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# Interface Mobilities for Low-Noise Design of Structure-Borne Sound Sources

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**Summary**
The source descriptor and coupling function have been introduced for a characterization of sources of structure-borne sound with single-point coupling to receiver structures. This concept offers a representation of an installation where a mismatching of the source and receiver mobilities can be studied directly for low-noise design. For the multi-point case, the source descriptor and coupling function can be reformulated by incorporating interface mobilities. Here, the forces, velocities and mobilities at the contact points are decomposed into interface orders by means of discrete Fourier series, thus yielding source descriptor and coupling function orders of equal number as there are contact points. In addition to the source and receiver mobility mismatching for low-noise design, this approach allows for a mismatching of the source descriptor and coupling function orders. The present paper introduces approaches for altering structure-borne sound sources in order to create or amplify a mismatching in terms of orders. Based on analytical solutions, such approaches are investigated numerically and the findings are corroborated by measurements on laboratory source structures.

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## 1. Introduction

For the majority of noise problems in engineering practice, the input data relates to sources of structure-borne sound. Vibrating machine parts, for example, radiate air-borne sound to the surrounding air, impart liquid-borne sound to liquid-filled systems such as pipes and transmit structure-borne sound to connected structures. Although it would be meaningful to reduce the noise levels directly at the source, few methods and approaches for low-noise design of structure-borne sound sources are yet available.

The concept of source descriptor $S$ and coupling function $C_f$ [1] is commonly recognized to form a rigorous basis for structure-borne sound source characterization, cf. Refs. [2, 3].

$$S = \frac{1}{2}\frac{|v_{FS}|^2}{Y_S^*}, \quad C_f = \frac{Y_S^* Y_R}{|Y_S + Y_R|^2} \tag{1}$$

The activity of the source is described by the free velocity $v_{FS}$, while the mobilities of the source $Y_S$ and the receiver $Y_R$ represent the passive dynamic properties. The source descriptor can be interpreted as the source's ability to deliver power, while the coupling function represents a filter which determines how much of the source descriptor results in power at the source-receiver interface.

The source descriptor and coupling function offer a representation of an installation where a mismatching of the source and receiver mobilities can be studied directly for low-noise design [1]. This concept is valid for sources and installations with a single contact point and a single component of motion, only. An extension to the more comprehensive multi-point and multi-component case is therefore expected to reveal additional physical insight that may be used to develop further procedures for low-noise design.

The characteristic power [3] represents a rigorous extension of the source descriptor for the multi-point and multi-component case. Due to the required matrix inversion, however, an insight to the underlying physics is obscured. An alternative approach is given by the concept of interface mobilities, where the velocities, forces and mobilities at the machine interface are decomposed into orders by means of a spatial Fourier series [4]. For structure-borne sound source characterization, the coupling between different orders is found to be negligible, yielding a series of source descriptor and coupling function orders [4]

$$S_p = \frac{1}{2}\frac{|\hat{v}_{p,FS}|^2}{\hat{Y}^*_{p-p,S}}, \quad C_{f,p} = \frac{\hat{Y}^*_{p-p,S}\hat{Y}_{p-p,R}}{|\hat{Y}_{p-p,S}+\hat{Y}_{p-p,R}|^2}. \quad (2)$$

Here, $\hat{v}_p$ represents the $p$-th order velocity and $\hat{Y}_{p-p}$ is the interface mobility that describes the coupling between force and velocity of order $p$.

In addition to the source and receiver mobility mismatching for low-noise design, the concept of interface mobilities allows to study a mismatching of the source descriptor and coupling function orders. For noise reduction, the dominating orders of the source descriptor should be matched with weak coupling function orders or vice versa.

The present paper introduces approaches for altering structure-borne sound sources in order to create or amplify a mismatching in terms of orders. After a revision of the concept of interface-mobilities in Sec. 2, such approaches are investigated numerically in Sec. 3.1 and the findings are corroborated by measurements on laboratory source structures in Sec. 3.2.

