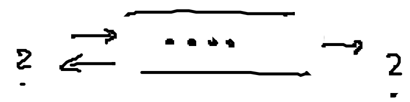
## d) Lösung des Wellenproblems

i) außen:



rechtslaufende Welle: $c_R(\zeta, y_-)$ $\quad c_L(\zeta, y_+)$

$$\partial_\zeta \tilde{C}_R = \frac{i}{v_g} \sum_i \delta(\zeta - z_i)\, \tilde{g}_{12}^{i-k_0}\, p_{12}^i\left(t = y_- + \frac{\zeta}{v_g}\right)$$

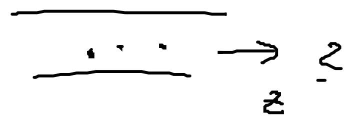
$\zeta > z_i$ $\qquad\qquad y_\mp = t \mp \frac{z}{v_g}$

integriere über $\int d\zeta$

$$\tilde{C}_R(\zeta, y) = \underbrace{\tilde{C}_R^0(y)}_{\substack{\text{homogene Lsg.,} \\ \text{physikalisch festlegen}}} + \underbrace{\frac{i}{v_g} \sum_i \tilde{g}_{12}^{i-k_0}\, p_{12}^i\left(y_- + \frac{z_i}{v_g}\right)}_{\text{inhomogene Lösg.}}$$

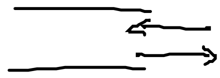
$$\tilde{C}_R(z,t) = \tilde{C}_R^0\left(t - \frac{z}{v_g}\right) + \frac{i}{v_g} \sum_i \tilde{g}_{12}^{i-k_0}\, p_{12}^i\left(t - \frac{z - z_i}{v_g}\right)$$

$z > z_i \quad \forall i$ :



Quelle $p_{12}^i$ benötigt Zeit $\frac{z - z_i}{v_g}$ um bei $z$ zu wirken analog: $\tilde{c}_L(z,t) = \tilde{c}_L^{0}\left(t + \frac{z}{v_g}\right) - \frac{i}{v_g}\sum_i \tilde{g}^{\,i+k_0-i}_{1L}\, p^{i}_{1L}\left(t + \frac{z-z_i}{v_g}\right)$

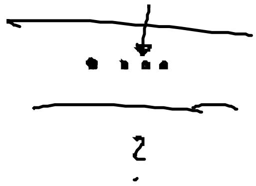
$\tilde{c}_L(z,t)$ $\quad \tilde{c}_R(z,t)$ :



$\tilde{c}_R(z,t)$ :

bestimmt z.B. Transmission $T(\omega)$

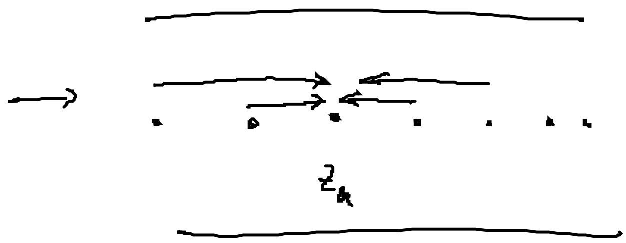
ii) innen



Feld bei $z_n$ ist nötig um Blochgleichg. zu lösen

$$\partial_z \tilde{c}_R = \frac{i}{v_g}\sum_i \delta(z - z_i)\, g^{i-k}_{1L}\, \tilde{p}^{\,i}_{1L}\left(t = t_- + \frac{z}{v_g}\right)$$

$\tilde{c}_R(z_n,t)$, $\tilde{c}_L(z_n,t) = ?$



$$\int_{-\infty}^{z_n} dz\; \delta(z - z_i)\ldots = \int_{-\infty}^{z_n' \to \infty} dz\; \theta(z_n - z)\,\delta(z - z_i)\ldots$$

$\hat{=}$ Beitrg $\forall z$ die $\leq z_n$

$$\tilde{c}_R(z_n,t) = \tilde{c}^0_R\left(t - \frac{z_n}{v_g}\right) + \frac{i}{v_g}\sum_{i<n} \underbrace{g^{i-k}_{12}\tilde{p}^i_{12}}\left(t - \frac{z_n - z_i}{v_g}\right)$$

$$+ \frac{i}{2 v_g}\, \underline{g^{n-k}_{n}\,\tilde{p}^n_{12}(t)}$$

$\Theta(0)$

$$\tilde{c}_L(z_n) = \tilde{c}^0_L\left(t + \frac{z_n}{v_g}\right) + \frac{i}{v_g}\sum_{i>n} \tilde{g}^{i+k_0}_{12}\,\tilde{p}^i_{12}\left(t - \frac{z_i - z_n}{v_g}\right)$$

$$+ \frac{i}{2 v_g}\, g^{n+k_0}_{12}\,\tilde{p}^n_{12}(t)$$

$\hat{=}$ Lösg. f. Lichtfeldoperatoren, $c^0$: physikalisch festlegen

einsetzen in Materialgleichg. aus letzter VL $(\dot{\Delta}_{12}, \dot{\tilde{p}}_{12})$

