

Wdh + Magnetostatik $\rightarrow \frac{\partial \rho}{\partial t} = 0 \Rightarrow \boxed{\nabla \cdot \underline{j} = 0}$
 $\frac{\partial \rho}{\partial t} + \nabla \cdot \underline{j} = 0$

+ mikroskop. Def. der SD

$$\underline{j} = \rho(\underline{r}) \cdot \underline{v}(\underline{r})$$

\uparrow Ladungsdichte \uparrow mittl. Geschwindigkeit

+ Konzept Stromfäden $\hat{=}$ Analogon zur PL in der ES

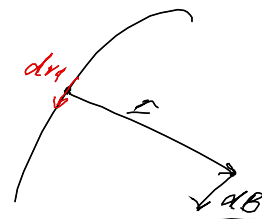
Stromfaden-Substitution

$$\underline{j} d^3r \Leftrightarrow \underline{I} d\underline{r}$$

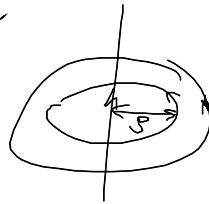
Volumenintegral über Stromdichte $\hat{=}$ Linienintegral entlang Stromfäden

+ Biot-Savart

$$\underline{dB} = \frac{\underline{I}}{c} \frac{d\underline{r}_1 \times \underline{r}}{|\underline{r}|^3}$$



radial lagere Kraft



$$\underline{B} = \frac{2\underline{I}}{c \cdot \rho} \cdot \underline{e}_z$$

+ Ampèresches Gesetz

Kraft auf ein \underline{v} Linienelement $d\underline{r}_1$ in ext. Feld

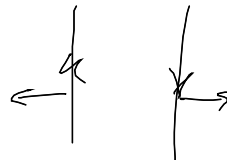
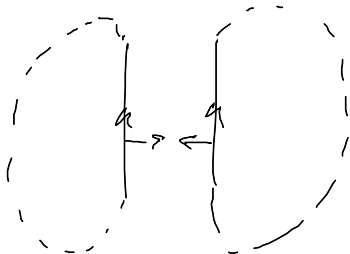
$$d\underline{F} = \frac{\underline{I}}{c} d\underline{r}_1 \times \underline{B}$$

\Rightarrow Kraft zwischen 2 geschlossenen Leitern

$$\underline{I}_2 = \underline{I}_1 \sqrt{\epsilon_0 \epsilon_0}$$

$$\mu_0 \cdot \epsilon_0 = \frac{1}{c^2}$$

$$\underline{F}_{12} = - \frac{\underline{I}_1 \cdot \underline{I}_2}{c^2} \oint_{C_1} \oint_{C_2} \frac{\underline{r}_1 - \underline{r}_2}{|\underline{r}_1 - \underline{r}_2|^3} d\underline{r}_1 \cdot d\underline{r}_2$$



$$\int j \, d^3v$$

4.4. Maxwell-Gleichungen der Magnetostatik

lokale $d\underline{B} = \frac{\underline{I}}{c} \frac{d\underline{r}_1 \times \underline{r}}{|\underline{r}|^3}$ \underline{r} = Vektor von $d\underline{r}_1$ zur Beobachtungspunkt

$B(\underline{r}) = \int d\underline{B} = \frac{\underline{I}}{c} \oint \frac{d\underline{r}' \times (\underline{r} - \underline{r}')}{|\underline{r} - \underline{r}'|^3} = \left(\frac{\underline{I}}{c} \int \frac{j(\underline{r}') \times (\underline{r} - \underline{r}')}{|\underline{r} - \underline{r}'|^3} d^3r' = B(\underline{r}) \right)$
 Beobachtungspunkt Gesetz von Biot-Savart (globale Form.)

