Wdh · Noether-Theorem für Felder $\partial_\mu \hat{j}^\mu = 0$

$x^\mu \to x^\mu + \delta x^\mu \to$ ETT

- Quellen $\longrightarrow$ Felder
  Felder $\longrightarrow$ Dynamik von PL

  (Felder, Quellen = $\sum$ PL)

- $L = \sum_i (-m_i c^2)\sqrt{1-\frac{\dot{q}_i^2}{c^2}} + \int \mathcal{L}_{ED}\, d^3r$

- Hamilton-Dichte $\hat{\mathcal{H}} = \frac{1}{8\pi}\left(\underline{E}^2 + \underline{B}^2\right) = 2\pi c^2\, \underline{\Pi}^2 + \frac{1}{8\pi}(\nabla\times\underline{A})^2$

- Vorb. Quantisierung
  Coulomb-Eichung $\nabla\cdot A = 0$
  Keine Quellen $\rho = 0 \quad \underline{j} = 0$ $\Big\}$ $\Phi = 0$, $\Box \underline{A} = 0$ $\qquad \underline{E} = -\frac{1}{c}\partial_t \underline{A}$, $\underline{B} = \nabla\times A$

$$\underline{A}(\underline{r},t) = \sum_{\underline{k}}\sum_{\lambda}\sqrt{\frac{2\pi\hbar c^2}{\omega(k)L^3}}\Big(\underbrace{A_\lambda(\underline{k})}_{\in\mathbb{C}} e^{i(\underline{k}\underline{r}-\omega(k)\cdot t)} + \text{c.c.}\Big)\,\underline{u}_\lambda^{\underline{k}}$$

Polarisationen

$$\underline{u}_\lambda^k\cdot\underline{k} = 0 \qquad u_\lambda^k\cdot u_\sigma^k = \delta_{\lambda\sigma} \qquad u_\lambda^{+k} = u_\lambda^{-k}$$

$$H = \frac{1}{8\pi}\int d^3r \left[\left(\frac{1}{c}\frac{\partial A}{\partial t}\right)^2 + (\nabla\times A)^2\right]$$

$$\boxed{H = \frac{1}{2}\sum_{k}\sum_{\lambda}\hbar\omega(k)\left[A_\lambda(k)A_\lambda^*(k) + A_\lambda^*(k)A_\lambda(k)\right]}$$

$\hookrightarrow$ entkoppelte harmon. Oszillatoren

8.4.2. Quantisierung

wollen $A \longrightarrow \hat{A}$ $\qquad \hat{A}_H = e^{+iHt}\hat{A}\, e^{-iHt}$

$$\frac{d}{dt}A_H = \frac{i}{\hbar}[H_H, A_H] \quad \text{(ohne expl. Z.A.)}$$

a.) einzelner Oszillator

$$\hat{H} = \frac{\hat{P}^2}{2m} + \frac{1}{2}m\omega^2\hat{q}^2$$

Leiter-Operatoren $\quad a = \frac{m\omega\cdot\hat{q} + i\hat{P}}{\sqrt{2m\hbar\omega}} \qquad a^+ = \frac{m\omega\hat{q} - i\hat{P}}{\sqrt{2m\hbar\omega}}$

$$[\hat{q},\hat{P}] = i\hbar \iff [a, a^+] = 1$$

$$H = \frac{\hbar\omega}{2}(a a^+ + a^+ a) = \hbar\omega(\underbrace{a^+ a}_{\text{TZ-Operator}} + 1/2) \qquad a^+ a|n\rangle = n\cdot|n\rangle$$

Nullpunktsenergie

$$H|n\rangle = (\hbar\omega(n+1/2))|n\rangle$$

$a^+|n\rangle = \sqrt{n+1}\,|n+1\rangle$ $\qquad a|n\rangle = \sqrt{n}\,|n-1\rangle$

"Erzeuger" $\qquad$ "Vernichter"

GZ $|0\rangle$ $\qquad \frac{(a^+)^n}{\sqrt{n!}}|0\rangle = |n\rangle$ $\qquad$ Fock-Raum

$a_H(t) = e^{+\frac{i}{\hbar}Ht}\, a\, e^{-\frac{i}{\hbar}Ht}$ $\qquad [H,a] = \hbar\omega\,[a^+a, a] = -\hbar\omega\, a$

$\frac{d}{dt} a_H(t) = \frac{i}{\hbar} e^{+iHt}[H,a]\, e^{-iHt}$

$= -i\omega \cdot e^{+iHt} a\, e^{-iHt} = -i\omega\, a_H(t) \rightsquigarrow a_H(t) = e^{-i\omega t} a$

$a_H^+(t) = e^{+i\omega t} a^+$

b.) elektrom. Feld

Quantisierung

$A_\lambda(k) \longrightarrow \hat{A}_\lambda(k) = a_{\lambda k}$ $\qquad A_\lambda^*(k) \longrightarrow a_{\lambda k}^+$

Pol. $\quad$ Wellenvekt.

