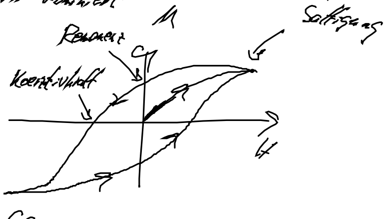
WdH

Suszeptibilität   magnet. Feldstärke

$$\underline{M} = \chi_m \underline{H} \qquad B = \underbrace{(1+4\pi\chi_m)}_{\mu\ \hat{=}\ \text{Permeabilität}} \cdot H$$

Magnetisierung

- Diamagnetismus: überall, Dipole werden induziert $\qquad \chi_m < 0$
- Paramagnetismus: perm. Dipole richten sich aus $\qquad \chi_m > 0$
- Kollektiver Magnetismus: Dipol-Dipol-WW dominiert



- $$\oint_{\partial F} \underline{E}\cdot d\underline{r} = -\frac{1}{c}\frac{d}{dt}\iint_F \underline{B}\cdot d\underline{F} \quad \Leftrightarrow \quad \nabla\times E = -\frac{1}{c}\dot{B}$$

"Lenz'sche Regel" Faraday-Gesetz

- MS: $\nabla\cdot j = 0 \quad \rightarrow$ jetzt $\frac{\partial\rho}{\partial t} + \nabla\cdot j = 0$

- $$\boxed{\begin{aligned} \nabla\cdot D &= 4\pi\rho \\ \nabla\cdot B &= 0 \\ \nabla\times E + \frac{1}{c}\partial_t B &= 0 \\ \nabla\times H - \frac{1}{c}\partial_t D &= \frac{4\pi}{c} j \end{aligned}}$$

$$\left.\begin{aligned} D &= \varepsilon\cdot E \\ B &= \mu\cdot H \\ j &= \sigma_c\cdot E \end{aligned}\right\} \text{für lin. Medien}$$

Leitfähigkeit

- Potentiale

$B = \nabla\times A$ (Vektorpotential) $\qquad E = -\nabla\Phi - \frac{1}{c}\partial_t A$

$A \rightarrow A + \nabla\Lambda(r,t)$

$$\left.\begin{aligned} \Box\Phi &= \{\Delta - \frac{1}{c^2}\partial_t^2\}\Phi = -4\pi\rho \\ \Box\underline{A} &= \{\Delta - \frac{1}{c^2}\partial_t^2\}\underline{A} = -\frac{4\pi}{c}\, j \end{aligned}\right\} \text{in Lorentz-Eichung } \nabla A + \frac{1}{c}\partial_t\Phi = 0$$

Lorentz-Eichung ist nicht eindeutig

5.3.2. Coulomb-Eichung

$\nabla\cdot A = 0$

$$-4\pi\rho = \Delta\Phi + \cancel{\frac{1}{c}\partial_t(\nabla\cdot A)}$$

$$-\frac{4\pi}{c}\, j = \cancel{-\nabla(\nabla\cdot A)} + \Delta A - \frac{1}{c^2}\partial_t^2 A - \frac{1}{c}\nabla\cdot\partial_t\Phi$$

$$\Delta\Phi = -4\pi\rho \longrightarrow \Phi(r,t) = \int \frac{\rho(r',t)}{|r-r'|}\, d^3r'$$ "instantane Eichung"

$$\Box A = -\frac{4\pi}{c}\cdot j + \frac{1}{c}\nabla\partial_t\Phi$$
$$= -\frac{4\pi}{c}\cdot j \underbrace{- \frac{1}{c}\nabla\int\frac{[\nabla'\cdot j(r',t)]}{|r-r'|}d^3r'}$$ aus Kontin.-Gl.

Stromdichte kontrast A $+\frac{4\pi}{c}\cdot j_\parallel$

$$j_\parallel = -\frac{1}{4\pi}\nabla\cdot\int\frac{\nabla'\cdot j(r',t)}{|r-r'|}d^3r'$$

$$j_\perp = \frac{1}{4\pi}\nabla\times\left(\nabla\times\int\frac{j(r',t)}{|r-r'|}d^3r'\right) \qquad \nabla\times(\nabla\times b) = \nabla(\nabla\cdot b) - \Delta b$$

