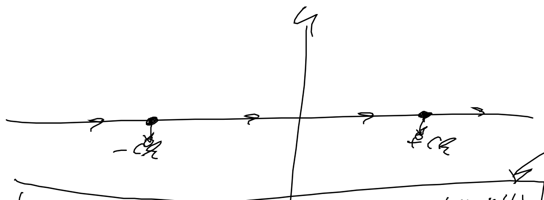
Wdh

- Kausalität als RB an die GF

$$\left\{\Delta - \frac{1}{c^2}\partial_t^2\right\} G(\underline{r}-\underline{r}', t-t') = -4\pi\,\delta(\underline{r}-\underline{r}')\,\delta(t-t')$$

FT $$\left[-k^2 + \frac{\omega^2}{c^2}\right] g(k,\omega) = -4\pi$$



$$t' = t - \frac{|r-r'|}{c} < t$$

"retardierte" GF

$$\Rightarrow \boxed{G(r-r', t-t') = \frac{\delta\left(t-t'-\frac{|r-r'|}{c}\right)}{|\underline{r}-\underline{r}'|}}$$

retard. Potentiale
$$\Phi(\underline{r},t) = \int \frac{\rho\left(\underline{r}', t-\frac{|r-r'|}{c}\right)}{|r-r'|}\, d^3r'$$

$$\underline{A}(r,t) = \frac{1}{c}\int \frac{\underline{j}\left(r', t-\frac{|r-r'|}{c}\right)}{|r-r'|}\, d^3r'$$

~~C. G~~

6.3. Pot. einer bewegten PL

$$\rho(r,t) = e\cdot\delta\left(\underline{r} - \underline{r}_0(t)\right) \qquad \underline{j}(r,t) = e\cdot\underline{v}_0(t)\,\delta\left(\underline{r}-\underline{r}_0(t)\right)$$

vorgegeben

$$v_0(t) = \frac{d r_0(t)}{dt}$$

wichtig $v_0(t)$ kann sich ändern

$$\Phi(r,t) = e\int d^3r'\,\frac{\delta^{(3)}\left(\underline{r}'-r_0\left(t-\frac{|r-\underline{r}'|}{c}\right)\right)}{|r-r'|}$$ schwierig

$$= e\int d^3r'\,dt'\,\frac{\delta^{(3)}(r'-r_0(t'))\cdot\delta\left(t'-t+\frac{|r-r'|}{c}\right)}{|r-r'|}$$

$$= e\int dt'\,\frac{\delta\left(\underline{t}'-t+\frac{|r-r_0(t')|}{c}\right)}{|\underline{r}-r_0(t')|}\,\frac{du}{du}$$ immer noch schwierig

$$u = t'-t+\frac{1}{c}\sqrt{(x-x_0(t'))^2+(y-y_0(t'))^2+(z-z_0(t'))^2} \overset{!}{=} 0$$ Subst.

$$\frac{du}{dt'} = 1 - \frac{(x-x_0(t'))\frac{dx_0}{dt'} + (y-y_0(t'))\frac{dy_0}{dt'} + (z-z_0(t'))\frac{dz_0}{dt'}}{c\sqrt{\ldots}} = 1-\frac{1}{c}\,\underline{n}(t')\cdot\underline{v}_0(t')$$

$$n(t') = \frac{r - r_0(t')}{|r - r_0(t')|}$$

$$\underline{\Phi}(r,t) = e \int du \, \frac{dt'}{du} \frac{1}{|r - r_0(t)|} \delta(u)$$

$$\boxed{\underline{\Phi}(r,t) = \frac{e}{|\underline{r} - \underline{r}_0(t')| \left[1 - \frac{1}{c} \underline{n}(t') \cdot \underline{v}(t')\right]} \quad \Bigg| \quad t' = t - \frac{|r - r_0(t)|}{c}}$$

Lienard-Wiechert Potentiale

$$\boxed{\underline{A}(r,t) = \frac{\frac{e}{c} \underline{v}(t')}{|\underline{r} - \underline{r}_0(t')| \left[1 - \frac{1}{c} \underline{n}(t') \cdot \underline{v}(t')\right]} \quad \Bigg| \quad t' = t - \frac{|r - r_0(t')|}{c}}$$

