

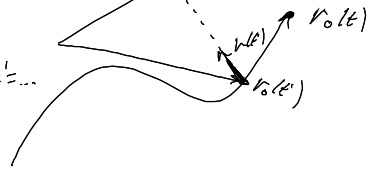
Wd4 • Liénard-Wiechert Potentiale

$$\underline{\Phi}(\underline{r}, t) = \frac{e}{|\underline{r} - \underline{r}_0(t')| \left[ 1 - \frac{1}{c} \underline{v}(t') \cdot \underline{v}(t') \right]} \quad \left| \begin{array}{l} \text{Ordnung rel. Zeit} \\ t' = t - \frac{|\underline{r} - \underline{r}_0(t')|}{c} \end{array} \right.$$

$$\underline{A}(\underline{r}, t) = \frac{\frac{e}{c} \cdot \underline{v}(t')}{|\underline{r} - \underline{r}_0(t')| \left[ 1 - \frac{1}{c} \underline{v}(t') \cdot \underline{v}(t') \right]} \quad \left| \begin{array}{l} t' = \dots \end{array} \right.$$

$$h(t') = \frac{r - r_0(t')}{|r - r_0(t')|}$$

$$t' = t - \frac{|\underline{r} - \underline{r}_0(t')|}{c}$$



• oszillierende Quellen

$$\dot{j}(z, t) = \text{Re } \dot{j}(z) e^{-i\omega t}$$

$$\rho(z, t) = \text{Re } \rho(z) e^{-i\omega t}$$

$$\underline{A}(\underline{r}) = \frac{1}{c} \int d^3r' \frac{\dot{j}(r') e^{i\omega(r-r')}}{|\underline{r}-r'|}$$

+ Nahfeld:  $\underline{A}(\underline{r}) = \frac{1}{c} \int d^3r' \frac{\dot{j}(r')}{|\underline{r}-r'|}$

$$e^{i\omega(r-r')} = e^{i\omega(r-r')/2} \approx 1$$

+ Fernfeld:  $r' < d \ll \lambda \ll r$   
 char. Ausdehnung

$$\underline{A}(r) = -i\omega \cdot \underline{P} \frac{e^{i\omega r}}{r} \quad \text{ausl. kugelwelle}$$

$$\underline{P} = \int \underline{r} \rho(z) d^3r \in \mathbb{C} \quad \text{stat. Dipolmoment}$$

$$\rho(t) = \int \underline{r} \rho(z, t) d^3r = \text{Re } \underline{P} \cdot e^{-i\omega t}$$

$$\underline{B} = \omega^2 (\underline{e}_r \times \underline{P}) \frac{e^{i\omega r}}{r}$$

$$\underline{E} = -\omega^2 \underline{e}_r \times (\underline{e}_r \times \underline{P}) \frac{e^{i\omega r}}{r}$$

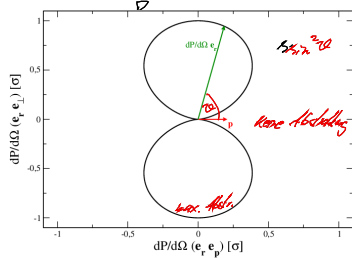
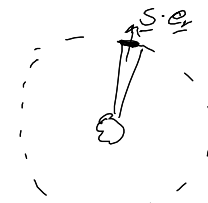
+ Poynting-Vektor  $\langle \underline{S} \rangle = \frac{c}{4\pi} \langle \underline{E} \times \underline{B} \rangle$

$$\frac{dP}{d\Omega} = r^2 \langle \underline{S} \rangle \cdot \underline{e}_r$$

$$= \frac{\omega^4}{8\pi c^3} (\underline{e}_r \times \underline{P}) \cdot (\underline{e}_r \times \underline{P}^*)$$

$$\underline{P} = \begin{pmatrix} |P_1| \\ |P_2| \\ |P_3| \end{pmatrix} e^{-i\delta}$$

$$\frac{dP}{d\Omega} = \frac{\omega^4}{8\pi c^3} |\underline{P}_\perp|^2 \sin^2 \vartheta$$



$$P = \int \frac{dP}{d\Omega} d\Omega = \frac{\omega^4}{8\pi c^3} |p_r|^2 \cdot 2\pi \int_0^\pi \sin^2 \vartheta \cdot \sin \vartheta d\vartheta$$

$$= \frac{\omega^4}{3c^3} |p_r|^2 \quad \text{ges. Strahlleistung} \quad \text{"Vierfaches!"}$$

a.) betr. oszill. PL

$$g(\underline{r}, t) = q \delta(r - r_0 \cdot \cos(\omega t))$$

$$P(\underline{r}, t) = \int g(\underline{r}, t) d^3r$$

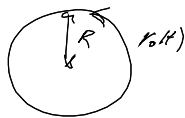
$$= q \cdot r_0 \cdot \cos(\omega t) = \text{Re } \underbrace{q \cdot r_0}_{\underline{p}} \cdot e^{-i\omega t}$$