ES $\underline{E}(\underline{r}) = \int \rho(\underline{r}') \frac{\underline{r} - \underline{r}'}{|\underline{r} - \underline{r}'|^3} d^3r'$

anschieben $B(\underline{r}) = \frac{\underline{I}}{c} \int d^3r' j(\underline{r}') \times \left(-\nabla \frac{1}{|\underline{r} - \underline{r}'|} \right) = + \frac{\underline{I}}{c} \int d^3r' \left(\nabla \cdot \frac{\underline{r}}{|\underline{r} - \underline{r}'|} \right) \times j(\underline{r}')$
 $= \nabla \times \left(\frac{\underline{I}}{c} \int \frac{j(\underline{r}')}{|\underline{r} - \underline{r}'|} d^3r' \right)$ B ist ein Rotationsfeld

$\Rightarrow \left(\nabla \cdot \underline{B} = 0 \right)$ $\Rightarrow \oint_{\partial V} \underline{B} \cdot d\underline{F} = 0$ *Kommutativgesetz*

$\nabla \times \underline{B} = \nabla \times \left(\nabla \times \left(\frac{\underline{I}}{c} \int \frac{j(\underline{r}')}{|\underline{r} - \underline{r}'|} d^3r' \right) \right)$
 $= \frac{\underline{I}}{c} \int \nabla \left(\nabla \cdot \frac{j(\underline{r}')}{|\underline{r} - \underline{r}'|} \right) d^3r' - \frac{\underline{I}}{c} \int \Delta \frac{j(\underline{r}')}{|\underline{r} - \underline{r}'|} d^3r'$

$= \frac{\underline{I}}{c} \int j(\underline{r}') \left(\nabla' \cdot \frac{\underline{r}}{|\underline{r} - \underline{r}'|} \right) d^3r' + \frac{\underline{I}}{c} j(\underline{r}') \Delta \frac{1}{|\underline{r} - \underline{r}'|} = -\frac{\underline{I}}{c} \int j(\underline{r}') \delta(\underline{r} - \underline{r}') d^3r' + \frac{\underline{I}}{c} j(\underline{r}') \Delta \frac{1}{|\underline{r} - \underline{r}'|}$

$= + \frac{\underline{I}}{c} \int \left(\nabla' \cdot \frac{j(\underline{r}')}{|\underline{r} - \underline{r}'|} \right) d^3r' - \frac{\underline{I}}{c} \int \nabla' \cdot \left(\frac{j(\underline{r}')}{|\underline{r} - \underline{r}'|} \right) d^3r' + \frac{\underline{I}}{c} j(\underline{r}') \Delta \frac{1}{|\underline{r} - \underline{r}'|}$

$= 0 - \frac{\underline{I}}{c} \underbrace{\oint_{\partial V} \frac{j(\underline{r}')}{|\underline{r} - \underline{r}'|} d\underline{F}'}_0 + \frac{\underline{I}}{c} j$

$$\nabla \times \underline{B} = \frac{4\pi}{c} \cdot \underline{j}(\underline{r})$$

zweites Maxwell'sches Gesetz

Maxwell

Maxwell-Gleichungen der MS

$$\nabla \cdot \underline{B} = 0$$

$$\nabla \times \underline{B} = \frac{4\pi}{c} \cdot \underline{j}(\underline{r})$$

lokale Form

$$\iint (\nabla \times \underline{B}) \cdot d\underline{F} = \oint_{C=\partial F} \underline{B} \cdot d\underline{r} = \frac{4\pi}{c} \iint \underline{j} \cdot d\underline{F} = \frac{4\pi}{c} \cdot \underline{I}_F$$

integrale Form

$$\oint \underline{B} \cdot d\underline{F} = 0$$

$$\oint_{\partial F} \underline{B} \cdot d\underline{r} = \frac{4\pi}{c} \cdot \underline{I}_F$$

4.5. Das Vektorpotential

$$\underline{B} = \nabla \times \underline{A} \quad \underline{A} \text{ "Vektorpotential"}$$

$$\underline{A} \rightarrow \underline{A} + \nabla \cdot \varphi(\underline{r}) \quad \text{"Eichfreiheit"}$$

↑
skalare Funktion

$$\nabla \times \underline{B} = \nabla \times (\nabla \times \underline{A}) = \nabla (\underbrace{\nabla \cdot \underline{A}}_0) - \Delta \underline{A}$$