- $v_0 = 0 \to$ konsistent $\quad \underline{\Phi}(r,t) \to \frac{e}{|r - r_0|} \quad \underline{A}(r,t) = 0$

  $r_0(t) = r_0 = \text{const}$
- $v_0 = \text{const} \quad \varrho(r,t) = e \cdot \delta(\underline{r} - v \cdot \underline{e_x} \cdot t)$

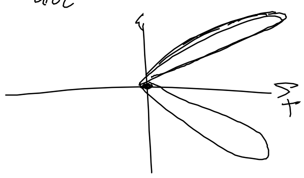
$\underline{\text{Löse}} \quad t' = t - \frac{1}{c}\sqrt{(x - v \cdot t')^2 + y^2 + z^2}$

$$t' = \frac{t - \frac{v \cdot x}{c^2} \pm \frac{1}{c}\sqrt{(x - v t)^2 + y^2 + z^2}}{1 - \frac{v^2}{c^2}}$$

$$\underline{\Phi}(r,t) = \frac{\gamma \cdot e}{\sqrt{(\gamma x - \beta\gamma \cdot ct)^2 + y^2 + z^2}} \qquad A_x(\underline{r},t) = \beta \cdot \underline{\Phi}(\underline{r},t)$$

$\underline{\text{Anwendung}} \quad \underline{A}, \underline{\Phi} \longrightarrow \underline{E} \,\&\, \underline{B} \longrightarrow \underline{S} = \frac{c}{4\pi} \underline{E} \times \underline{B} \quad$ (Volumen)

$$\frac{dP}{d\Omega} = R^2 \cdot (\underline{S} \cdot \underline{e}_R)$$ beschreibt Energiefluss in Raumwinkel $d\Omega$



6.4. Strahlung zeitlich oszillierender Quellen

$\underline{\text{Wdh}} \quad \underline{\text{mikroskop. MWG}}$

$$\nabla E = 4\pi \varrho \qquad \nabla \times E + \frac{1}{c} \partial_t B = 0$$
$$\nabla B = 0 \qquad \nabla \times B - \frac{1}{c} \partial_t E = \frac{4\pi}{c} j$$

sind linear in $\underline{E} \,\&\, \underline{B}$ und $\varrho \,\&\, \underline{j}$

$\varrho, j \in \mathbb{C} \quad \rightsquigarrow \underline{E}, \underline{B} \in \mathbb{C}$

$\to$ wir können die Felder & Quellen komplex ansetzen

$E(r,t) = \text{Re}\{E\} \in \mathbb{R} \quad \in \mathbb{C} \qquad \underline{E}(r), E \in \mathbb{C}$

Quellen $\rho(r,t) = \mathrm{Re}\, \underset{\sim}{\rho}(r)\cdot e^{-i\omega t}$

$$j(r,t) = \mathrm{Re}\, \underset{\in \mathbb{C}}{\underline{j}(r)} \cdot e^{-i\omega t}$$

$$A(r,t) = \frac{1}{c}\int d^3r' \int dt' \frac{j(r',t')\cdot \delta\left(t'-t+\frac{|r-r'|}{c}\right)}{|r-r'|}$$

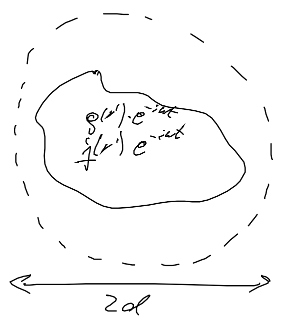
$$= \frac{1}{c}\cdot \mathrm{Re}\int d^3r' \frac{j(r')\, e^{+i\frac{\omega}{c}|r-r'|}}{|r-r'|}\; e^{-i\omega t}$$

$$= \mathrm{Re}\, \underset{\in \mathbb{C}}{A(r)}\; e^{-i\omega t}$$

$$\boxed{\underline{A}(r) = \frac{1}{c}\int d^3r' \frac{\underline{j}(r')\, e^{ik|r-r'|}}{|r-r'|}}$$

räuml. Komp. des Vektorpots.