## 2. Revision of the concept of interface mobilities

As described in the literature, the concept of interface mobilities has been to consider a continuous interface between a source and a receiver that passes all contact points, see for example Ref. [4]. Along such an interface the field variables such as forces and velocities can be considered continuous and strictly periodic. By means of a continuous Fourier series, the forces and velocities can be decomposed into force and velocity orders, respectively. For the case of a spatial velocity distribution $v(s)$ along an interface of length $C$, the corresponding transform pair can be written as

$$v(s) = \sum_{p=-\infty}^{\infty} \hat{v}_p e^{jk_p s}, \quad k_p = \frac{2p\pi}{C}, \quad p \in \mathbb{Z} \quad (3)$$

and

$$\hat{v}_p = \frac{1}{C}\int_0^C v(s) e^{-jk_p s}\,\mathrm{d}s, \quad (4)$$

where $s$ is the coordinate along the contour. The decomposition is illustrated in Fig. 1, where a continuous velocity distribution along a circular contour is described by means of the continuous velocity orders. For simplicity, the positive and negative first-order velocities have been combined. Here, the zero-order velocity can be interpreted as a translational motion, while the first orders describe a rotation similar to rigid body vibrations. It is important to note that for the description of a continuous distribution in general, an infinite number of orders is required, see Eq. (3).

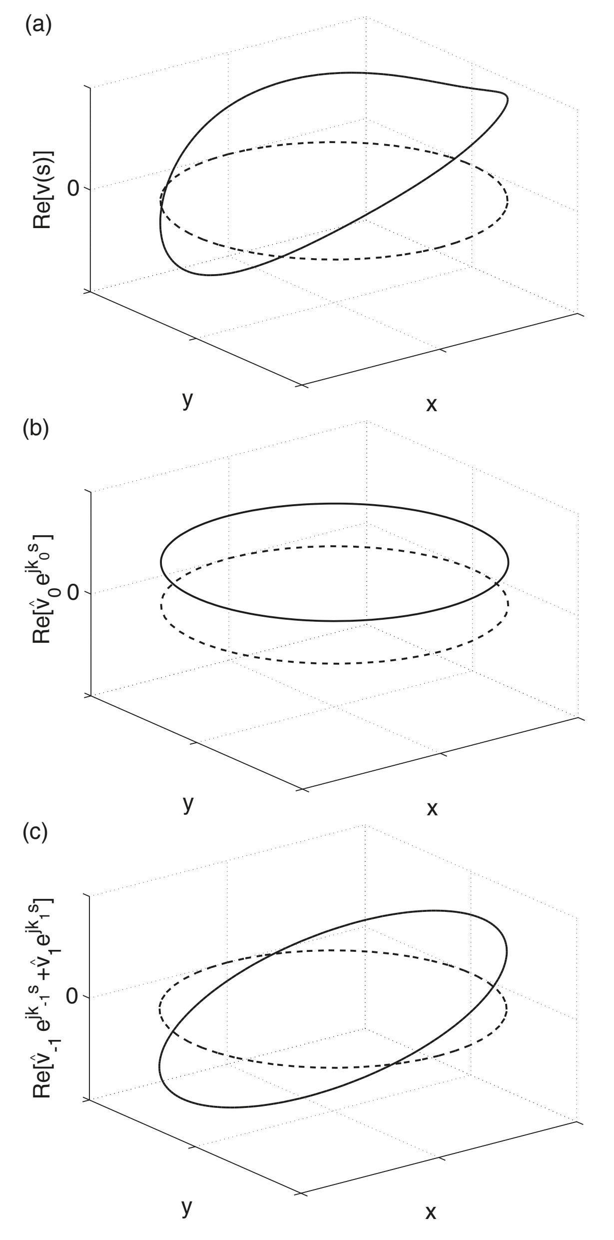


Figure 1. Illustration of the order decomposition for a continuous velocity distribution along a circular contour: (a) spatial velocity; (b) zero-order velocity; (c) first-order velocity. — velocity; – – – contour.

The above definition of the interface-order decomposition poses a few problems regarding practicality and interpretation. An interface with known length $C$ has to be defined and the integral in Eq. (4) has to be solved for a theoretically infinite number of orders. As will be shown below, moreover, the interpretation of the interface orders following Fig. 1 can be misleading for installations with multiple contact points. Analytical solutions for the integral representation of the Fourier orders in Eq. (4) are seldom available which is why the interface has to be discretized.