## e) Selbstkonsistente Materialgleichg.

$$\dot{\tilde{P}}_{12}^{i}(t) = i\underbrace{(\omega_{12}^{i} + \omega_{k_0})}_{\delta}\tilde{p}_{12}^{i}(t) \quad \hat{=} \text{ Verstimmung.}$$

$$-\gamma_{rad}^{i}\,\tilde{p}_{12}^{i}(t) \quad \hat{=} \text{ aus Selbst-WW}$$

$$\gamma_{rad}^{i} = |\vec{d}_{12}^{\,i}\cdot\vec{e}_1|^2 \frac{\omega_{40}}{2\varepsilon_0\hbar A}\frac{1}{v_g}$$

Dämpfg. d. Dipols

(rad $\hat{=}$ radiativ)

$$+\,i\,\Delta_{12}^{i}(t)\left(\tilde{g}_{12}^{*\,i,-k_0}\,\tilde{c}_R^{\,0}\left(t-\frac{z_i}{v_g}\right) + \tilde{g}_{12}^{*\,i,+k_0}\,\tilde{c}_L^{\,0}\left(t+\frac{z_i}{v_g}\right)\right)$$

$\uparrow$

Inversion — Quelle durch das extern aufgeprägte Feld $\tilde{c}^{\,0}$.

$$-\sum_{j\neq i}\gamma_{rad}^{ij}\,\Delta_{12}^{i}(t)\,\tilde{p}_{12}^{j}\left(t-\frac{|z_i-z_j|}{v_g}\right)e^{ik_0|z_j-z_i|}$$

Quelle die durch Dipole $j\neq i$ hervorgerufen wird

$\gamma_{rad}^{ij}$ : WW-Matrix zwischen Dipolen

$$= \left(\vec{d}_{12}^{\,i}\cdot\vec{e}_1\right)\left(\vec{d}_{12}^{\,j}\cdot\vec{e}_1\right)\frac{\omega_{k_0}}{2\varepsilon_0\hbar A}$$

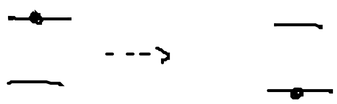
$e^{ik_0(z_j-z_i)}$ $\hat{=}$ phasengerechte Einkopplg.

enthält Effekte v. Sub und Superradianz

$$\dot{\Delta}^i_{12}(t) = -2i\tilde{p}^i_{12}(t)\left(\tilde{g}^{*\,i-k_0}_{12}\,\tilde{c}^0_R\left(t-\frac{z_i}{v_g}\right) + \tilde{g}^{*\,i+k_0}_{12}\,\tilde{c}^0_L\left(t+\frac{z_i}{v_g}\right)\right)$$

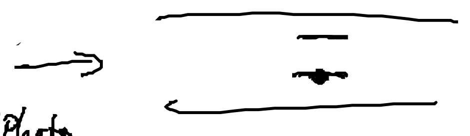
$+\ h.a.$ von außen angelegt

$$-2\gamma^i_{rad}\left(\Delta^i_{12}(t)-1\right)$$ spontane Emission in den Wellenleiter



$$+2\sum_{j\neq i}\gamma^{ij}_{rad}\underbrace{\tilde{p}^i_{21}(t)\,\tilde{p}^j_{12}\left(t-\frac{|z_i-z_j|}{v_g}\right)}_{\text{Dipol-Dipol-WW: Vielteilchenproblem}}e^{ik_0|z_i-z_j|}+h.a.$$

## 5.2. Eine Anwendung: Transmission eines Photons durch ein ZNS




$$T(\omega)_{z>0} = \frac{|\langle \tilde{c}^{+}_R(\omega)\,\tilde{c}_R(\omega)\rangle|}{\underbrace{|\langle \tilde{c}^{0+}_R(\omega)\,\tilde{c}^0_R(\omega)\rangle|}_{\text{Verhältnis v. Photonspektren mit und ohne ZNS}}}$$

Verhältnis v. Photonspektren mit und ohne ZNS:

$$\tilde{c}^{+}_{R}(z,\omega)\,\tilde{c}_{R}(z,\omega) \;\hat{=}\; \text{Observablen}$$

$$\tilde{c}_R(z,t) = \tilde{c}^{0}_R\left(t - \frac{z}{v_g}\right) + i v_g^{-1}\, \tilde{g}_{12}\, \tilde{p}_{12}\left(t - \frac{z}{v_g}\right)$$

$$\tilde{c}_R(z,\omega) = \tilde{c}^{0}_R(\omega)\, e^{i\omega \frac{z}{v_g}} + i v_g^{-1}\, \tilde{g}_{12}\, \tilde{p}_{12}(\omega)\, e^{i\omega \frac{z}{v_g}}$$