Coulomb-Eichung: Wähle $\varphi(\underline{r})$ so, dass $\nabla \cdot \underline{A} = 0$

$$\nabla \times \underline{B} = -\Delta \underline{A} \quad \text{in Coulomb-Eichung}$$

$$\Rightarrow \Delta \underline{A} = -\frac{4\pi}{c} \cdot \underline{j} \quad \Delta A_i = -\frac{4\pi}{c} \cdot j_i$$

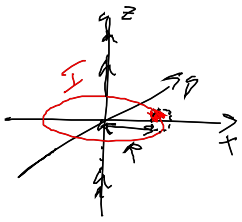
Poisson-Gleichung für jede Komponente

ohne RB z. Endlichen \underline{A} gibt

$$\underline{A}(\underline{r}) = \frac{1}{c} \int \frac{\underline{j}(\underline{r}')}{|\underline{r}-\underline{r}'|} d^3r'$$

$$\underline{B}(\underline{r}) = \nabla \times \frac{1}{c} \int \frac{\underline{j}(\underline{r}')}{|\underline{r}-\underline{r}'|} d^3r' = \nabla \times \underline{A}$$

4.6. Vektorfeld einer Stromschleife



in Zylinderkoordin.

$$\underline{e}_\rho = \begin{pmatrix} \cos\varphi \\ \sin\varphi \\ 0 \end{pmatrix} \quad \underline{e}_\varphi = \begin{pmatrix} -\sin\varphi \\ \cos\varphi \\ 0 \end{pmatrix} \quad \underline{e}_z = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\Rightarrow \underline{e}_\varphi \times \underline{e}_z = \underline{e}_\rho \quad \underline{e}_\rho \times \underline{e}_\varphi = -\underline{e}_z$$

$$\underline{j}(\underline{r}') = \underset{\substack{\uparrow \\ \text{Stromstärke}}}{I} \cdot \delta(z') \cdot \delta(\rho' - R) \cdot \underline{e}_\varphi$$

$$\text{MF} \quad B(\underline{r}) = \frac{I}{c} \int \int \int \underline{j}(\underline{r}') \times \frac{\underline{r} - \underline{r}'}{|\underline{r} - \underline{r}'|^3} d^3r'$$

$$B(z, \underline{e}_z) = \frac{I}{c} \int_0^{2\pi} \int_0^{\infty} \int_{-\infty}^{\infty} \delta(\rho' - R) \delta(z') \frac{\underline{e}_\varphi \times (z - z') \underline{e}_z - \rho' \underline{e}_\rho}{[(z - z')^2 + (\rho')^2]^{3/2}}$$

$$= \frac{I}{c} \int_0^{2\pi} \int \frac{(z - z') \underline{e}_\rho + \rho' \underline{e}_z}{[\dots]^{3/2}}$$

$$= \frac{I}{c} \int_0^{2\pi} \frac{z \cdot \underline{e}_\rho + R \cdot \underline{e}_z}{(z^2 + R^2)^{3/2}} \cdot R \int_0^{2\pi} \sin(\varphi) d\varphi = 0$$

$$= \frac{2\pi I}{c} \cdot \frac{R^2}{(z^2 + R^2)^{3/2}} \cdot \underline{e}_z$$

oder $B(z, \underline{e}_z) = \frac{I}{c} \oint \underline{dr}' \times \frac{z \underline{e}_z - R \underline{e}_\rho}{(z^2 + R^2)^{3/2}} \quad \underline{dr}' = R \cdot \underline{e}_\varphi \cdot d\varphi'$

$$= \frac{I}{c} \int_0^{2\pi} \frac{R \cdot \underline{e}_\varphi \times (z \underline{e}_z - R \underline{e}_\rho)}{(z^2 + R^2)^{3/2}} d\varphi' = \frac{2\pi I}{c} \frac{R^2}{(z^2 + R^2)^{3/2}} \cdot \underline{e}_z$$

