Wdh

- Quanten-Ising-Modell

1d $\hat{H}_{QA} = -g \sum_i \hat{\sigma}^x_i - J \sum_i \hat{\sigma}^z_i \hat{\sigma}^z_{i+1}$

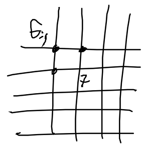
$\hat{\sigma}^z_i |z_i\rangle = (-1)^{z_i} |z_i\rangle$

$|z_1 \dots z_N\rangle$

$E_{\{\sigma\}} = -g \sum_i \sigma_i - J \sum_i \sigma_i \sigma_{i+1} \qquad \sigma_i \in \{-1, +1\}$

1d

$H^{1d}_{QA} \Leftrightarrow H^{2d}_{cl} (g=0) \qquad \beta_c^{\text{class 2d}} \cdot J_{\text{class}} = \frac{1}{2} \ln [1 + \sqrt{2}]$



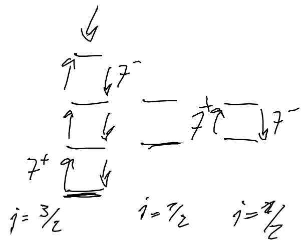
- kollektiver Spin $J^\alpha = \frac{1}{2} \sum_{i=1}^N \sigma^\alpha_i \qquad \alpha \in \{x, y, z\}$

$J^z |j, m\rangle = m |j\, m\rangle \qquad J_\pm = J^x \pm i J^y \qquad \sigma^\pm = \frac{1}{2}(\sigma^x \pm i \sigma^y)$

$J^2 |j\, m\rangle = j(j+1) |j\, m\rangle \qquad \sigma^+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$

$J^\pm |j\, m\rangle = \sqrt{j(j+1) - (m)(m \pm 1)}\, |j, m \pm 1\rangle \qquad \sigma^- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$

N+1 Zust.



$\Rightarrow$ Lipkin-Meshkov-Glick (Curie-Weiss)

$\hat{H} = -h \hat{J}^z - \frac{\gamma}{N} (\hat{J}^x)^2$

$= \dots = \Delta E(h, \gamma) \cdot c^\dagger c + E_0(h, \gamma) \cdot \mathbb{1}$

$\rightsquigarrow$ QPT bei $h = \gamma$

2^N Zust.

bisher: Gleichgewicht

4. Nicht-GG-TD

4.1. Ratengleichungen

4.1.1. Allg. Eigenschaften

- System mit N diskreten Zuständen
  $E_0 \le E_1 \le \dots \le E_{N-1}$
  $N_i \ge 0 : \; 0 \le i \le N-1$

$P_n(t)$ sei die WS, das System bei Energie E_n & TZ N_n zur Zeit t anzutreffen

$$\frac{d}{dt} P_n = \sum_\ell \left[W_{n\ell} \cdot P_\ell - W_{\ell n} P_n \right] \Leftrightarrow \boxed{\underline{\underline{W}}\, \underline{P} = \dot{\underline{P}}} \Rightarrow \frac{d}{dt} \sum_n P_n = 0$$

$\sum_n P_n = \text{const}$

Rate Übergangs-WS für einen Übergang von $\ell \to n$ pro Zeit $\qquad W_{n \neq \ell} \ge 0$

$$\underline{\underline{W}} = \begin{pmatrix} -\sum_{j \neq 0} W_{j0} & W_{01} & W_{02} & \dots & W_{0N-1} \\ W_{10} & -\sum_{j \neq 1} W_{j1} & & & \\ W_{20} & & \ddots & & \\ \vdots & & & & \\ W_{N-1\,0} & & & & -\sum_{j \neq N-1} W_{j\,N-1} \end{pmatrix}$$

Spaltensumme verschwindet

- $W_{\ell n} \ge 0 : \ell \neq n$
- $W_{\ell\ell} = -\sum_{n \neq \ell} W_{n\ell}$
- $W_{n\ell}(t)$ ist möglich
- $W_{\ell n} \neq W_{n\ell}$ i.A.

