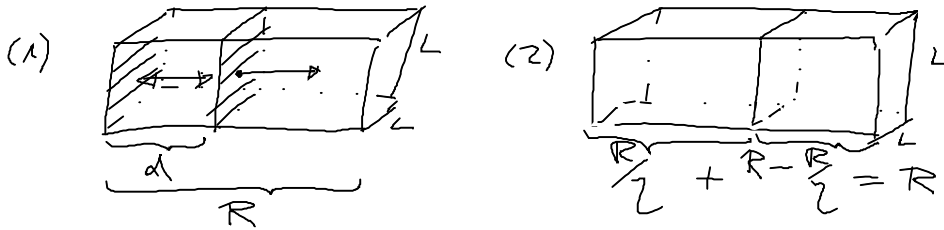


Casimir-Effekt: Rechnung über den Casimir-Kollen (oder Hohlraum)

aus Feiner, Special topics in Quantum Mechanics und William M.R. Simpson,
 „Surprises in Theoretical Casimir Physics“, Springer (2015); published Phd-thesis



$$U(d, L) = \frac{\hbar c}{2} \sum_{m, n, l=0}^{\infty} \mathcal{E}_{lmn}(d, L, L) \Theta(n, m, l) e^{-\lambda \mathcal{E}_{lmn}}$$

cut-off
funktion

Physikalische Motivation: elektromagnetisch
 induzierte Transparenz
 (bei hohen Frequenzen werden die Leiterplatten
 transparent für das Licht)

$$\Theta(n, m, l) = \begin{cases} 1, & n, m, l = 0 \\ 2, & n, m, l \neq 0 \end{cases}$$

$$U(d, L) = \frac{\hbar c}{2} \sum_{l=0}^{\infty} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{\Delta m}{\Delta m} \frac{\Delta n}{\Delta n} \sqrt{\left(\frac{2m\pi}{d}\right)^2 + \left(\frac{n\pi}{L}\right)^2 + \left(\frac{l\pi}{L}\right)^2} e^{-\lambda \mathcal{E}_{lmn}} \Theta(n, m, l)$$

$(\Delta m = \frac{2\pi}{d})$

$$= \frac{\hbar c L^2}{2\pi^2} \sum_{l=0}^{\infty} \int_0^{\infty} dm \int_0^{\infty} dn \sqrt{\left(\frac{2m\pi}{d}\right)^2 + m^2 + n^2} e^{-\lambda \mathcal{E}_l(m, n)} \Theta(l)$$

$d \ll L$
 $= \mathcal{E}_l(m, n)$

$$= \frac{\hbar c L^2}{2\pi^2} \left\{ \int_0^{\infty} dm \int_0^{\infty} dn \sqrt{m^2 + n^2} e^{-\lambda \mathcal{E}_0(m, n)} \Theta(0) \right. \\ \left. + \Theta(1) \sum_{l=1}^{\infty} \int_0^{\infty} dm \int_0^{\infty} dn \sqrt{\left(\frac{2m\pi}{d}\right)^2 + m^2 + n^2} e^{-\lambda \mathcal{E}_l(m, n)} \right\}$$

$U_0(x)$ das ist unabhängig von d, L (Offset)

$$U(d, \lambda) = \frac{\Theta(\lambda) \hbar c L^2}{2\pi^2} \sum_{l=1}^{\infty} \int_0^{\infty} dm \int_0^{\infty} dn \sqrt{\left(\frac{l\pi}{d}\right)^2 + m^2 + n^2} e^{-\lambda R_l(m, n)}$$

$$= \frac{\Theta(\lambda) \hbar c L^2}{2\pi^2} \sum_{l=1}^{\infty} \iint \left(\frac{l\pi}{d}\right) \sqrt{1 + m^2 \left(\frac{d}{l\pi}\right)^2 + n^2 \left(\frac{d}{l\pi}\right)^2} e^{-\lambda \frac{l\pi}{d} \sqrt{1 + \dots}}$$

$$m = \frac{l\pi}{d} \sqrt{z} \cos \varphi$$

$$n = \frac{l\pi}{d} \sqrt{z} \sin \varphi$$

$$= \frac{\Theta(\lambda) \hbar c L^2}{2\pi^2} \sum_{l=1}^{\infty} \int_0^{\frac{\pi}{2}} d\varphi \int_0^{\infty} dz \left| \frac{\partial(m, n)}{\partial(z, \varphi)} \right| \frac{l\pi}{d} \sqrt{1+z} e^{-\lambda \frac{l\pi}{d} \sqrt{1+z}}$$

$$= \frac{\Theta(\lambda) \hbar c L^2}{2\pi^2} \sum_{l=1}^{\infty} \left(\frac{l\pi}{d}\right)^3 \frac{\pi}{4} \int_0^{\infty} dz \sqrt{1+z} e^{-\lambda \frac{l\pi}{d} \sqrt{1+z}}$$