oder Vektorpotential

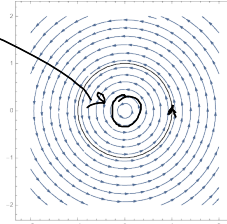
$$A(z, \underline{e}_z + \rho \underline{e}_\rho) = \frac{I}{c} \int \frac{\underline{j}(\underline{r}')}{|\underline{r} - \underline{r}'|} d^3r' = \frac{I}{c} \oint \frac{d\underline{r}'}{|\underline{r} - \underline{r}'|}$$

$$= \frac{I \cdot R}{c} \int_0^{2\pi} \frac{\underline{e}_\varphi d\varphi'}{|\rho \underline{e}_\rho + z \underline{e}_z - R \underline{e}_\rho|}$$

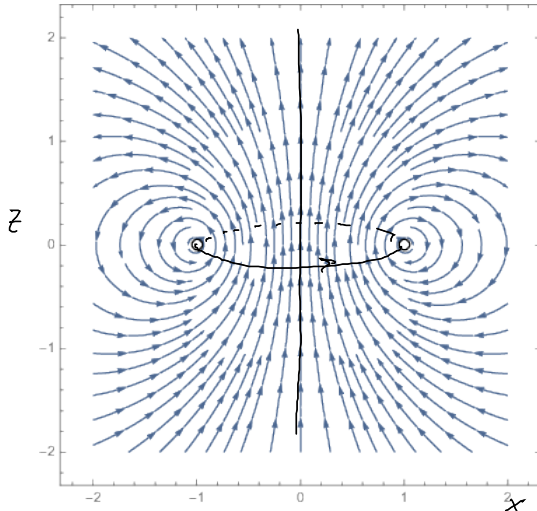
$$= \frac{I \cdot R}{c} \int_0^{2\pi} \frac{d\varphi' \underline{e}_\varphi}{\sqrt{\rho^2 z^2 + R^2 - 2R\rho(\cos\varphi \cos\varphi' + \sin\varphi \sin\varphi')}} \quad \text{mit } \varphi = \varphi'$$

Entwicklung für kleine ρ

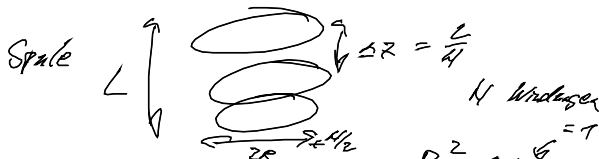
$$A = \frac{\mu_0 I}{c} \frac{R^2 \rho}{(R^2 + z^2)^{3/2}} \cdot e_\phi + \mathcal{O}(\rho^2)$$



Vektorpot. in xy -Ebene



Magnetfeld in der xz -Ebene



$$B(z, e_z) = \frac{\mu_0 I}{c} \cdot e_z \int_{-L/2}^{L/2} \frac{R^2 dz'}{\sqrt{(z - z')^2 + R^2}^{3/2}}$$

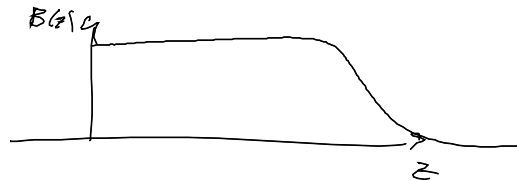
$$\approx \frac{\mu_0 I}{c} \cdot e_z \int_{-L/2}^{L/2} \frac{R^2 dz'}{[(z - z')^2 + R^2]^{3/2}}$$

$$= \frac{\mu_0 I}{c} \cdot e_z \frac{N}{L} \int_{-L/2}^{L/2} \frac{R^2 dz'}{\sqrt{(z - z')^2 + R^2}^{3/2}}$$

$$dz' = dz \cdot \frac{L}{N}$$

$$z' = z - \frac{L}{N} \cdot k$$

$$\xrightarrow{L \rightarrow \infty} \frac{\mu_0 I}{c} \cdot \frac{N}{L} \cdot e_z$$



$$B = \frac{\mu_0}{c} \int j(r') \times \frac{r - r'}{|r - r'|^3} d^3 r'$$