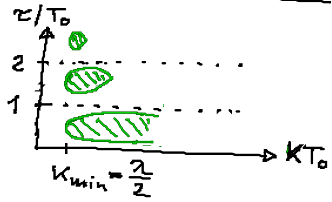
English Summary:

Stabilization of unstable fixed point by time-delayed feedback

Stab. boundaries : $\mathrm{Re}\,\Lambda = 0$ $\Rightarrow$



multiple feedback (ETDAS)

$$u = -K \sum_{n=0}^{\infty} R^n \left[x(t-n\tau) - x(t-(n+1)\tau) \right]$$

$$\Rightarrow \boxed{\Lambda + K \frac{1 - e^{-\Lambda\tau}}{1 - Re^{-\Lambda\tau}} = \lambda + i\omega} \qquad (0 \leq R < 1)$$

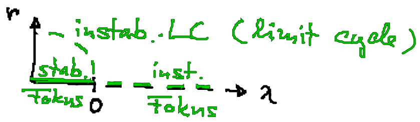
2.2.2 Stabilisierung instabiler periodischer Orbits

Normalform einer subkrit. Hopf-Bif.:

$$\boxed{\dot{z} = (\lambda + i\omega + (1+i\gamma)|z|^2) z + b(z(t-\tau) - z(t))} \qquad z \in \mathbb{C}$$

$\lambda < 0$, $\omega = 1$, $\gamma > 0$, $b = b_0 e^{i\beta} \in \mathbb{C}$

ohne Kontrolle :



$$z = re^{i\phi} : \quad \dot{r} = (\lambda + r^2) r$$
$$\dot{\phi} = \omega + \gamma r^2$$

LC: $r^2 = -\lambda$ ex. für $\lambda < 0$

$$\dot{\phi} = \omega - \gamma\lambda \Rightarrow T = \frac{2\pi}{\omega - \gamma\lambda}$$

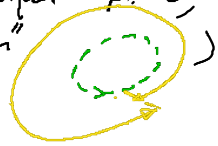
Periode des UPO (unstable periodic orbit)

nichtinvasive Kontrolle (oBdA: $\omega = 1$):

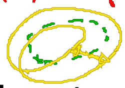
wähle $\tau \overset{!}{=} nT = \frac{2\pi n}{1 - \gamma\lambda}$ $\qquad n \in \mathbb{N}$

(Pyragas-Kurve in der (τ, λ)-Ebene)

Odd-number orbit (= ungerade Zahl von reellen Floquet-Exp. > 0, hier $n=1$; orbit ohne „Torsion")



$\longleftrightarrow$ flip-Bifurk. (Perioden-verdoppl.)



Nakajima (1997) : Stabilisierung von odd-number orbits durch zeitverzögerte Rückkopplung nicht möglich („Odd-number Theorem")

Fiedler, Flunkert, Georgi, Hövel, Schöll: PRL 98, 114101 (2007)
„Odd-number Theorem" gilt nicht !

Just, Fiedler, Georgi, Flunkert, Hövel, Schöll: PRE 76, 026210 (2007)

Gegenbeispiel : subkrit. Hopf-Bifurkation (Orbit ohne Torsion !)
Wähle b_0, β geeignet !

Fiedler, Yanchuk, Flunkert, Hövel, Wünsche, Schöll : PRE 77, 066207 (2008)
Sattel-Knoten-Bif. von Grenzzyklen

Kehrt, Hövel, Flunkert, Dahlem, Rodin, Schöll: Eur. Phys. J. B 68, 55 (2009)
raum-zeitl. Muster

Erzgräber, Just : Physica D 238, 1680 (2009)
Brown, Postlethwaite, Silber : Physica D 240, 859 (2011)
Flunkert, Schöll : PRE 84, 016214 (2011)

Exp. mit el. Stromkreis : Loewenich, Benner, Just : PRE 82, 036204 (2010)

Laserexp.: Schikora, Wünsche, Henneberger : PRE 83, 026203 (2011)

Stabilisierung instab. period. Orbits :

nichtinvasive Kontrolle : $\tau \overset{!}{=} nT = \frac{2\pi n}{1-\gamma\lambda}$ (T = Periode der UPO)

Pyragas-Kurven in der (τ, λ)-Ebene

Hopf-Kurve : lin. Stab. des Fixp. $z(t) \sim e^{\Lambda t}$

$$\Lambda + b\,(1-e^{-\Lambda\tau}) = \lambda + i \qquad \text{vgl. } 2.2.1$$

Hopf-Bif.: $\Lambda = i\omega$:
$$0 = \lambda + b\,[\cos(\beta-\omega\tau) - \cos\beta] \qquad (1)$$
$$\omega - 1 = b_0\,[\sin(\beta-\omega\tau) - \sin\beta] \qquad (2)$$

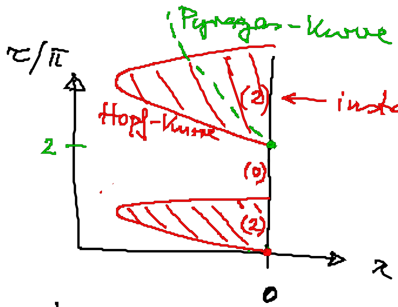
(1) $\Rightarrow \omega\tau = \pm \arccos\left(\frac{b_0 \cos\beta - \lambda}{b_0}\right) + \beta + 2\pi n$

$$\left.\begin{aligned} \left(\frac{b_0\cos\beta - \lambda}{b_0}\right)^2 &= \cos^2(\beta - \omega\tau) \\ \left(\frac{\omega - 1 + b_0 \sin\beta}{b_0}\right)^2 &= \sin^2(\beta - \omega\tau) \end{aligned}\right\} \cos^2 + \sin^2 = 1 \quad (3)$$

