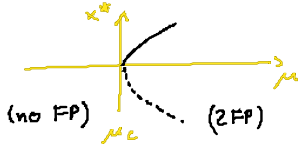


English Summary 1-

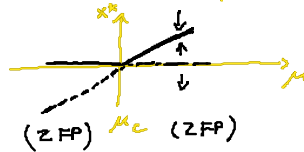
1.3. Bifurcations

(A) Zero-eigenvalue bifurcations: $\lambda < 0 \rightarrow \lambda > 0$
 $\det > 0 \quad \det < 0$

saddle-node bif.



transcritical bif.



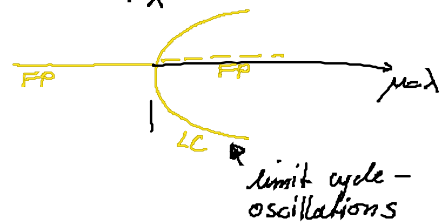
pitchfork bif.



(B) Hopf-bifurcation: $\lambda_{1,2} = \lambda_0 \pm i\omega$

$\lambda_0 < 0 \rightarrow \lambda_0 > 0$
 $r \uparrow$

$$\dot{z} = (\lambda + i\omega \mp (r + i\gamma)|z|^2)z$$

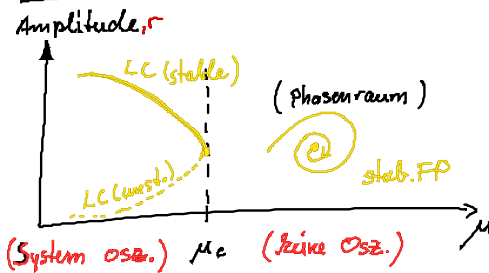


osc. amplitude $r \rightarrow 0$
 frequency $\neq 0$

(C) Lokale Bifurkationen von Grenzyklen

Startpunkt jetzt: Grenzyklus, nicht FP
 keine einfache lin. Stabilitätsanalyse

(C1) Sattel-Knoten-Bifurkation eines Grenzyklus



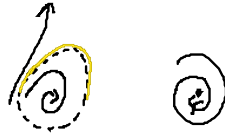
Amplitude $\neq 0$

Frequenz $\neq 0$

Phasenraum:



$\mu < \mu_c$



$\mu = \mu_c$ $\mu > \mu_c$

[Floquet-Theorie nötig]

Englisch:
condensation of paths
fold bifurcation of LC

(CZ) Perioden - Verdopplung (flip - bifurcation)



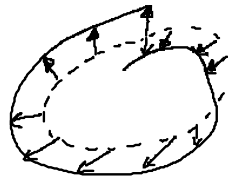
Periode - 1
Grenzzyklus



Periode - 2
Grenzzyklus

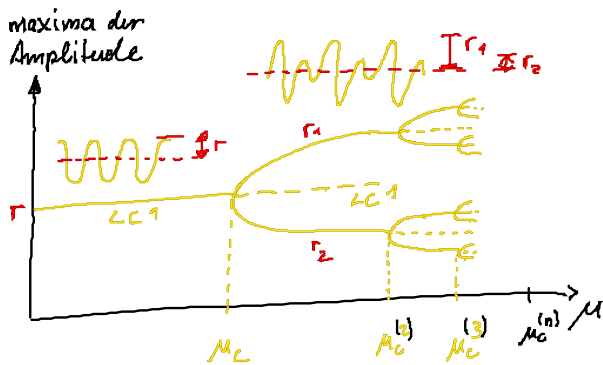
Möbius-band

mind. 3D Phasenraum



phase flip π
nach einem Umlauf

Torsion benachbarter Trajektorien



$n=0$
 T

$n=1$
 $2T$

$n=2$
 $4T$

n
 $2^n T$

Floquet-Exp. $\Lambda = \lambda + i\omega$

Bif.: $\lambda = 0$
 $\omega T = \pi$

\Rightarrow Floquet-Multiplikatoren
 $\mu = e^{\Lambda T} = e^{i\pi} = -1$

häufig Periodenverdopplungs-Kaskade
ins Chaos (Feigenbaum-Szenario)

