

Waldh. - Maxwell-Theorie für Felder $\partial_\mu \tilde{F}^{\mu\nu} = 0$

$x^\mu \rightarrow x^\mu + dx^\mu \rightarrow E, \vec{T}$

- Quellen \rightarrow Felder
- Felder \rightarrow Dynamik von PL

(Felder
Quellen = $\sum PL$)

$L = \sum_i (-i\omega_i c^2) \int_{ED} \frac{\vec{q}_i^2}{c^2} d^3r$

• Hamilton-Dichte $\tilde{H} = \frac{1}{8\pi} (\underline{E}^2 + \underline{B}^2) = 2\pi c^2 \underline{H}^2 + \frac{1}{8\pi} (\nabla \times \underline{A})^2$

• Verb. Quantisierung

Coulomb-Eichung $\nabla \cdot \underline{A} = 0$
Keine Quellen $\rho = 0, \vec{j} = 0$ } $\Phi = 0, \underline{B} = \nabla \times \underline{A}$
 $\underline{E} = -\frac{1}{c} \dot{\underline{A}}$
 $\underline{B} = \nabla \times \underline{A}$

$\underline{A}(\underline{r}, t) = \sum_{\underline{k}} \sum_{\lambda} \sqrt{\frac{2\pi\hbar c^2}{\omega_{\underline{k}}}} \left(\frac{A_{\underline{k}}(\lambda)}{EC} e^{-i(\underline{k}\cdot\underline{r} - \omega(\lambda)t)} + c.c. \right) \underline{u}_{\underline{k}\lambda}$

$\underline{u}_{\underline{k}} \cdot \underline{k} = 0, \quad \underline{u}_{\underline{k}} \cdot \underline{u}_{\underline{k}'} = \delta_{\lambda\lambda'}, \quad \underline{u}_{\underline{k}}^{\lambda} = \underline{u}_{\underline{k}}^{-\lambda}$ (Polarisationen)

$H = \frac{1}{8\pi} \int d^3r \left[\left(\frac{1}{c} \frac{\partial \underline{A}}{\partial t} \right)^2 + (\nabla \times \underline{A})^2 \right]$

$H = \frac{1}{2} \sum_{\underline{k}} \sum_{\lambda} \hbar \omega_{\underline{k}} [A_{\underline{k}}(\lambda) A_{\underline{k}}^*(\lambda) + A_{\underline{k}}^*(\lambda) A_{\underline{k}}(\lambda)]$

entkoppelte harmon. Oszillatoren

8.4.2. Quantisierung

wollen $A \rightarrow \hat{A}$ $\hat{A}_\mu = e^{i\hbar t} A e^{-i\hbar t}$

$\frac{d}{dt} \hat{A}_\mu = \frac{i}{\hbar} [\hat{H}_\mu, \hat{A}_\mu]$ (ohne expl. P.A.)

a.) Einzelner Oszillator

$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2} m \omega^2 \hat{q}^2$

Leiter-Operatoren $a = \frac{m\omega \hat{q} + i\hat{p}}{\sqrt{2\hbar m \omega}}, \quad a^\dagger = \frac{m\omega \hat{q} - i\hat{p}}{\sqrt{2\hbar m \omega}}$

$[\hat{q}, \hat{p}] = i\hbar \Leftrightarrow [a, a^\dagger] = 1$

$H = \frac{\hbar\omega}{2} (a a^\dagger + a^\dagger a) = \hbar\omega \left(\underbrace{a^\dagger a}_{\text{Tz-Operator}} + \frac{1}{2} \right)$ $a^\dagger a |n\rangle = n |n\rangle$

$H |n\rangle = (\hbar\omega(n + \frac{1}{2})) |n\rangle$ ↑ Nullpunktsenergie

$$a^+ |n\rangle = \sqrt{n+1} |n+1\rangle \quad a |n\rangle = \sqrt{n} |n-1\rangle$$