$$j_\parallel + j_\perp = +\frac{1}{4\pi}\nabla\left(\nabla\cdot\int\frac{j(r',t)}{|r-r'|}d^3r'\right) + j(r,t) \underline{- \frac{1}{4\pi}\nabla\cdot\int\frac{\nabla' j(r',t)}{|r-r'|}d^3r'}$$
$$= \cancel{-\frac{1}{4\pi}\nabla\int j(r',t)\left(\nabla'\frac{1}{|r-r'|}\right)d^3r'} + j(r,t)$$
$$\underline{-\frac{1}{4\pi}\nabla\int\nabla'\left(\frac{j(r',t)}{|r-r'|}\right)d^3r' + \cancel{\frac{1}{4\pi}\nabla\int j(r',t)\left(\nabla'\frac{1}{|r-r'|}\right)d^3r'}}$$
$$= j(r,t) \qquad \text{Gauss}$$

$$\Rightarrow \Box A = -\frac{4\pi}{c}j_\perp$$ „transversale Eichung"

z.B. $\rho = 0 \qquad j = 0 \quad \rightarrow \Phi = 0 \qquad \Box A = 0$

dann: $B = \nabla\times A \quad E = -\frac{1}{c}\partial_t A$

jetzt $A' = A + \nabla\Lambda \qquad \Phi' = \Phi - \frac{1}{c}\frac{\partial\Lambda}{\partial t}$

$$\nabla\cdot A' = 0 = \nabla\cdot A + \nabla(\nabla\Lambda) = \nabla\cdot A + \Delta\Lambda = 0$$
$$\Delta\Lambda = -\nabla\cdot A$$

5.4. Erhaltungssätze und Poynting-Vektor

5.4.1. Energie-Erhaltung

$$\underline{F} = q\left(\underline{E} + \frac{1}{c}\underline{v}\times\underline{B}\right) \qquad v = \frac{dr}{dt}$$

$$dW = \underline{F}\cdot d\underline{r} = q\cdot\underline{E}\cdot d\underline{r} \qquad \text{für PL}$$

$$q \to \rho(r,t)\cdot d^3r$$

Kraftdichte $\underline{f}(r,t) = \rho(r,t)\left(\underline{E} + \frac{1}{c}\cdot\underline{v}\times\underline{B}\right)$

Leistungsdichte $\underline{f}(r,t)\cdot v(r,t) = \rho(r,t)(\underline{v}\cdot\underline{E}) = (j\cdot E)$

Leistung in Volumen V: $\frac{dW_V^{(mech)}}{dt} = \int_V (j\cdot E)d^3r$

nutze $\nabla\times H - \frac{1}{c}\partial_t D = \frac{4\pi}{c}\cdot j$

$$\rightarrow (E\cdot j) = \frac{c}{4\pi}\underline{E\cdot(\nabla\times H)} - \frac{1}{c}E\cdot\frac{\partial D}{\partial t} \qquad \varepsilon_{ijk} = \varepsilon_{kij} = -\varepsilon_{jik}$$

$$\nabla\cdot(E\times H) = \underline{H\cdot(\nabla\times E)} - E(\nabla\times H) = \sum_{ijk}\varepsilon_{ijk}\,\partial_i(E_j\cdot H_k)$$

$$(E\cdot j) = \nabla\cdot(E\times H) - \frac{1}{4\pi}E\cdot\frac{\partial D}{\partial t} - \frac{1}{4\pi}H\cdot\frac{\partial B}{\partial t}$$

$$\frac{dW_V}{dt} = -\frac{1}{4\pi}\int d^3r\left[c\,\nabla\cdot\left(E\times H + \underline{E\cdot\partial_t D + H\cdot\partial_t B}\right)\right]$$

$$\boxed{\text{Poynting-Vektor}\quad S = \frac{c}{4\pi}(E\times H)}$$

$$\boxed{w = \frac{1}{8\pi}(E\cdot D + B\cdot H)}$$

Def. der Energiedichte des EM-Feldes

für lineare Medien

$$H\cdot\partial_t B = \frac{1}{2}\partial_t(H\cdot B)$$

$$E\cdot\partial_t D = \frac{1}{2}\partial_t(E\cdot D)$$

$$\frac{dW_V^{\text{mech}}}{dt} = \int_V j\cdot E\,d^3r = -\int_V\left(\frac{\partial w}{\partial t} + \underline{\nabla\cdot S}\right)d^3r$$

$$\boxed{\frac{\partial w}{\partial t} + \nabla\cdot S = -j\cdot E}$$

mech. Leistungsdichte

Poynting-Theorem

EM Leistungsdichte

Energiefluss durch ∂V

$$\frac{d}{dt}\left(W_V^{\text{mech}} + W_V^{\text{feld}}\right) = -\oint_{\partial V}\underline{S}\cdot d\underline{F}$$