### 2.1. Discretization

For the discretization, the sampling theorem has to be observed, where at least two samples per wavelength are required for a proper solution [5]. Consequently, a given number of sampling points along the contour results in a frequency limit of the order representation. At frequencies above that limit, the original distribution at locations between the sampling points cannot be reproduced properly without any further knowledge of the system under consideration.

For sources of structure-borne sound with a multi-point coupling to receiver structures, the connection points between the source and the receiver can be viewed as a predetermined discretization of the theoretically continuous interface. By means of a normalized impulse train employing the Dirac delta function, the continuous velocity can be constructed from the velocities at the $N$ contact points

$$v(s) = \frac{C}{N} \sum_{n=1}^{N} v_n \delta(s - nC/N). \quad (5)$$

Here, the interface is assumed to be formed in a way yielding an equi-distant spacing between the contact points. After substitution in Eq. (4) and solving the integral, the velocity orders are found to be given by

$$\hat{v}_p = \frac{1}{N} \sum_{n=1}^{N} v_n e^{-j2\pi pn/N}. \quad (6)$$

The above equation represents the discrete Fourier transform of the velocity at the contact points and thus allows the employment of fast Fourier transform algorithms for the calculation of the velocity orders [5].

Owing to the periodicity of the complex exponential function, the coefficients of a discrete Fourier transform are repeated in accordance with the number of sampling points, i.e. $\hat{v}_p = \hat{v}_{p \pm N}$. This effect is illustrated in Fig. 2 for the order zero and a four-point sampling. The repetition is seen to result from the fact that the exponential functions of the corresponding Fourier coefficients describe identical amplitude distributions at the sampling points.

With Eq. (6) it is possible to calculate an infinite number of velocity orders which allows to accommodate the sum in Eq. (3). As the repeated orders do not yield additional information at the contact points, however, the series can be truncated as

$$v_n = \sum_{p=-\lfloor N/2 \rfloor}^{\lceil N/2-1 \rceil} \hat{v}_p e^{j2\pi pn/N}. \quad (7)$$