"Erzeuger" "Vernichtet"

$$GZ |0\rangle \quad \frac{(a^+)^n}{\sqrt{n!}} |0\rangle = |n\rangle \quad \text{Fock-Raum}$$

$$a(t) = e^{\frac{i}{\hbar} H t} a e^{-\frac{i}{\hbar} H t} \quad [H, a] = \hbar \omega [a^+ a, a] = -\hbar \omega a$$

$$\frac{d}{dt} a(t) = \frac{i}{\hbar} e^{\frac{i}{\hbar} H t} [H, a] e^{-\frac{i}{\hbar} H t} = -i\omega e^{\frac{i}{\hbar} H t} a e^{-\frac{i}{\hbar} H t} = -i\omega a(t) \quad \rightarrow a(t) = e^{-i\omega t} a$$

$$a^+(t) = e^{+i\omega t} a^+$$

b.) elektrom. Feld

Quantisierung

$$A_2(\mathbf{r}) \rightarrow \hat{A}_2(\mathbf{r}) = \sum_{\mathbf{k}} \hat{a}_{\mathbf{k}2} e^{i\mathbf{k}\cdot\mathbf{r}} \quad A_2(\mathbf{r}) \rightarrow a_{2\mathbf{k}}$$

↑ ↑
Pa. Wellenvekt.

$$\frac{d}{dt} \hat{A}_\mu = \frac{i}{\hbar} [H, \hat{A}_\mu] \quad [a_{\mathbf{k}2}, a_{\mathbf{k}'2}^+] = \delta_{\mathbf{k}\mathbf{k}'}$$

$$H = \sum_{\mathbf{k}} \sum_{\lambda} \frac{\hbar \omega_{\mathbf{k}}}{2} [a_{\mathbf{k}\lambda}^+ a_{\mathbf{k}\lambda} + a_{\mathbf{k}\lambda} a_{\mathbf{k}\lambda}^+] \quad \left(\text{fermionisch } \{C_i, C_i^+\} = C_i C_i^+ + C_i^+ C_i = \delta_{ii} \right)$$

$$\left[a_{\mathbf{k}2}, a_{\mathbf{k}'2}^+ \right] = \delta_{\mathbf{k}\mathbf{k}'} \delta_{22} \quad [a_{\mathbf{k}2}, a_{\mathbf{k}'2}] = 0$$

$$\left[a_{\mathbf{k}2}^+, a_{\mathbf{k}'2}^+ \right] = 0$$

"bosonische Kommutator-Relationen"

$$H = \sum_{\mathbf{k}} \sum_{\lambda} \hbar \omega_{\mathbf{k}} \left(a_{\mathbf{k}\lambda}^+ a_{\mathbf{k}\lambda} + \frac{1}{2} \right)$$

$$\frac{d}{dt} a_{\mathbf{k}2}(t) = e^{-i\omega_{\mathbf{k}} t} \frac{i}{\hbar} [H, a_{\mathbf{k}2}] e^{+i\omega_{\mathbf{k}} t} = -i\omega_{\mathbf{k}} a_{\mathbf{k}2}(t) \quad \rightarrow a_{\mathbf{k}2}(t) = e^{-i\omega_{\mathbf{k}} t} a_{\mathbf{k}2}$$

$$\hat{A}_\mu = \sum_{\mathbf{k}} \sum_{\lambda} \sqrt{\frac{\hbar \omega_{\mathbf{k}}}{L^3 \epsilon_0}} \left[a_{\mathbf{k}\lambda}(t) e^{i\mathbf{k}\cdot\mathbf{r}} + \underbrace{a_{\mathbf{k}\lambda}^+(t) e^{-i\mathbf{k}\cdot\mathbf{r}}}_{\text{h.c.}} \right] \cdot \hat{h}_{\lambda\mu}$$