$\frac{d}{dt}\hat{A}_H = \frac{i}{\hbar}[H, \hat{A}_H]$ $\qquad [a_{k\lambda}, a_{k\lambda}^+] = 1$

$H = \sum_k \sum_\lambda \frac{\hbar\omega(k)}{2}\left[a_{k\lambda}^+ a_{k\lambda} + a_{k\lambda} a_{k\lambda}^+\right]$

(fermionisch $\{c_k, c_{k'}^+\} = c_k c_{k'}^+ + c_{k'}^+ c_k = \delta_{kk'}$)

$$\boxed{[a_{k\lambda}, a_{k'\lambda'}^+] = \delta_{kk'}\delta_{\lambda\lambda'} \qquad [a_{k\lambda}, a_{k'\lambda'}] = 0 \qquad [a_{k\lambda}^+, a_{k'\lambda'}^+] = 0}$$

"bosonische Kommutator-Relationen"

$H = \sum_k \sum_\lambda \hbar\omega_k\,(a_{k\lambda}^+ a_{k\lambda} + 1/2)$

$\frac{d}{dt} a_{k\lambda,H}(t) = e^{+iHt}\frac{i}{\hbar}[H, a_{k\lambda}]\, e^{-iHt} = -i\omega_k\, a_{k\lambda,H}(t) \rightsquigarrow a_{k\lambda,H}(t) = e^{-i\omega(k)\cdot t} a_{k\lambda}$

$\underline{\hat{A}}_H = \sum_k \sum_\lambda \sqrt{\frac{2\pi\hbar c^2}{L^3\omega(k)}}\left[a_{k\lambda,H}(t)\cdot e^{i\underline{k}\cdot\underline{r}} + \underbrace{a_{k\lambda,H}^+(t)\, e^{-i\underline{k}\cdot\underline{r}}}_{\text{h.c.}}\right]\cdot \underline{n}_\lambda^k$

Eigenschaften

- $\frac{d}{dt} A_H = \frac{i}{\hbar}[H, A_H]$
- Felder $\hat{E} = -\frac{1}{c}\partial_t \hat{A}$ $\qquad \hat{B} = \nabla\times\hat{A}$

$$\underline{E}_H = i\sum_k \sum_\lambda \sqrt{\ldots}\left(\hat{a}_{k\lambda,H}(t)\, e^{+i\underline{k}\cdot\underline{r}} - h.c.\right) = \underline{E}_H^+$$

$[E_{H,i}, B_{H,j}] \neq 0$ im Allgemeinen

∘ ein Photon wird gebildet durch

$$a^+_{k\lambda}|0,\ldots,0\rangle \hat{=} a^+_{k\lambda}|0\rangle$$

$$= |0,\ldots,0,\overset{k\lambda}{1},0,\ldots 0\rangle$$

∘ mehrere Photonen der gleichen Mode

$$|0,\ldots,0,n_{k\lambda},0\ldots 0\rangle \hat{=} |n_{k\lambda}\rangle = \frac{\left(a^+_{k\lambda}\right)^{n_{k\lambda}}}{\sqrt{n_{k\lambda}!}}|0\ldots 0\rangle$$

∘ mehrere Phot. versch. Moden

$$|\underline{n}\rangle = |\{n_{k\lambda}\}\rangle = |n_{k_1\lambda_1}, n_{k_1\lambda_2}, n_{k_2\lambda_1},\ldots\rangle$$

$$= \frac{1}{\sqrt{\prod_k\prod_\lambda n_{k\lambda}!}}\prod_k\prod_\lambda \left(a^+_{k\lambda}\right)^{n_{k\lambda}}|0,\ldots 0\rangle$$

eigentlich Tensorprodukt

2 Moden $|n_1 n_2\rangle \hat{=} |n_1\rangle\otimes|n_2\rangle$

qubits 2 qubits n qubits

$$\left.\begin{array}{l}|00\rangle = |0\rangle\otimes|0\rangle\\ |01\rangle\\ |10\rangle\\ |11\rangle\end{array}\right\} 2^n \text{ Basiszust.}$$

$$\langle \underline{n}|\underline{m}\rangle = \delta_{\underline{n},\underline{m}} = \prod_k\prod_\lambda \delta_{(n_{k\lambda}),(m_{k\lambda})}$$

• $\langle \underline{n}|\hat{E}|\underline{n}\rangle = 0 = \langle \underline{n}|\hat{B}|\underline{n}\rangle$

Fock-Zustände sind nicht klassisch

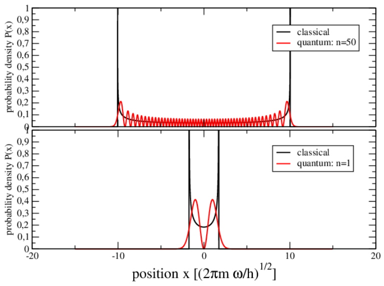
$a^+_{k\lambda}a_{k\lambda}|\underline{n}\rangle = n_{k\lambda}|\underline{n}\rangle$