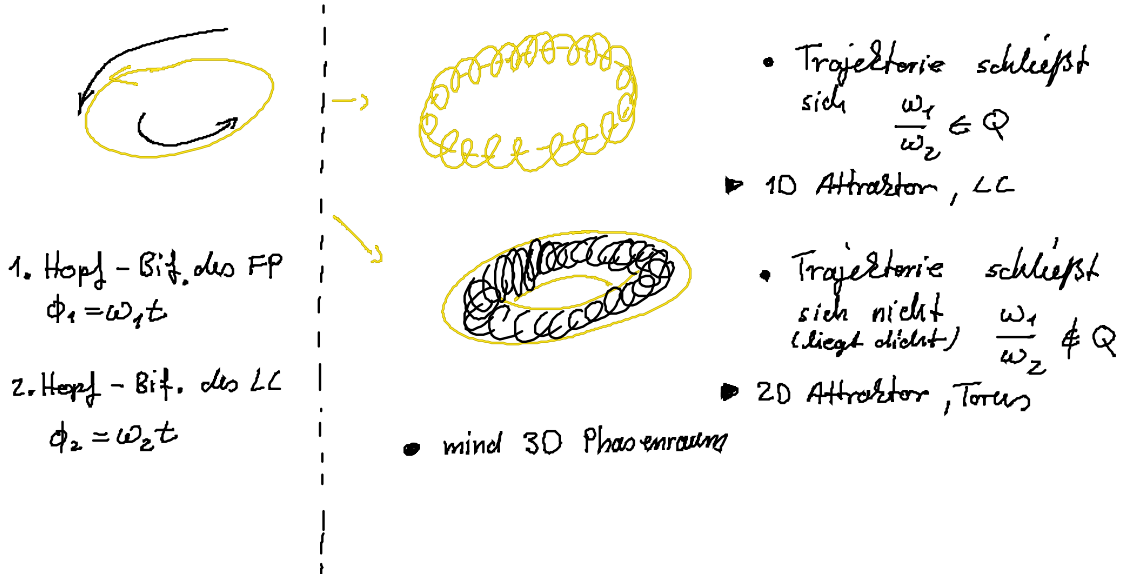
\rightarrow unendlich viele instabile periodische Orbits der Periode $2^n T$
($n=0, 1, 2, \dots$)



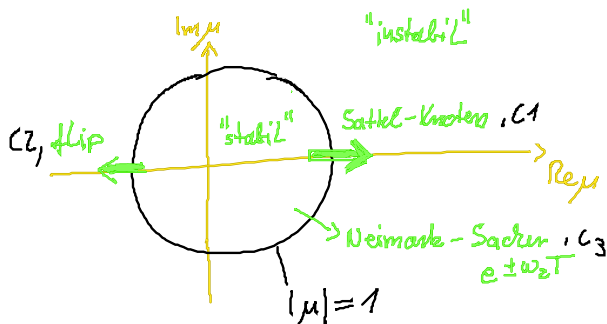
chaos

(C3) Sackur - Neimark - Bifurkation

(Sekundäre Hopf - Bif. eines Grenzzykluses)



[Fall $\frac{\omega_1}{\omega_2} \in \mathbb{Q}$: frequency locking]

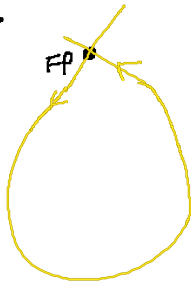


Floquet Multiplikator $\mu = e^{\lambda T}$

(D) Globale Bifurkationen vom Grenzzyklus

globale qualitative Änderung des Phasenportraits

z.B.