nimm an:

$\tilde{P}_n(t) = 0 \qquad P_\ell(t) \ge 0$

$\frac{d}{dt} \tilde{P}_n = \sum_{\ell \neq n} W_{n\ell} \cdot P_\ell \ge 0$

$\underline{\underline{W}}\, v_n = \lambda_n v_n$ (EW: λ_n; rechter EV: v_n) $\longrightarrow v_n^{*,T}\, \underline{\underline{W}}^{+} = v_n^{*,T} \cdot \lambda_n^{*}$

$\Rightarrow \forall \lambda_n \;\exists \lambda_n^{*}$ EW · kommen in komplex konj. Paaren · oder sind reell

Gershgorin: $Re(\lambda_n) \le 0$

linken EV $(1, \dots, 1)\, \underline{\underline{W}} = (0, \dots, 0) = 0$ mit EV 0

$\Rightarrow \exists$ rechter EV mit $\underline{\underline{W}}\, \bar{P} = 0$ $\bar{P}$ ist "stationäre Verteilung"

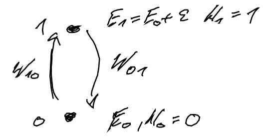
$$\boxed{W_{k\ell} \cdot \bar{P}_\ell = W_{\ell k} \cdot \bar{P}_k}$$
"detailliertes Gleichgewicht"

Bsp.: $\frac{d}{dt}\begin{pmatrix} P_0 \\ P_1 \end{pmatrix} = \begin{pmatrix} -W_{10} & W_{01} \\ W_{10} & -W_{01} \end{pmatrix}\begin{pmatrix} P_0 \\ P_1 \end{pmatrix}$ $N = 2$

$P_0 = 1 - P_1 \rightsquigarrow \dot{P}_1 = W_{10}[1 - P_1] - W_{01} \cdot P_1$

$P_1(t) = P_1^0 \cdot e^{-(W_{01}+W_{10}) \cdot t} + \frac{W_{10}}{W_{01}+W_{10}}\left[1 - e^{-(W_{01}+W_{10}) \cdot t}\right]$ $P_0(t) = 1 - P_1(t)$

$\bar{P}_1 = \frac{W_{10}}{W_{01}+W_{10}}$ $\bar{P}_0 = \frac{W_{01}}{W_{01}+W_{10}}$



4.1.2. Ableitung der Raten

Fermis Gold. Regel

$H = H_0 + V$ $H_0 |i\rangle = E_i |i\rangle$

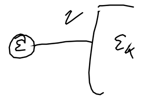
$H_0 |f\rangle = E_f |f\rangle$

$R_{f,i} = \frac{2\pi}{\hbar}\, \delta(E_f - E_i)\, |\langle f | V | i \rangle|^2$

Fermis Gold. Regel

$H_0 = H_S + H_B$ (System, Bad) $H = H_0 + V$ (WW zwischen System & Bad)

Beispiel $N = 2$ QD



$H_S = \varepsilon\, c^{\dagger} c$

$H_B = \sum_k \varepsilon_k\, c_k^{\dagger} c_k$

$V = \sum_k \left[t_k\, c^{\dagger} c_k + t_k^{*}\, c_k^{\dagger} c\right]$

betrachte Rate für Besetzung des Systems

$|i\rangle = |0, n_1^i, \dots, n_k^i, \dots n_K^i\rangle$ $n_k^i \in \{0, 1\}$

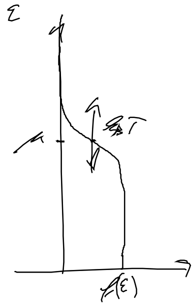
$|f\rangle = |1, n_1^f = n_1^i, \dots, n_k^i - 1, \dots, n_K^f = n_K^i\rangle$