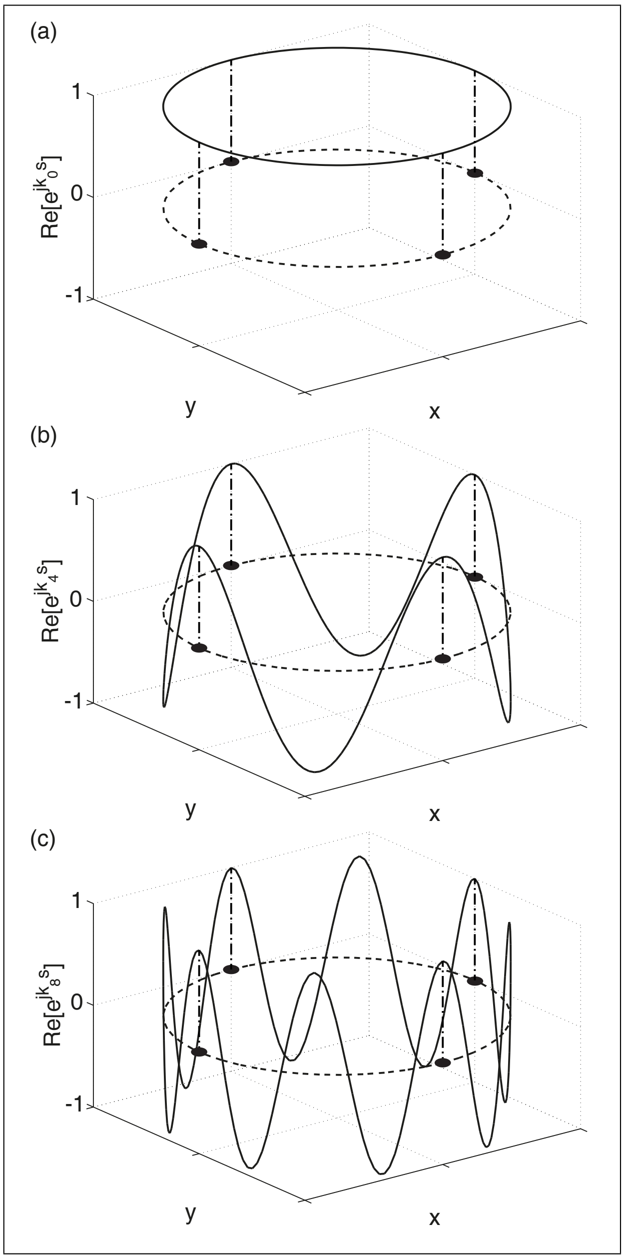


Figure 2. Illustration of the order decomposition for a continuous velocity distribution along a circular contour: (a) zero order; (b) forth order; (c) eighth order. — order distribution; − − − contour; − · − · − amplitude at sampling point; • sampling point.

### 2.2. Conclusion

Eqs. (6) and (7) form the transform pair of a discrete Fourier series [6] and allow for a simplified definition of the interface mobility approach. Hence, the interface mobilities are given by

$$\hat{Y}_{pq} = \frac{1}{N^2} \sum_{n=1}^{N} \sum_{m=1}^{N} Y_{nm} e^{-j2\pi pn/N} e^{-j2\pi qm/N}. \quad (8)$$

The data at the contact points between a source and a receiver are fully sufficient for source characterization as well as the description of the transmission process. With regard to the transmitted power for instance, there is no need to calculate the field variables at additional locations because a transfer of vibrational power between the source and the receiver only occurs at the contact points. The predetermined sampling of the interface contour due to the connection points, therefore, does not represent a frequency limit. Hence, for the description of multi-point vibrational sources and installations, Eq. (7) yields exact results at all frequencies. With equally many orders as there are contact points, the total source descriptor follows as

$$S = \sum_{p=-\lfloor N/2 \rfloor}^{\lceil N/2-1 \rceil} S_p. \tag{9}$$

The use of discrete Fourier series for the multi-point case clarifies the interpretation of the interface orders. The zero-order velocity, for instance, does not only describe a translation along the interface as shown in Fig. 1(b). It also covers higher-order vibrations where the contact points act in phase, see Fig. 2.

## 3. Manipulation of the source descriptor orders

The source descriptor orders consist of the free velocity orders and the interface mobilities of the source, while the coupling function orders represent a ratio of source and receiver interface mobilities, see Eq. (2). In a first attempt to create or amplify a mismatching in terms of orders, a manipulation of the source descriptor orders is presented here. In order not to affect the coupling function orders, furthermore, this investigation considers approaches for altering the source activity only.

### 3.1. Numerical study

For a numerical study of approaches for altering the source descriptor orders, a structure-borne sound source has been modeled as a rectangular finite plate with free-free boundary conditions connected to a cuboidal rigid mass via four points. The combination of plate and mass facilitates three characteristic frequency regions. The mobilities of the source model show a mass-like behavior below approximately 10 Hz, followed by a frequency band governed by the plate stiffness until around 100 Hz after which the source is controlled by the resonances of the plate. Three contact points on the plate structure at locations remote from the connection points between the mass and the plate represent the source-receiver interface. As discussed in the previous section, a three-point interface yields the three orders −1, 0 and 1.

The mobilities of the rigid mass are obtained from Ref. [7] and those of the finite plate from Ref. [8]. By application of force equilibrium and continuity of the velocity, the combined system can be calculated. In order to obtain the free velocity, the internal source mechanisms of the source are modeled as a point excitation located on the rigid mass. For a force excitation at an off-center position, the corresponding source descriptor orders are plotted in Fig. 3(a). In analogy with Fig. 1(c), the positive and negative first-order source descriptors have been combined.