Eigenwerte

• $\frac{d}{dt} \hat{A}_\mu = \frac{i}{\hbar} [H, \hat{A}_\mu]$

• Felder $\vec{E} = -\frac{1}{c} \partial_t \vec{A} \quad \vec{B} = \nabla \times \vec{A}$

$$\underline{E_H} = i \sum_k \sum_\lambda \sqrt{\frac{\hbar \omega_k}{2 \epsilon_0 V}} (\hat{a}_{k\lambda} e^{i\mathbf{k}\cdot\mathbf{r}} - h.c.) = \underline{E_H}^+$$

$$[E_{H_i}, B_{H_j}] \neq 0 \quad \text{in Abgesen}$$

• ein Photon wird gebildet durch

$$\hat{a}_{k\lambda}^+ |0, \dots, 0\rangle \stackrel{!}{=} \hat{a}_{k\lambda}^+ |0\rangle \\ = |0, \dots, 0, \overset{k\lambda}{1}, 0, \dots, 0\rangle$$

• mehrere Photonen der gleichen Mode

$$|0, \dots, 0, k\lambda, 0, \dots, 0\rangle \stackrel{!}{=} |k\lambda\rangle = \frac{(\hat{a}_{k\lambda}^+)^{k\lambda}}{\sqrt{(k\lambda)!}} |0, \dots, 0\rangle$$

• mehrere Phot. versch. Moden

$$|\underline{k}\rangle = |\{k\lambda\}\rangle = |k_{\lambda_1}, k_{\lambda_2}, k_{\lambda_3}, \dots\rangle \\ = \frac{1}{\sqrt{\prod_{k\lambda} (k\lambda)!}} \prod_{k\lambda} (\hat{a}_{k\lambda}^+)^{k\lambda} |0, \dots, 0\rangle$$

essentiell Tensorprodukt

$$\text{zwei Phot. } |k_1, k_2\rangle \stackrel{!}{=} |k_1\rangle \otimes |k_2\rangle$$

2 photons	2 photons	4 photons	
	$ 00\rangle = 0\rangle \otimes 0\rangle$	} 2^4 Basiszust.	
	$ 01\rangle$		
	$ 10\rangle$		
	$ 11\rangle$		

$$\langle \underline{k} | \underline{k} \rangle = \delta_{k_1, k_1} \delta_{k_2, k_2} \\ = \prod_{k\lambda} \prod_{k'\lambda'} \delta_{(k\lambda), (k'\lambda')}$$

$$\bullet \langle \underline{k} | \hat{E} | \underline{k} \rangle = 0 = \langle \underline{k} | \hat{B} | \underline{k} \rangle$$

Fock-Zustände sind nicht klassisch

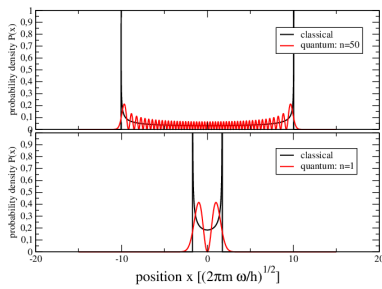
$$\hat{a}_{k\lambda}^+ \hat{a}_{k\lambda} |\underline{k}\rangle = k\lambda |\underline{k}\rangle$$

$$\bullet \langle \underline{k} | \hat{H} | \underline{k} \rangle = \sum_k \sum_\lambda \hbar \omega_k (k\lambda + \frac{1}{2})$$

$$\bullet \omega = \frac{1}{8\pi\epsilon_0} \langle \underline{k} | (\underline{E}^2 + \underline{B}^2) | \underline{k} \rangle = \sum_k \sum_\lambda \frac{\hbar \omega_k}{\epsilon_0 V} (k\lambda + \frac{1}{2})$$

$$\bullet \text{Feldimpuls } \underline{P} = \frac{1}{4\pi c} \int \underline{E} \times \underline{B} d^3r = \sum_k \sum_\lambda \hbar \underline{k} \hat{a}_{k\lambda}^+ \hat{a}_{k\lambda}$$

8.4.3. Korrespondenz & kohärente Zustände



$$\psi_n(x) \rightarrow P(x) = |\psi_n(x)|^2 = |\langle x | \psi_n \rangle|^2$$

klassisch: $x(t) = A \cos(\omega t - \delta)$

$$P(x, t) = \delta\left(x - A \cdot \cos\left(\frac{\omega t - \delta}{g}\right)\right)$$

$$P(x) = \frac{1}{2\hbar} \int_0^{\frac{1}{g}} P(x, g) dg = \begin{cases} \frac{1}{\hbar} \frac{1}{\sqrt{A^2 - x^2}} & : x^2 < A^2 \\ 0 & : \text{sonst} \end{cases}$$