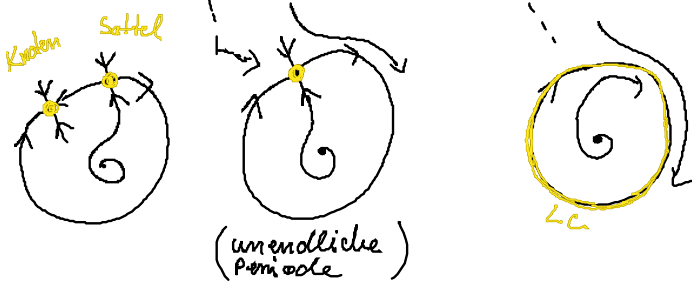
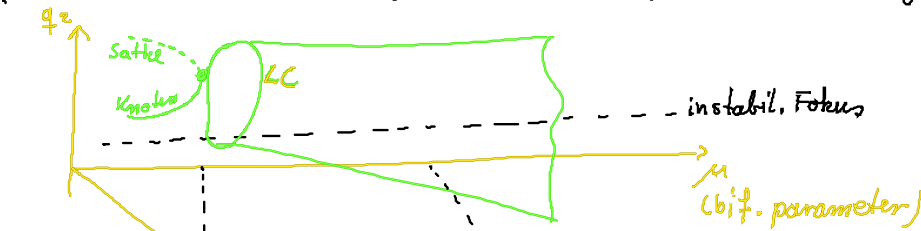


homoklinur
Orbit
(an der Bif.)



kein loop
(nach Bif.)

(D1) Sattel-Knoten-Bif. auf einem Grenzzyklus (Omega explosion)



Amplitude $\neq 0$
Frequenz $\rightarrow 0$

Saddle-node infinite period (SNIPER)
saddle-node on invariant cycle (SNIC)

Einfaches generisches Modell für SNIPER

$$\begin{aligned}\dot{x} &= x(1-x^2-y^2) + y(x-b) \\ \dot{y} &= y(1-x^2-y^2) - x(x-b)\end{aligned}$$

Dizinger, Ning, Hu, PRE 50, 3508 (1994)

Lasor: Ziemann, Aurd, Lingnan
Schöll, Lüdge
Europhys. Lett 103 (2013)

in Polarkoordinaten $x = r \cos \varphi$
 $y = r \sin \varphi$

$$\begin{cases} \dot{r} = r(1-r^2) \\ \dot{\varphi} = b - r \cos \varphi \end{cases}$$

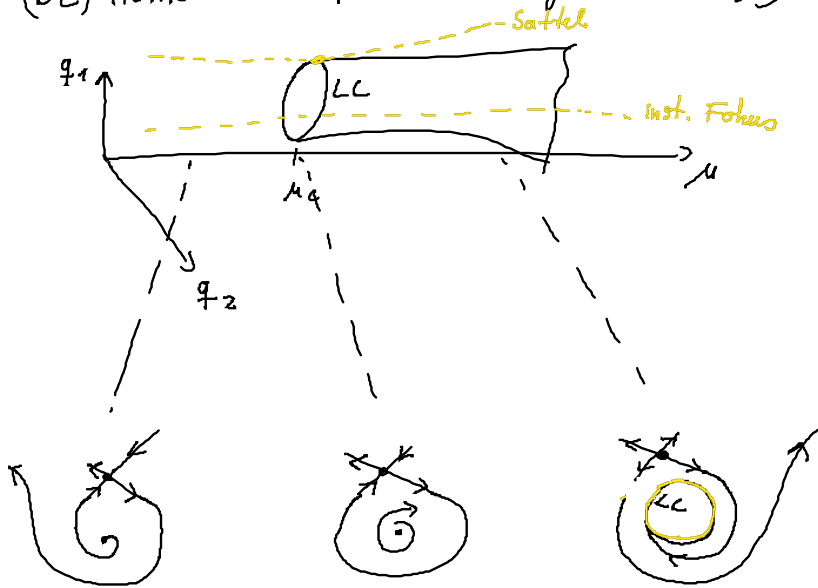
Fixpunkte: $r=0$ \leftarrow Fokus im Ursprung
 $r=1, b = \cos \varphi \Rightarrow \varphi = \arccos b \quad (b < 1)$
 \rightarrow Sattler & Knoten SNIPER bei $b=1$

$b > 1$: LC mit $r=1 \quad \dot{\varphi} = b - \cos \varphi > 0 \quad \forall \varphi$

$$T = \int_0^T dt = \int_0^{2\pi} \frac{d\varphi}{b - \cos \varphi} = \frac{2\pi}{\sqrt{b^2 - 1}}$$

$\rightarrow T \rightarrow \infty$ für $b \rightarrow 1$
 (skaliert mit $\frac{1}{\sqrt{b^2 - 1}}$)

(DZ) Homokline Bif. (blue-sky catastrophe)



Amplitude $\neq 0$
 Frequenz $\rightarrow 0$