$\underline{p} \longrightarrow \underline{p} \in \mathbb{R}^3$  (Spezialfall)

$$P = \frac{\omega^4 q^2 r_0^2}{3c^3}$$

b.) Kreisbewegung

$$g(\underline{r}, t) = q \delta(r - R \begin{pmatrix} \cos \omega t \\ \sin \omega t \\ 0 \end{pmatrix})$$



$$\underline{p}(t) = q \cdot R \begin{pmatrix} \cos \omega t \\ \sin \omega t \\ 0 \end{pmatrix} = q \cdot R \cdot \text{Re} \begin{pmatrix} 1 \\ i \\ 0 \end{pmatrix} e^{-i\omega t}$$

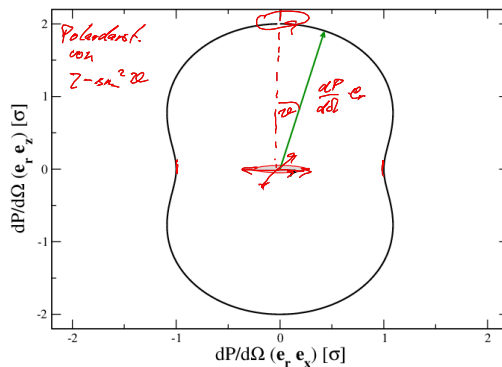
$$\Rightarrow \underline{p} = q \cdot R \begin{pmatrix} 1 \\ i \\ 0 \end{pmatrix}$$

$$\frac{d\underline{p}}{dt} = \frac{\omega^4}{8\pi c^3} (\underline{e}_\vartheta \times \underline{p}) \cdot (\underline{e}_\vartheta \times \underline{p}^*)$$

$$|\underline{e}_\vartheta \times \underline{p}| = q^2 R^2 [2 - \sin^2 \vartheta]$$

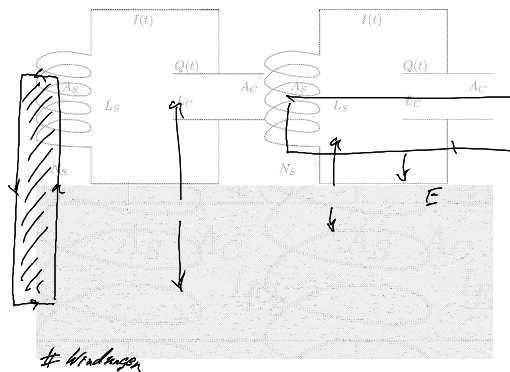
ges. Leistung

$$P = \frac{2}{3} \frac{\omega^4 R^2 q^2}{c^3}$$



## 6.7. Schwingkreis

Kondensator: Elektrostatik  
Spule: Magnetostatik



$$\underline{E} \cdot \underline{D} \cdot E = \underline{E} \cdot \underline{E}$$

$$\int_V \nabla \cdot \underline{E} d^3r = \oint_{\partial V} \underline{E} \cdot d\underline{A} = 4\pi Q = E \cdot A_c$$

$$U = E \cdot L_c = 4\pi Q \cdot \frac{L_c}{A_c} = \frac{1}{C} \cdot Q$$

↑  
Kapazität  $C = \frac{1}{4\pi} \frac{A_c}{L_c}$

$$\underline{NS} : \nabla \times \underline{B} = \frac{4\pi}{c} \cdot \underline{j}$$

$$L_s \cdot \underline{B} = \frac{4\pi}{c} \underline{I} \cdot \underline{NS}$$

magn. Fluss  $\overline{\Phi}_S = A_S \cdot \underline{B} =$   
 $= A_S \cdot \frac{4\pi}{c} \cdot \frac{N_S}{L_s} \cdot \underline{I}$   
B

$$\overline{\Phi}_S \sim \underline{I}$$

$$\underline{I} = \frac{N_S}{c \cdot L} \cdot \overline{\Phi}_S \quad L \hat{=} \text{Induktivität}$$

$$L = \frac{N_S}{c} \cdot \frac{\overline{\Phi}_S}{\underline{I}} = 4\pi \cdot A_S \frac{N_S^2}{L_s \cdot c^2}$$

### phänomenologisch

einfach Gleichungen rekursiv machen

$$Q(t) \approx C \cdot U(t)$$

$$\underline{I}(t) \approx \frac{N_S}{c \cdot L} \cdot \overline{\Phi}_S(t)$$

$$\nabla \times \underline{E} + \frac{1}{c} \partial_t \underline{B} = 0$$

$$\underbrace{\oint \underline{E} \cdot d\underline{r}}_{U} = - \frac{1}{c} \partial_t \underbrace{\iint \underline{B} \cdot d\underline{F}}_{\overline{\Phi}_S}$$

$$U(t) = - \frac{N_S}{c} \cdot \partial_t \overline{\Phi}_S \quad \overline{\Phi}_S = A_S \cdot B$$

$$\frac{d}{dt} Q(t) = \underline{I}(t)$$

$$U(t) = -L \cdot \frac{dI}{dt} = -L \cdot \frac{d^2 Q}{dt^2} = -L \cdot C \cdot \frac{d^2 U}{dt^2}$$

$$\frac{d^2 U}{dt^2} + \frac{1}{LC} \cdot U(t) = 0 \quad \omega = \frac{1}{\sqrt{LC}} \quad \text{Schwingkreis-Frequenz}$$