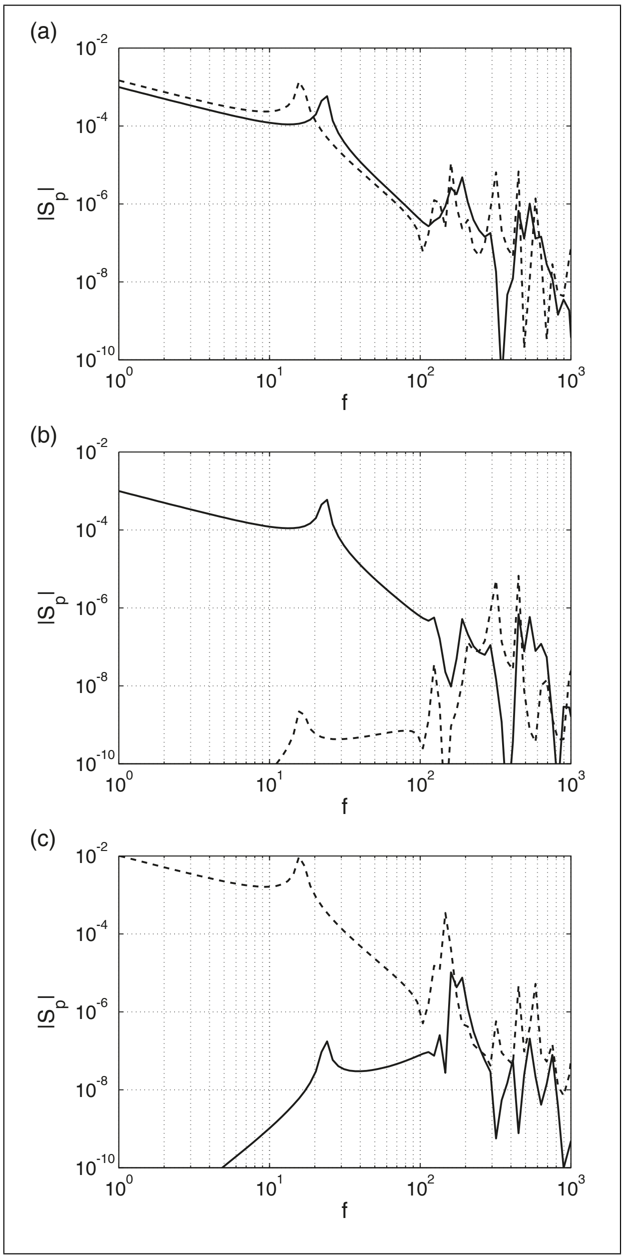


Figure 3. Numerical results for a manipulation of the source descriptor orders: (a) source with off-center force excitation; (b) source with force excitation at the center of mass; (c) source with off-center moment excitation. — zero-order source descriptor; – – – first-order source descriptor.

In Fig. 3(a), the zero- and first-order source descriptors are found to be of the same order of magnitude. Upon recalling that the zero-order velocity manages to fully describe the translational motion and the first-