geometrische Reihe $\bar{x} = \pi l$

$$\frac{d^3}{d\bar{x}^3} e^{-\bar{x} \frac{l\pi}{d} \sqrt{1+z}} = \left(-\frac{l\pi}{d} \sqrt{1+z}\right)^3 e^{-\bar{x} \frac{l\pi}{d} \sqrt{1+z}}$$

$$= -\frac{\Theta(\lambda) \hbar c L^2}{8} \pi^2 \sum_{l=1}^{\infty} \int_0^{\infty} \frac{dz}{1+z} \left(\frac{d^3}{d\bar{x}^3} e^{-\bar{x} \frac{l\pi}{d} \sqrt{1+z}} \right)$$

$$= -\frac{\Theta(\lambda) \hbar c L^2}{8} \pi^2 \frac{d^3}{d\bar{x}^3} \int_0^{\infty} \frac{dz}{1+z} \sum_{l=1}^{\infty} e^{-\bar{x} \frac{l\pi}{d} \sqrt{1+z}}$$

$$= \frac{1}{e^{\bar{x} \frac{u}{d}} - 1}$$

$$= \frac{1}{1-q} - 1 = \frac{q}{1-q}$$

$$u = \sqrt{1+z} \quad du = \frac{1}{2\sqrt{1+z}} dz$$

$$= - \frac{\Theta(1) \hbar c L^2 \pi^2}{4} \frac{d^3}{d\bar{x}^3} \int_1^\infty du \frac{1}{u} \frac{1}{e^{\bar{x} \frac{u}{d}} - 1}$$

$$= \Theta(1) \frac{\hbar c L^2 \pi^2}{4} \frac{1}{d} \frac{d^2}{d\bar{x}^2} \int_1^\infty du \frac{e^{\bar{x} \frac{u}{d}}}{(e^{\bar{x} \frac{u}{d}} - 1)^2}$$

$$y = e^{\bar{x} \frac{u}{d}} \quad dy = \frac{\bar{x}}{d} e^{\bar{x} \frac{u}{d}} du$$

$$y|_{u=1} = e^{\bar{x}/d} \quad y|_{u \rightarrow \infty} = \infty$$

$$= \Theta(1) \frac{\hbar c L^2 \pi^2}{4d} \frac{d^2}{d\bar{x}^2} \int_{e^{\bar{x}/d}}^\infty dy \frac{d}{\bar{x}} \frac{1}{(y-1)^2}$$

$$\bar{y} := y-1 \quad d\bar{y} = dy$$

$$= \Theta(1) \frac{\hbar c L^2 \pi^2}{4d} \frac{d^2}{d\bar{x}^2} \int_{e^{\bar{x}/d}-1}^\infty d\bar{y} \frac{d}{\bar{x}} \frac{1}{\bar{y}^2}$$

$$= \Theta(\eta) [\dots] \frac{d^2}{d\bar{x}^2} \left\{ \frac{d}{d\bar{x}} \frac{d}{d\bar{x}} \frac{(\bar{x}/d)}{e^{\bar{x}/d} - 1} \right\}$$

Bernoulli-Formel $e^{\bar{x}/d} - 1 = \sum_{n=0}^{\infty} \frac{B_n}{n!} (\bar{x}/d)^n$

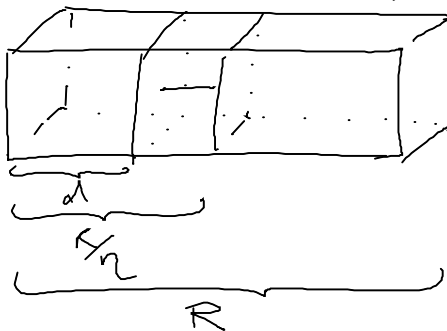
$\left[\begin{array}{l} B_0 = 1, B_1 = -\frac{1}{2}, B_2 = \frac{1}{6}, B_3 = 0 \\ B_4 = -\frac{1}{30} \end{array} \right]$ special points

$$= \Theta(\eta) [\dots] \frac{d^2}{d\bar{x}^2} \left\{ \sum_{n=0}^{\infty} \frac{B_n}{n!} \left(\frac{\bar{x}}{d}\right)^{n-2} \right\}$$

$$U(d, \bar{x}, L) = U_0(\bar{x}) + \Theta(\eta) \frac{\hbar c L^2 \pi^2}{2} \frac{d^2}{d\bar{x}^2} \left\{ \frac{1}{d} \sum_{n=0}^{\infty} \frac{B_n}{n!} \left(\frac{\bar{x}}{d}\right)^{n-2} \right\}$$