∘ $\langle n|H|n\rangle = \sum_k\sum_\lambda \hbar\omega_k\left(n_{k\lambda}+1/2\right)$

∘ $w = \frac{1}{8\pi}\langle n|\left(\hat{\underline{E}}^2+\hat{\underline{B}}^2\right)|n\rangle = \sum_k\sum_\lambda \frac{\hbar\omega_k}{L^3}\left(n_{k\lambda}+1/2\right)$

• Feldimpuls $\hat{\underline{P}} = \frac{1}{4\pi c}\int \hat{\underline{E}}\times\hat{\underline{B}}\, d^3r = \sum_k\sum_\lambda \hbar\underline{k}\, a^+_{k\lambda}a_{k\lambda}$

8.4.3. Korrespondenz & kohärente Zustände



$$\psi_n(x) \longrightarrow P(x) = |\psi_n(x)|^2 = |\langle x|n\rangle|^2$$

klassisch: $x(t) = A\cos(\omega t - \delta)$

$$P(x,t) = \delta_{2\pi}(x - A\cdot\cos(\underbrace{\omega t-\delta}_{\varphi}))$$

$$P(x) = \frac{1}{2\pi}\int_0^{2\pi} P(x,\varphi)\,d\varphi = \begin{cases} \frac{1}{\pi}\frac{1}{\sqrt{A^2-x^2}} & : x^2 < A^2 \\ 0 & : \text{sonst} \end{cases}$$

Glauber-Zustände

$$|\alpha\rangle = \exp\left\{-\frac{|\alpha|^2}{2} + \alpha\cdot a^\dagger\right\}|0\rangle \qquad \langle\alpha|\alpha\rangle = 1 \text{ aber } \langle\alpha|\alpha'\rangle \neq 0$$

$$= e^{-|\alpha|^2/2}\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle \qquad \Rightarrow a|\alpha\rangle = \alpha|\alpha\rangle$$

$$\langle\alpha|a^\dagger = \langle\alpha|\cdot\alpha^*$$

$$\left.\begin{aligned}\Delta x^2 &= \langle x^2\rangle - \langle x\rangle^2 = \frac{\hbar}{2m\omega}\\ \Delta p^2 &= \langle p^2\rangle - \langle p\rangle^2 = \frac{m\hbar\omega}{2}\end{aligned}\right\} \Delta x\cdot\Delta p = \frac{\hbar}{2}$$

minimieren die USR

$$\langle H\rangle_\alpha = \hbar\omega\left(\langle\alpha|a^\dagger a|\alpha\rangle + 1/2\right) = \hbar\omega\left(|\alpha|^2 + 1/2\right)$$

$$\langle x\rangle_\alpha = \sqrt{\frac{\hbar}{2m\omega}}\,\langle\alpha|e^{+iHt}(a+a^\dagger)e^{-iHt}|\alpha\rangle$$

$$= \sqrt{\frac{\hbar}{2m\omega}}\,\langle\alpha|a e^{-i\omega t} + a^\dagger e^{+i\omega t}|\alpha\rangle \qquad \alpha = |\alpha|e^{-i\delta}$$

$$= \underbrace{\sqrt{\frac{\hbar}{2m\omega}}\,2\cdot|\alpha|}_{A}\cos(\omega t - \delta)$$

$$\boxed{P_\alpha(x,t) = |\langle x|\psi_\alpha\rangle|^2 = \frac{1}{\sqrt{2\pi}\cdot\sigma}\exp\left\{-\frac{[x - 2|\alpha|\cdot\sigma\cos(\omega t-\delta)]^2}{2\sigma^2}\right\} \qquad \sigma = \sqrt{\frac{\hbar}{2m\omega}}}$$

$$M(z) = \langle\alpha|e^{iz\cdot\hat{x}}|\alpha\rangle \Rightarrow (-i\partial_z)^n M(z)\Big|_{z=0} = \langle\alpha|\hat{x}^n|\alpha\rangle = \langle x^n\rangle$$

$$= \int e^{izx}P_\alpha(x,t)\,dx$$

Momenten-generierende Fkt

$$\longrightarrow P_\alpha(x,t) = \frac{1}{2\pi}\int M(z)\,e^{-iz\cdot x}\,dz$$

b.) EM-Feld

Fock

vorher $|\underline{n}\rangle = |n_{k_1,-}\rangle \otimes \ldots \otimes |n_{k_K,t}\rangle$

Glauber

jetzt $|\alpha\rangle = |\alpha_{k_1,-}\rangle \otimes \ldots \otimes |\alpha_{k_K,t}\rangle$

$$A = \langle\alpha|\hat{A}_k|\alpha\rangle = \sum_k \sum_\lambda \sqrt{\ldots}\left(\alpha_{k\lambda}\, e^{i(kx-\omega t)} + \alpha_{k\lambda}^*\, e^{-i(kx-\omega t)}\right) \epsilon_{k\lambda}$$

$\Rightarrow \langle\alpha|\hat{E}_{\parallel}|\alpha\rangle \neq 0 \quad \hat{=}$ klassisches Licht