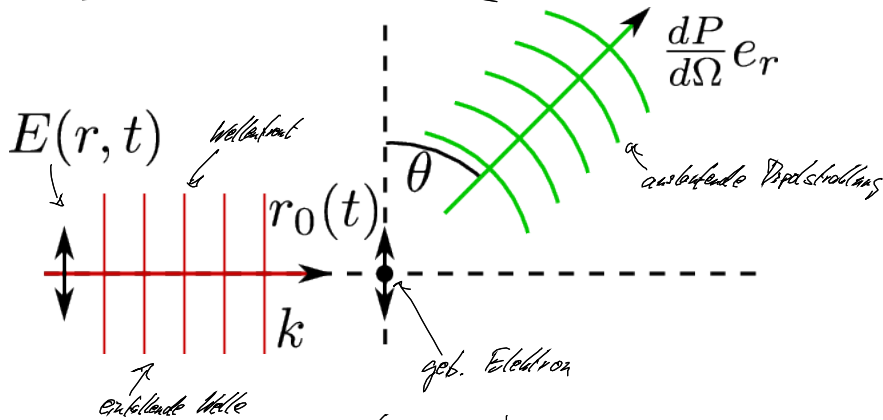
analog:  $\ddot{Q} + \frac{1}{LC} Q = 0$   
 mit Widerstand  $\ddot{Q} + \frac{R}{L} \dot{Q} + \frac{1}{LC} Q = 0$

$$\left. \begin{aligned} \nabla \times \underline{E} &= -\frac{1}{c} \dot{\underline{B}} \approx 0 && \text{(Kondensator)} \\ \nabla \times \underline{B} &= \frac{1}{c} \dot{\underline{E}} + \frac{4\pi}{c} \underline{j} \approx \frac{4\pi}{c} \underline{j} && \text{(Spule)} \end{aligned} \right\} \frac{\omega \cdot l}{c} \ll 1$$

charakt. Länge  
↓

$$\frac{\omega \cdot l}{c} \rightarrow \frac{1}{\kappa_S} \ll 1$$

### 6.8. Streuung von Licht



$$\underline{E}(\underline{r}, t) = \text{Re} \underline{E}_0 \cdot e^{i(\underline{k} \cdot \underline{r} - \omega t)} \quad \underline{B}(\underline{r}, t) = \frac{\underline{k}}{|\underline{k}|} \times \underline{E}(\underline{r}, t)$$

BWGL für geb. Elektron: nichtrelativistisch

$$m_e \ddot{\underline{r}}_0(t) + m_e \Gamma \dot{\underline{r}}_0(t) + m_e \omega_0^2 \underline{r}_0(t) = -e \text{Re} \underline{E}_0 \cdot e^{i(\underline{k} \cdot \underline{r}_0(t) - \omega t)} + \dots$$

↑  
Rückstellkraft

Dämpfung: + Stoßprozesse  
+ Abstrahlung

Langwelle-Näherung  $\lambda \gg \underline{r}_0(t) \quad e^{i(\underline{k} \cdot \underline{r}_0(t))} \approx 1$

$$\Downarrow m_e \ddot{\underline{r}}_0 + m_e \Gamma \dot{\underline{r}}_0 + m_e \omega_0^2 \underline{r}_0 = -e \text{Re} \underline{E}_0 \cdot e^{-i\omega t}$$

Lösung: homog. Lösung + 1 spez. der inhomog. DGL

fällt für  $t \rightarrow \infty$  ab

$\Rightarrow$  für asymptot. langzeitl. los reicht es, die erzwungene Schw. zu betrachten

Ansatz  $r_0(t) = \text{Re } \underline{q} \cdot e^{-i\omega t}$

$$(-\omega^2 - i\gamma\omega + \omega_0^2) \cdot \underline{q} = \frac{-e}{m_e} \underline{E}_0 \rightarrow \underline{q} = \left( \dots \right) \underline{E}_0$$

$$\underline{p}(t) = -e \cdot \dot{r}_0(t) = -e \cdot \text{Re} \cdot \dot{\underline{q}} \cdot e^{-i\omega t} \\ = \text{Re} \frac{e^2 / m_e}{\omega_0^2 - \omega^2 - i\gamma\omega} \underline{E}_0 e^{-i\omega t}$$

$$\underline{p} = \frac{e^2 / m_e}{\omega_0^2 - \omega^2 - i\gamma\omega} \underline{E}_0 = \alpha_p(\omega) \cdot \underline{E}_0$$

↑  
"dielektr. Polarisierbarkeit"

$$\frac{dP}{d\Omega} = \frac{c}{8\pi} \left( \frac{e^2}{4\pi\epsilon_0 c^2} \right)^2 \frac{\omega^4}{(\omega^2 - \omega_0^2)^2 + \gamma^2 \omega^2} |\underline{E}_0|^2 \cdot \sin^2 \vartheta$$

$\rightarrow$  Streuung von Licht an Atomen