FT bzgl. $t$

$$\tilde{c}^{+}_{R}(z,\omega)\,\tilde{c}_{R}(z,\omega) = \left(c^{+0}_R(\omega) - i v_g^{-1}\, \tilde{g}_{12}^{*}\, \tilde{p}_{21}(\omega)\right)\left(c^{0}_R(\omega) + i v_g^{-1}\, \tilde{g}_{12}\, \tilde{p}_{12}(\omega)\right)$$

a, b, c, d

a) $\langle c^{+0}_R(\omega)\, \tilde{c}^{0}_R(\omega) \rangle$ mit $|$1 Photon rechtslaufend$\rangle$

$\cdot\, |$ZNS im Grundzustand$\rangle$

$$\equiv |1P\rangle\, |GZ\rangle$$

$$\langle 1P|\langle GZ|\; c^{+0}_R(\omega)\; c^{0}_R(\omega)\; |GZ\rangle\, |1P\rangle$$

$c^{+0}_R(\omega)$ an FT: $\underbrace{c^{+0}_R(t) = w(t)\, c^{+0}_R(0)}$

idd Lösg. zu $H_0$

$$c_R^{+0}(\omega) = w(\omega)\, c_R^{+0}$$

Wellpaket

$$\underbrace{\langle Gz | Gz \rangle}_{1} \langle 1P | \underbrace{c_R^{+0} c_R^{0} | 1P \rangle}_{1} |w(\omega)|^2$$

a) $= |w(\omega)|^2$

b) $\langle c_R^{+0}(\omega)\, \tilde{p}_{12}(\omega) \rangle = w^*(\omega) \langle c_R^{+0}\, \tilde{p}_{12}(\omega) \rangle$

$$= \langle Gz | \langle 1P | c_R^{+0}\, \tilde{p}_{12}(\omega) | Gz \rangle | 1P \rangle\, w^*(\omega)$$

$$= \langle Gz | \underbrace{\langle 0P | \tilde{p}_{12}(\omega) | 1P \rangle}_{?} | Gz \rangle\, w^*(\omega)$$

$w(t)\, \tilde{c}_R^{0}$

$$\tilde{p}_{12}(\omega) = \frac{FT_\omega\left( i\, \tilde{g}_{12}\, \Delta_{1L}(t)\, \tilde{c}_R^{0}(t) \right)}{\left( -i\left( \omega - \omega_{12} - \omega_{K_0} \right) + \gamma^{rad} \right)}$$

Fourtrafo aus

flgl. v. $\dot{\tilde{p}}_{12} = \ldots$

$$\langle 0P|\tilde{p}_{12}(\omega)|1P\rangle = \frac{\langle 0P|FT_\omega(i\bar{g}_{12}\Delta_{12}(t)\, w(t))|0P\rangle}{(\quad \dots \quad)}$$

$\langle 0P|\Delta_{12}|0P\rangle = ?$, aus $\langle 0P|\dot{\Delta}_{12}|0P\rangle =$

$-2\gamma_{rad}\left(\langle 0P|\Delta_{12}|0P\rangle - 1\right)$

$\langle 0P|\Delta_{12}|0P\rangle = 1$, wenn Atom im Grundzustand

$$b = -\frac{\gamma_{rad}|w(\omega)|^2}{-i(\omega+\delta)+\gamma_{rad}}$$

c) $c = b^*$, siehe oben

$$b + c = -\frac{2\gamma_{rad}^2|w(\omega)|^2}{(\omega+\delta)^2+\gamma_{rad}^2}$$

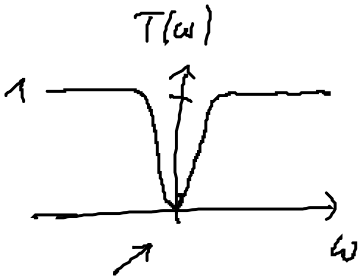
d) $\langle \tilde{p}_{21}(\omega)\,\tilde{p}_{12}(\omega)\rangle = \left|\ \tilde{p}_{12}(\omega) \text{ von oben übernehmen}\ \right|$

$$\frac{1}{(\omega+\delta)^2+\gamma_{rad}^2}\,|\bar{g}_{12}|^2 \langle \underset{GZ}{0P} |\, FT_\omega\big(\underset{\Downarrow\,1}{A_{12}}(t)\, w(t)\big)\, FT_\omega\big(\underset{\Downarrow\,1}{A_{12}}(t)\, w^*(t)\big) | \underset{GZ}{0P} \rangle$$

$$= \frac{|w(\omega)|^2}{(\omega+\delta)^2+\gamma_{rad}^2}\,|\bar{g}_{12}|^2$$

$$d = \frac{\gamma_{rad}^2\,|w(\omega)|^2}{(\omega+\delta)^2+\gamma_{rad}^2}$$

$$T(\omega) = a+b+c+d = 1 - \frac{\gamma_{rad}^2}{(\omega+\delta)^2+\gamma_{rad}^2}$$




$\delta = 0$

T verschwindet bei Resonanz