Glauber-Zustände

$$|\alpha\rangle = \exp\left\{-\frac{|\alpha|^2}{2} + \alpha \cdot a^\dagger\right\} |0\rangle \quad \langle \alpha | \alpha \rangle = 1 \text{ aber } \langle \alpha | \alpha' \rangle \neq 0$$

$$= e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle \Rightarrow a |\alpha\rangle = \alpha |\alpha\rangle$$

$$\langle \alpha | a^\dagger = \langle \alpha | \cdot \alpha^*$$

$$\left. \begin{aligned} \Delta x^2 &= \langle x^2 \rangle - \langle x \rangle^2 = \frac{\hbar}{2m\omega} \\ \Delta p^2 &= \langle p^2 \rangle - \langle p \rangle^2 = \frac{\hbar m \omega}{2} \end{aligned} \right\} \Delta x \cdot \Delta p = \frac{\hbar}{2} \quad \text{minimale die GSR}$$

$$\begin{aligned} \langle H \rangle_\alpha &= \hbar\omega \left(\langle \alpha | a^\dagger a | \alpha \rangle + \frac{1}{2} \right) \\ &= \hbar\omega \left(|\alpha|^2 + \frac{1}{2} \right) \end{aligned}$$

$$\begin{aligned} \langle x \rangle_\alpha &= \sqrt{\frac{\hbar}{2m\omega}} \langle \alpha | e^{i\hat{x}/\delta} (a+a^\dagger) e^{-i\hat{x}/\delta} | \alpha \rangle \\ &= \sqrt{\frac{\hbar}{2m\omega}} \langle \alpha | a e^{-i\hat{x}/\delta} + a^\dagger e^{i\hat{x}/\delta} | \alpha \rangle \end{aligned}$$

$$\alpha = |\alpha| e^{-i\delta}$$

$$= \sqrt{\frac{\hbar}{2m\omega}} 2 \cdot |\alpha| \cos(\omega t - \delta)$$

$$P_\alpha(x, t) = |\langle x | \psi_\alpha \rangle|^2 = \frac{1}{\sqrt{2\pi} \cdot \delta} \exp\left\{-\frac{[x - 2|\alpha| \cdot \delta \cos(\omega t - \delta)]^2}{2 \delta^2}\right\} \quad \delta = \sqrt{\frac{\hbar}{2m\omega}}$$

$$\mu(z) = \langle \alpha | e^{z \cdot \hat{x}} | \alpha \rangle \Rightarrow (-i\partial_z)^n \mu(z) \Big|_{z=0} = \langle \alpha | \hat{x}^n | \alpha \rangle = \langle x^n \rangle$$

$$= \int e^{izx} P_\alpha(x, t) dx \quad \text{Momentum-generierende Fkt}$$

$$\rightarrow P_\alpha(x, t) = \frac{1}{\hbar} \int \mu(z) e^{-izx} dz$$

b) E-Feld

vorher $|u\rangle = |u_{k_1, -}\rangle \otimes \dots \otimes |u_{k_n, +}\rangle$

Fock

jetzt $|u\rangle = |\alpha_{k_1, -}\rangle \otimes \dots \otimes |\alpha_{k_n, +}\rangle$

Glauber

$$A = \langle \hat{A} | \hat{A} | \alpha \rangle = \sum_k \sum_{\lambda} \left(\dots \left(\alpha_{k, \lambda} e^{-i(k \cdot r - \omega t)} + \alpha_{k, \lambda}^* e^{-i(k \cdot r - \omega t)} \right) \right) \frac{1}{\epsilon_0}$$

$$\Rightarrow \langle \alpha | \hat{E} | \alpha \rangle \neq 0 \quad \Rightarrow \text{klassisches Licht}$$