$R_{0\to 1} = \frac{2\pi}{\hbar} \sum_k |t_k|^2 \cdot n_k^i \longrightarrow \Gamma(\varepsilon) \cdot f(\varepsilon)$

im GG

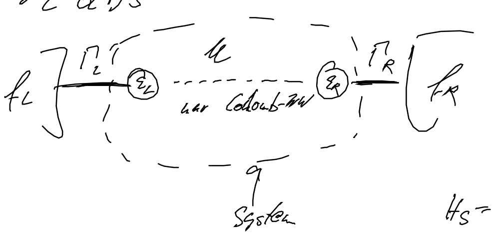
$$R_{1\to 0} = \Gamma(\varepsilon)\left[1-f(\varepsilon)\right]$$

$$\underline{\underline{W}} = \Gamma(\varepsilon)\begin{pmatrix} -f(\varepsilon) & 1-f(\varepsilon) \\ f(\varepsilon) & -[1-f(\varepsilon)] \end{pmatrix} \qquad \bar{P}_1 = f(\varepsilon), \quad \bar{P}_0 = 1-f(\varepsilon)$$

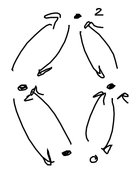


$$W_{01}\cdot \bar{P}_1 = W_{10}\cdot \bar{P}_0 \longrightarrow \text{det. GG.}$$

• 2 QD's



$$H_S = \varepsilon_L d_L^\dagger d_L + \varepsilon_R d_R^\dagger d_R + U d_L^\dagger d_L d_R^\dagger d_R$$



P_0: beide unbesetzt — $E_0 = 0$

P_L: linker dot besetzt — $E_L = \varepsilon_L$

P_R: rechter —"— — $E_R = \varepsilon_R$

P_2: beide besetzt — $E_2 = \varepsilon_L + \varepsilon_R + U$

$$W = \Gamma_L \begin{array}{c} \begin{array}{cccc} P_0 & P_L & P_R & P_2 \end{array} \\ \begin{pmatrix} -f_L & 1-f_L & 0 & 0 \\ f_L & -(1-f_L) & 0 & 0 \\ 0 & 0 & -f_L^U & 1-f_L^U \\ 0 & 0 & f_L^U & -(1-f_L^U) \end{pmatrix} \end{array} + \Gamma_R \begin{pmatrix} -f_R & 0 & 1-f_R & 0 \\ 0 & -f_R^U & 0 & 1-f_R^U \\ +f_R & 0 & -(1-f_R) & 0 \\ 0 & f_R^U & 0 & -(1-f_R^U) \end{pmatrix}$$

$$f_\alpha = \frac{1}{e^{\beta_\alpha(\varepsilon_\alpha - \mu_\alpha)}+1} \qquad f_\alpha^U = \frac{1}{e^{\beta_\alpha(\varepsilon_\alpha + U - \mu_\alpha)}+1}$$

$\alpha \in \{L, R\}$

Bosonen

$$R^{em} = \Gamma\left[\underbrace{1}_{\text{spont. Em.}} + \overbrace{n_B(\Delta E)}^{\text{stim. Emission}}\right] \qquad R^{ab} = \Gamma\, n_B(\Delta E) \qquad n_B = \frac{1}{e^{\beta\omega}\underbrace{-1}_{\text{Bosonen}}}$$

$$\text{det. GG: } W_{ij}\bar{P}_j = W_{ji}\cdot\bar{P}_i \rightsquigarrow \frac{\bar{P}_i}{\bar{P}_j} = \frac{W_{ij}}{W_{ji}}$$

μ kürzt sich raus

$$\boxed{\frac{n_B(\Delta E)}{1 + n_B(\Delta E)} = e^{-\beta \cdot \Delta E} \qquad \frac{f(\varepsilon)}{1 - f(\varepsilon)} = e^{-\beta(\varepsilon - \cancel{\mu})}}$$

nach Fermis Goldener Regel folgt bei nur einem Res. det. GG $\longrightarrow \bar{P} = \bar{P}_0 / \bar{P}_{gc}$

- vernachlässigt: WW-Energie zwischen System & Bad