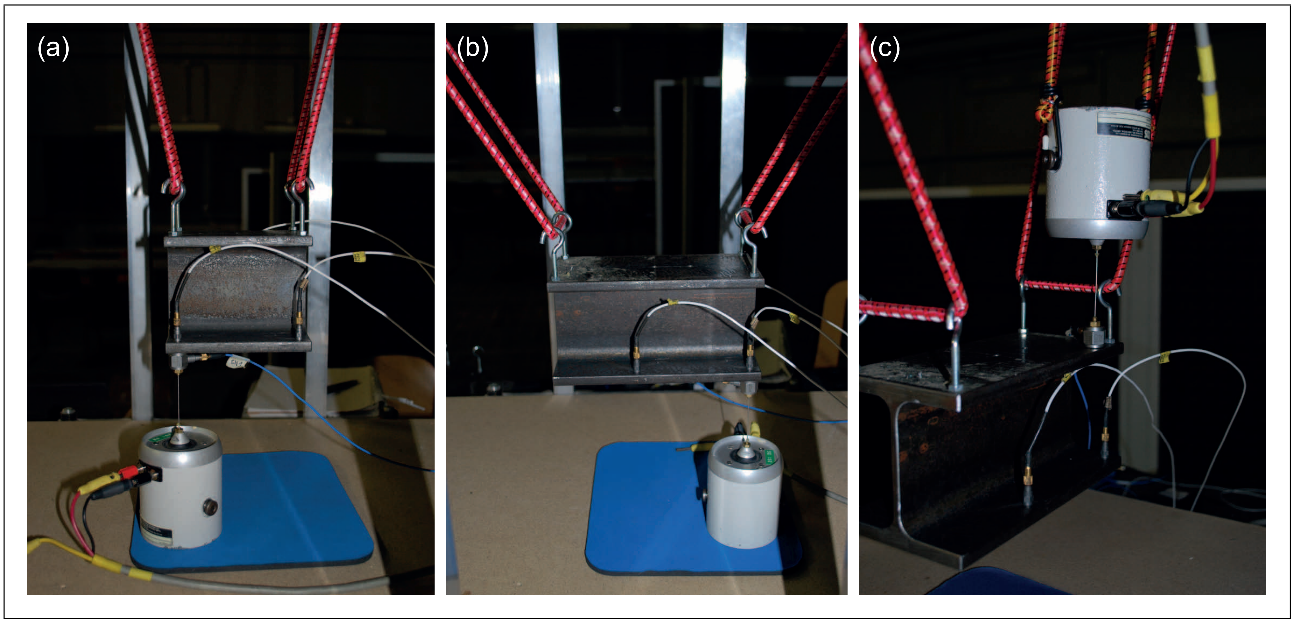


Figure 4. Experimental setup for measurements on laboratory source structures: (a) mobility measurement; (b) mobility measurement on enlarged source; (c) free-velocity measurement.

order velocities the rotational motion of a rigid body, see Sec. 2, possible modifications for altering the orders can be devised. By changing the location of the force excitation on the rigid mass, the translational and rotational components of motion will be shifted relative to one another. For the case of a force excitation at the center of mass, the source will mainly exhibit a translational motion. As shown in Fig. 3(b), therefore, the first-order source descriptor will be reduced.

The reduction of the first-order source descriptor due to a force excitation at the center of the source, however, is confined to the mass- and stiffness-controlled frequency regions. In the resonance-controlled region, the zero- and first-order velocities also describe higher-order vibrations, see Fig. 2. Although these higher orders also are affected by a modification of the excitation location, the effect is found to be frequency dependent in this upper region.

An alternative approach for altering the source descriptor orders is to change the type of excitation. In Fig. 3(c) are shown the resultant orders for the case where the initial force excitation from Fig. 3(a) is changed into a moment excitation. The source now mainly exhibits a rotational motion and the zero-order source descriptor is reduced. As for the previous case, however, the effect is mainly observed in the frequency regions governed by the mass and the stiffness.

### 3.2. Experimental study

For an experimental study of approaches for altering the source descriptor orders, a compact laboratory source structure consisting of plate-like elements is selected, see Fig. 4. Made from double-T steel, the mobilities of the source structure exhibit a distinct mass-controlled frequency region up until approximately 200 Hz, followed by a narrow-band stiffness-like behavior and resonances. As for the numerical study, the source has a three-point interface, yielding the orders $-1$, 0 and 1.

The measurement setup for obtaining the mobilities is shown in Fig. 4(a), where the source structure is freely suspended and excited with a shaker. A similar setup is used for the free-velocity measurements, see Fig. 4(c). Here, the transfer mobilities from the different points of excitation to the contact points at the interface are measured. By multiplying such transfer mobilities with a unit force, the location of the excitation yielding the free velocity can be changed without altering the induced force. The range of validity of the measurement results is limited to 30 – 3000 Hz due to the eigenfrequency of the suspension and the coherence function.

The zero- and first-order source descriptors of the laboratory source for a force excitation at an off-center location are shown in Fig. 5(a), where $S_{-1}$ and $S_1$ have been combined. When moving the point of excitation for the free velocity to the center of mass, the first-order source descriptor is reduced, see Fig. 5(b).