(3)
$\Longrightarrow \omega = g(\lambda, b_0, \beta) \overset{\text{elim.}}{\Longrightarrow} \tau = h(\lambda, b_0, \beta)$ Hopf-Kurven in (τ, λ) Ebene

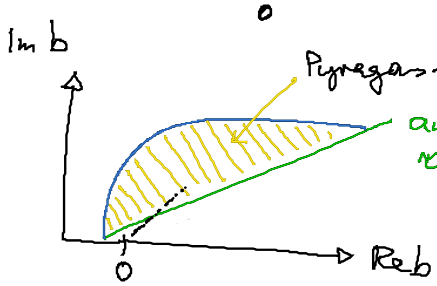
ohne Kontrolle $(b = 0)$ subkrit. Hopf

mit Kontrolle $b = b_0 e^{i\beta}$ superkrit. Hopf entlang der Pyragaskurve



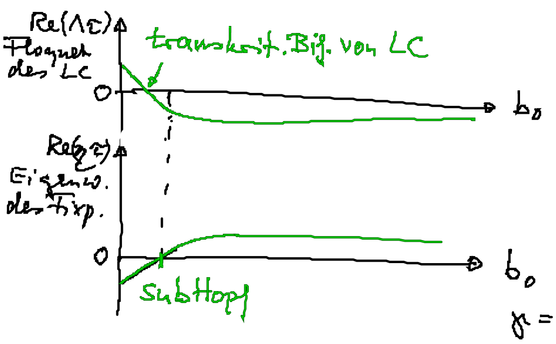
← instabile Dim. des Fixpktes

Fiedler et al, PRL (2007)



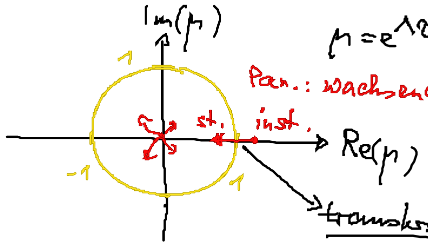
Pyragas-Kontrolle funktioniert (Kopplungsphase $\beta \neq 0$!)

analyt. Bed.: $\tau_{Hopf}(\lambda) < \tau_{Pyragas}(\lambda)$



für festes $\beta = \frac{\pi}{4}$

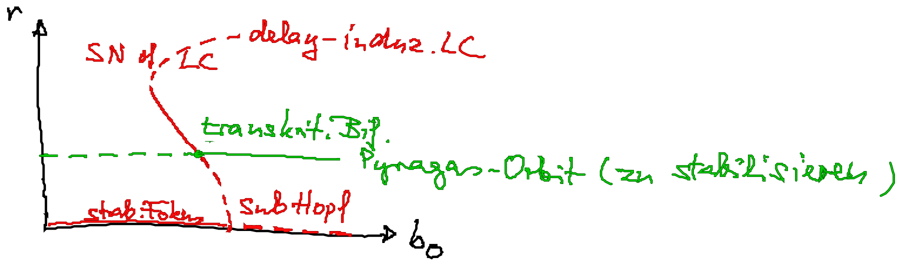
$\gamma = -10, \quad \tau = \frac{2\pi}{1 - \gamma\lambda}, \quad \lambda = -0.005$



$\mu = e^{\Lambda\tau}$ (Floquet-Mult.)

Par.: wachsendes b_0

transkrit. Bif. eines LC



korrektes Odd-Number-Theorem: E.W. Hooton, A. Amann, PRL 109, 154101 (2012)

Analytical limitations for time-delayed feedback control in autonomous systems

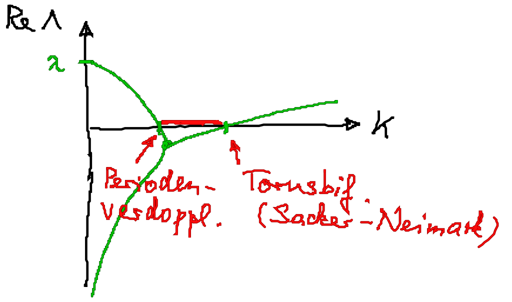
A. Amann, E.W. Hooton: Phil. Trans. R. Soc. A (2013)

2.2.3 Chaos-Kontrolle durch zeitverzögerte Rückkopplung

Pyragas, Phys. Lett. A 170, 421 (1992)

$$\boxed{\dot{x} = f(x) + KA[x(t-\tau) - x(t)]} \quad x \in \mathbb{R}^n, \quad n \times n \text{ Kopplungsmatrix } A$$

$\dot{x} = f(x)$: chaot. Attraktor mit unendl. vielen UPOs



unkontroll. Floquet-Problem

$\delta x = e^{\lambda t} u(t)$, $u(t) = u(t+T)$

$(\lambda + i\omega) u + \dot{u} = Df\, u$

kontr. (diagonale Kontrolle)

$\Lambda u + \dot{u} = Df\, u + (Ke^{-\Lambda\tau} - 1) u$

$$\overset{\text{Just}}{\Longrightarrow} \quad \boxed{\Lambda + K(1 - e^{-\Lambda\tau}) = \lambda + i\omega}$$

Schöll u. Schuster (eds.): Handbook of Chaos Control (2008)