Argumentation für $B_{02}, B'_{02}, B''_{02}$ wie in (a)

$$\Rightarrow \boxed{\frac{E'_{\parallel}}{E_{\parallel}} = \frac{\tan(\gamma-\gamma'')}{\tan(\gamma+\gamma'')}\,, \quad \frac{E''_{\parallel}}{E_{\parallel}} = \frac{2\sin\gamma''\cos\gamma}{\sin(\gamma+\gamma'')\cos(\gamma''-\gamma)} \qquad R_{\parallel} = 1-T_{\parallel} = \frac{\tan^2(\gamma''-\gamma)}{\tan^2(\gamma''+\gamma)}}$$

Bem.: (i) Bei Reflexion und Brechung wird i.a. die Polarisationsrichtung gedreht!

Speziell für:

$\gamma''+\gamma = \frac{\pi}{2} \Rightarrow \tan(\gamma''+\gamma) \to \infty \Rightarrow \boxed{R_{\parallel}=0}$

⇒ reflektierte Welle ist vollständig polarisiert ⊥ Einfallsebene

γ = Brewster-Winkel γ_B mit $\tan\gamma_B = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$

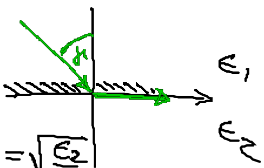
denn:

$$\tan\gamma = \frac{\sin\gamma}{\cos\gamma} = \frac{\sin\gamma}{\sin(\frac{\pi}{2}-\gamma)} = \frac{\sin\gamma}{\sin\gamma''} = \frac{n_2}{n_1} = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$$

$\cos\gamma = \sin(\frac{\pi}{2}-\gamma)$

(ii) Totalreflexion

Sei $\epsilon_2 < \epsilon_1 \Rightarrow$

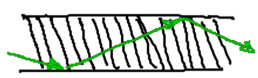


für $\gamma = \gamma_G$ mit $\sin\gamma_G = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$

(Grenzwinkel der Totalreflexion)

$\gamma'' = \frac{\pi}{2}\,, \quad R_{\perp} = R_{\parallel} = 1$

$T_{\perp} = T_{\parallel} = 0$



für $\gamma > \gamma_G \Rightarrow k_3'' = \frac{i}{d}$ imaginär

evaneszente Welle $\underline{E}'' = \underline{E}_0'' \underbrace{e^{-|x_3|/d}}_{\text{evaneszent}} e^{i(k_1'' x_1 - \omega t)}$