$S \neq 0$ muss nicht immer mit einem Energiestrom korrespondieren
denn $S \to S + \nabla\times b$ lässt die Bilanz invariant

Beispiel: homogene Felder

$$S = \frac{c}{4\pi}\begin{pmatrix}E\\0\\0\end{pmatrix}\times\begin{pmatrix}0\\0\\H\end{pmatrix} = \frac{c}{4\pi}\begin{pmatrix}0\\-E\cdot H\\0\end{pmatrix} \neq 0$$

aber: $\underline{\nabla\cdot S} = 0 \longrightarrow$ kein Energiestrom

5.4.2. Impulsbilanz

$$F = \frac{d}{dt} p^{mech} = q\left(E + \frac{1}{c} v \times B\right) \quad \text{für 1. PL}$$

$$\underbrace{\frac{d}{dt} P_V^{mech}}_{\text{Gesamtimpuls in } V} = \int_V \left(\rho E + \frac{1}{c} j \times B\right) d^3r \qquad \nabla \cdot D = 4\pi \rho, \quad \nabla \times H - \frac{1}{c}\partial_t D = \frac{4\pi}{c} j$$

$$= \frac{1}{4\pi} \int_V \left(E (\nabla \cdot D) - B \times (\nabla \times H) + \frac{1}{c} B \times \frac{\partial D}{\partial t}\right) d^3r$$

$$= \frac{1}{4\pi} \int_V \left[E(\nabla D) + H(\nabla B) - B \times (\nabla \times H) - \underline{\frac{1}{c}\partial_t (B \times D)} - D \times (\nabla \times E)\right] d^3r$$

im Vakuum $D = E \quad B = H$

$$\underline{P}_V^{Feld} = \frac{1}{4\pi c} \int_V (\underline{E} \times \underline{B})\, d^3r = \frac{1}{c^2} \int \underline{S}\, d^3r$$

$$\frac{d}{dt}\left(P_V^{mech} + P_V^{Feld}\right) = \frac{1}{4\pi} \int_V \left[E(\nabla \cdot E) + B(\nabla B) - B \times (\nabla \times B) - E \times (\nabla \times E)\right] d^3r$$

$$\underbrace{\left[E(\nabla \cdot E) - E \times (\nabla \times E)\right]_i}_{\sum_j \partial_j F_{ij}} = \sum_j \partial_j \left[E_i E_j - \delta_{ij}/2 \sum_k E_k E_k\right]$$

$$\sum_k \varepsilon_{ijk} \varepsilon_{kmn} = \delta_{im}\delta_{jn} - \delta_{in}\delta_{jm} \qquad \text{auch für } E \to B$$

$$\boxed{\begin{array}{l}\text{Maxwell'scher Spannungstensor} \\ T_{ij} = \frac{1}{4\pi}\left[E_i E_j + B_i B_j - \frac{1}{2}\delta_{ij} \sum_k \left(E_k E_k + B_k B_k\right)\right]\end{array}}$$

$$\sum_i T_{ii} = -w$$

$$T_{ij} = T_{ji}$$

$$\frac{d}{dt}\left(P_V^{mech} + P_V^{Feld}\right)_i = \int_V \left(\sum_j \partial_j T_{ij}\right) d^3r \qquad \underline{t}_i = \begin{pmatrix} T_{i1} \\ T_{i2} \\ T_{i3} \end{pmatrix}$$

$$\Rightarrow \sum_j \partial_j T_{ij} = \nabla \cdot t_i$$

$$= \oint_{\partial V} \underline{t}_i \cdot \underline{n}\, dF = \oint_{\partial V} \sum_j T_{ij} \cdot n_j\, dF$$

Kraft auf eine Randfläche der Größe ΔF

$$F_i = -\sum_j T_{ij} \cdot n_j \cdot \Delta F$$

$$\underline{F} \cdot \underline{n} = \sum_i F_i \cdot n_i = -\sum_{ij} T_{ij}\, n_i n_j \Delta F$$

$$\boxed{\begin{array}{l} P_{Strahlung} = \frac{F \cdot n}{\Delta F} = -\sum_{ij} T_{ij} n_i n_j \\ \text{Strahlungsdruck}\end{array}}$$