Berechnet wird die Energiedifferenz

$$\Delta U(d, \bar{x}, R) = \underline{U(d, \bar{x})} + \underline{U(R-d, \bar{x})} - \left[\underline{U\left(\frac{R}{2}, \bar{x}\right)} + \underline{U\left(R - \frac{R}{2}, \bar{x}\right)} \right]$$



$$L^2 = A \rightarrow \infty$$

$$R \rightarrow \infty$$

$$C_{\alpha} = \Theta(\eta) \frac{\hbar c L^2 \pi^2}{4}$$

$$\Delta U = U_0 + C_{\alpha} \frac{d^2}{d\bar{x}^2} \left\{ B_0 \frac{d}{\bar{x}^2} + B_1 \frac{1}{\bar{x}} + B_2 \frac{1}{2d} + B_3 \frac{\bar{x}^3}{d^2} + B_4 \frac{\bar{x}^4}{d^3} \right\}$$

$$\begin{aligned}
& + U_0 + c_\alpha \frac{d^4}{d\bar{x}^2} \left\{ B_0 \frac{R-d}{\bar{x}^2} + B_1 \frac{1}{\bar{x}} + B_2 \frac{1}{2(R-d)} + \dots \right\} \\
& - U_0 - c_\alpha \frac{d^2}{d\bar{x}^2} \left\{ \dots \right\} \\
& - U_0 - c_\alpha \frac{d^4}{d\bar{x}^2} \left\{ B_0 \frac{R-\bar{x}}{\bar{x}^2} + B_1 \frac{1}{\bar{x}} + B_2 \frac{1}{2(R-\bar{x})} + \frac{B_3 \bar{x}}{6(R-\bar{x})} \right\}
\end{aligned}$$

$\underbrace{\hspace{15em}}_{B_2 \text{ Terme, weil unabhängig von } \bar{x}^2}$

$$\begin{aligned}
\frac{\Delta U}{c_\alpha}(d, \bar{x}, R) &= \frac{B_4 \cdot 2 \cdot 1}{24} \left[\frac{1}{d^3} + \frac{1}{(R-d)^3} - \frac{1}{(R/2)^3} - \frac{1}{(R-R/2)^3} \right] \\
&+ \frac{B_5 \cdot 3 \cdot 2}{5!} \left[\frac{\bar{x}}{d^4} + \frac{\bar{x}}{(R-d)^4} - \frac{\bar{x}}{(R/2)^4} - \frac{\bar{x}}{(R-R/2)^4} \right] + \frac{B_6 \dots}{6!} \dots
\end{aligned}$$

$$\lim_{\bar{x} \rightarrow 0} \frac{U}{c_\alpha}(d, \bar{x}, R) = \frac{B_4}{12} \left[\frac{1}{d^3} + \frac{1}{(R-d)^3} - \frac{1}{(R/2)^3} - \frac{1}{(R-R/2)^3} \right]$$

$$\frac{U(d)}{c_\alpha} = \lim_{R \rightarrow \infty} \frac{U(d, R)}{c_\alpha} = \frac{B_4}{12} \frac{1}{d^3} = \frac{1}{360} \frac{1}{12} \frac{1}{d^3}$$

$$U(d) = -c_\alpha \frac{1}{360} \frac{1}{d^3} = -\frac{4c\pi^2}{720} \frac{\hbar^2}{d^3} \underbrace{\Theta(1)}_{=1 \text{ (Experiment)}}$$

damit Casimir-Effekt hergeleitet

- ohne Maclaurin-Fehler
- ohne Zeta-Riemann
- ohne heikle Berücksichtigung der verschwindenden zweiten Polarisationsmode
- und unabhängig vom cut-off \hbar !!

- Milonni leitet sogar Ausdruck

$$H = \frac{\hbar c}{2} \sum_{\mathbf{m}, \mathbf{n}, \mathbf{p}} \mathcal{R}_{\mathbf{m}, \mathbf{n}}(d, L, L) \Theta(\mathbf{m}, \mathbf{n}, \mathbf{p})$$

über den klassischen Strahlungsdruck
aus der Drehimpulserhaltung des
Maxwellfeldes her

- alles leitet Schwinger allein aus
der Van-der-Waals WW der leitenden
Platten her

(1.) Unscharferelation : Quantenmetrologie
(Atomuhren, Sensing)

(2.) EPR-Paradoxon, Bell'schen Ungleichungen:
Quanteninformation
(Kryptografie - Protokolle)

(3.) Anomaler Bohm Effekt: Quanten
Topologie
(fractionalen QHE, TI)
[Strom - Normale - Definition]

(4.) Casimir - Effekt : Energie der Vakuum
(Energiequelle ?)
[Quantum sensing]

(5.) Hansury Brown - Tavis Effekt :