$$B(r,t) = \mathrm{Re}\left(\nabla\times A(r)\cdot e^{-i\omega t}\right)$$



$\rho(r>d) = 0$

$j(r>d) = 0$

$\underline{E}$? $\underline{B}$?

$$\nabla\times B - \frac{1}{c}\partial_t E = \frac{4\pi}{c} j \xrightarrow{\text{im Fernfeld}} 0$$

$$\partial_t \underset{\in\mathbb{C}}{\underline{E}} = c\cdot\nabla\times\left(\nabla\times \underset{\in\mathbb{C}}{\underline{A}(r)}\right)\cdot e^{-i\omega t} \quad : r>d$$

$$\underline{E} = \frac{i}{k}\,\nabla\times\left(\nabla\times \underline{A}(r)\right)\cdot e^{-i\omega t} \quad : r>d$$

$\underline{E}(r,t) = \mathrm{Re}\{\underline{E}\}$ $\longrightarrow$ Vorgabe von $j(r',t')$ ist ausreichend

$$\lambda = \frac{2\pi c}{\omega} = \frac{2\pi}{k}$$

a) Strahlungs-Nahzone

$r, r' \ll \lambda$

$$e^{ik|r-r'|} = e^{i\cdot 2\pi\frac{|r-r'|}{\lambda}} \approx 1$$

$$A(r) = \frac{1}{c}\int d^3r' \frac{j(r')}{|r-r'|}$$

Retardierung hat keinen Effekt

b) Fernzone $\quad d \ll r \qquad d \ll \lambda$

↑ "Langwellen-Näherung"

$$|\underline{r}-\underline{r}'| = |\underline{r}|\sqrt{1+\left(\frac{r'}{r}\right)^2 - 2\,\underline{n}\cdot\frac{\underline{r}'}{r}} \approx r\left(1-\frac{1}{r}\,\underline{n}\cdot\underline{r}' + \ldots\right)$$

$$n = \frac{\underline{r}}{|\underline{r}|}$$

$$\to e^{ik|r-r'|} \approx e^{ik\cdot r}\, e^{-ik\cdot \underline{n}\cdot \underline{r}'} = e^{ikr}\left[1 - ik\cdot(\underline{n}\cdot\underline{r}') + \ldots\right]$$

$$\frac{1}{|r-r'|} \approx \frac{1}{r}\left(1+\frac{1}{r}\cdot\underline{n}\cdot\underline{r}' + \ldots\right)$$

$$\frac{e^{ik|r-r'|}}{|r-r'|} = \frac{e^{ikr}}{r}\left[1+(\underline{n}\cdot\underline{r}')\left(\frac{1}{r}-ik\right)+\ldots\right]$$

↑ Kugelwelle (auslaufende)

$$\underline{A}(\underline{r}) = \underbrace{\frac{1}{c}\,\frac{e^{ikr}}{r}\int d^3r'\, j(r')}_{\text{elektr. Dipolterm}} + \underbrace{\frac{1}{c}\left(\frac{1}{r}-ik\right)\frac{e^{ikr}}{r}\int j(r')\,(\underline{n}\cdot\underline{r}')\,d^3r'}_{\text{elektr. Quadrupol \& magnet. Dipol}} + \ldots$$

↑ hier

6.5. Dipolstrahlung

Kont.-Gleichung $\quad \partial_t\rho + \nabla\cdot j = 0 \longrightarrow -i\omega\cdot\underset{\in\mathbb{C}}{\rho(r)} + \nabla\cdot \underset{\in\mathbb{C}}{j(r)} = 0$

$$\int d^3r\, j(r) = \int d^3r\,(j\cdot\nabla)\,\underline{r} \overset{\text{part. Integ.}}{\underset{\in\mathbb{C}}{=}} -\int d^3r\,\underline{r}\,(\underline{\nabla}\cdot j)$$

$$= -i\omega\int d^3r\cdot\underline{r}\;\rho(r) = -i\omega\,\underline{p}$$

$$\underset{\in\mathbb{C}}{\underline{p}} = \int d^3r\; r\cdot\rho(r) \qquad \text{stat. Dipolmoment}$$

$$\underline{p}(t) = \int d^3r\; r\cdot\rho(r,t) = \mathrm{Re}\left\{p\cdot e^{-i\omega t}\right\}$$