Due to the complexity of a moment excitation, this case is not studied experimentally. However, a different approach for a reduction of the first-order source descriptor is presented here. Instead of changing the location of the force excitation to the center of mass, the source structure can be modified in order to move the center of mass to the point of excitation. This is done for the source structure shown in Fig. 4(b), where a longer source structure is used to simulate

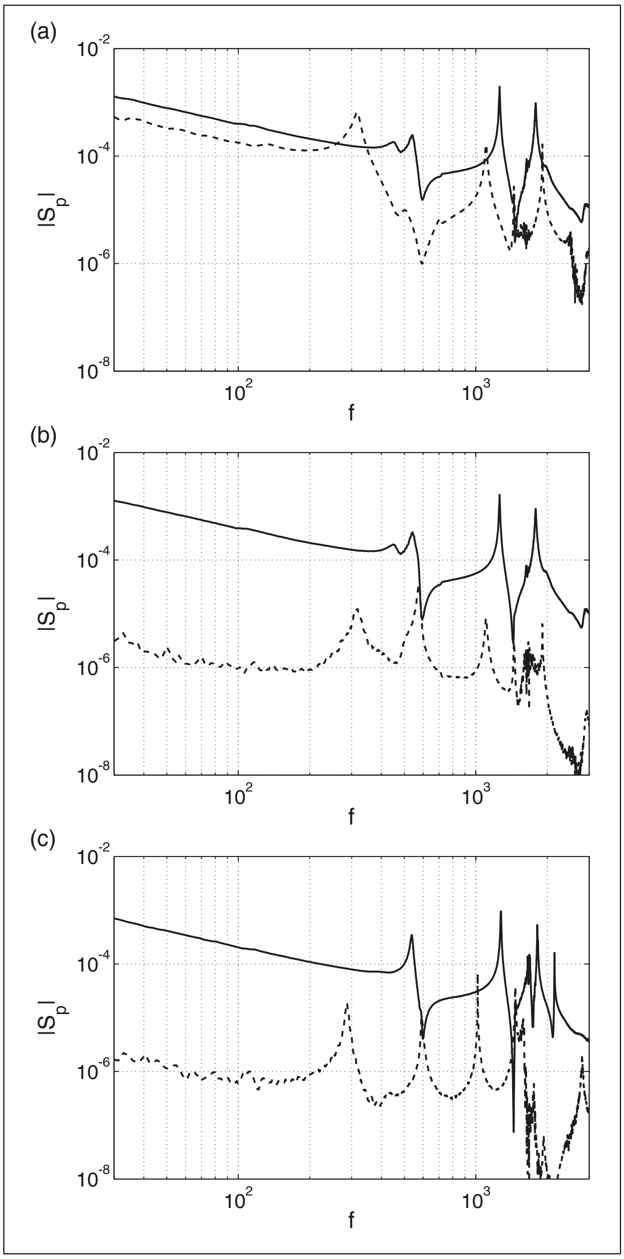


Figure 5. Experimental results for a manipulation of the first-order source descriptor: (a) source with off-center force excitation; (b) source with force excitation at the center of mass; (c) source with attached structure. — zero-order source descriptor; – – – first-order source descriptor.

an attached structure. The resultant reduction of the first-order source descriptor can be seen when comparing Figs. 5(a) and 5(c).

## 4. Concluding remarks

The present paper discusses the interface mobility approach and its application for low-noise design of structure-borne sound sources. The use of a discrete Fourier series as a basis for the concept of interface mobilities is shown to yield substantial simplifications with regard to the implementation of the approach. For the multi-point case, moreover, a discrete Fourier series leads to a clarification of the interface-order interpretation.

In addition to existing methods for low-noise design of vibrational sources, the concept of interface mobilities allows for a mismatching of the source descriptor and coupling function orders. Approaches for a creation or amplification of such a mismatching by manipulating the active properties of the source descriptor orders are introduced. In numerical and experimental studies on simplified source models a reduction of the source descriptor orders of several orders of magnitude is achieved. Although the results can be considered promising, the efficiency of such approaches remain to be tested on practical source structures.

Future work regarding low-noise design by means of a order mismatching is to include the interface mobilities of both the source and the receiver. Here, the inverse dependence of the source descriptor orders on the interface mobilities is to be considered. With approaches for altering the orders of both the source descriptor and the coupling function, a potentially effective tool for low-noise design might emerge.

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