$$A(r) = -ik\,\underline{p}\cdot\frac{e^{ikr}}{r} + \ldots \qquad A(r,t) = \mathrm{Re}\;A(r)\cdot e^{-i\omega t}$$

nur Dipolterm

$$\partial_r\,\frac{e^{ikr}}{r} = ik\,\frac{e^{ikr}}{r} - \underbrace{\frac{e^{ikr}}{r^2}}_{\text{vernachl.}} \approx ik\,\frac{e^{ikr}}{r}$$

$$k = \frac{2\pi}{\lambda} \qquad \lambda \ll r$$

$$\boxed{d \ll \lambda \ll r}$$

Ersetzung im Fernfeld: $\nabla \to i k \cdot e_r$

$$\boxed{\begin{aligned} B(r) &\approx k^2 (e_r \times p) \frac{e^{ikr}}{r} \\ E(r) &= \frac{i}{k} \nabla \times B(r) \approx -k^2\, e_r \times (e_r \times p) \frac{e^{ikr}}{r} \end{aligned}}$$

Fernfeld eines oszillierenden Dipols (räumliche Komp.)

$$B = \nabla \times A(r) \approx i k \cdot e_r \times A(r)$$

$$E(r) \perp B(r) \perp e_r$$

Poynting-Vektor

zeitl. Mittel

$$S(r) = \frac{c}{4\pi} \langle E(r,t) \times B(r,t) \rangle = \frac{c}{4\pi} \langle [\mathrm{Re}\, E(r) \cdot e^{-i\omega t}] \times [\mathrm{Re}\, B(r) e^{-i\omega t}] \rangle$$

betrachte $a(t) = \mathrm{Re}\, a_0 e^{-i\omega t}$ $\qquad b(t) = \mathrm{Re}\, b_0 \cdot e^{-i\omega t}$

$$= \frac{1}{2} a_0 e^{-i\omega t} + \frac{1}{2} a_0^* e^{+i\omega t}$$

$$\langle a(t) \cdot b(t) \rangle = \frac{1}{4} \langle (a_0 e^{-i\omega t} + a_0^* e^{+i\omega t})(b_0 e^{-i\omega t} + b_0^* e^{+i\omega t}) \rangle$$

$$= \frac{1}{4} (a_0 b_0^* + a_0^* b_0) = \frac{1}{2} \cdot \mathrm{Re}\{a_0 \cdot b_0^*\}$$

$\Rightarrow S(r)$ $\qquad$ Leistung in Raumwinkelelement

$$dP = (\langle S \rangle \cdot e_r)\, r^2 \cdot d\Omega \qquad d\Omega = \sin\vartheta\, d\vartheta\, d\varphi$$

$$\frac{dP}{d\Omega} = r^2 \langle e_r \cdot S \rangle$$

$$\frac{dP}{d\Omega} = \frac{c r^2}{4\pi} e_r \cdot \langle E(r,t) \times B(r,t) \rangle = \frac{c \cdot r^2}{4\pi} \frac{1}{2} e_r \cdot \left(\mathrm{Re}\, [E(r) \times B^*(r)] \right)$$

$$= \frac{c \cdot k^4}{8\pi} e_r \cdot [[(e_r \times p) \times e_r] \times (e_r \times p^*)]$$

$$= \frac{\omega^4}{8\pi c^3} [(e_r \times p) \times e_r] \cdot [(e_r \times p^*) \times e_r]$$

$$= \frac{\omega^4}{8\pi c^3} (e_r \times p) \cdot (e_r \times p^*) \equiv \frac{\omega^4}{8\pi c^3} |e_r \times p|^2$$

Strahlungsleistung pro Raumwinkel

falls $p = \underbrace{\begin{pmatrix} |p_1| \\ |p_2| \\ |p_3| \end{pmatrix}}_{p_r} \cdot e^{i\delta}$

$$\boxed{\frac{dP}{d\Omega} = \frac{\omega^4}{8\pi c^3} |p_r|^2 \cdot \sin^2\vartheta}$$

Strahlungschar. eines Dipols

$$\vartheta = \sphericalangle(p_r, e_r)$$