Wdh 4er Vektoren $(x^\mu) = \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$ $(x_\mu) = \begin{pmatrix} ct \\ -x \\ -y \\ -z \end{pmatrix}$

kontravarianten

$x_\mu = \eta_{\mu\nu} x^\nu$ $(\eta_{\mu\nu}) = \begin{pmatrix} +1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$

Summenkonvention spez. Konvention

$x_\mu y^\mu = \eta_{\mu\nu} x^\mu y^\nu = x^\mu y_\mu$ $v = v \cdot e_x$ ist Geschw. von K' in K

$x'^\mu = \Lambda^\mu{}_\nu x^\nu$ $(\Lambda^\mu{}_\nu) = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

in K' in K

$\boxed{\eta = \Lambda \eta \Lambda}$ $\longrightarrow$ Skalarprod. ist invariant

- 4er Ableitung $(\partial_\mu) = \left(\frac{\partial}{\partial x^\mu}\right) = \begin{pmatrix} \frac{1}{c}\partial_t \\ \partial_x \\ \partial_y \\ \partial_z \end{pmatrix}$
- 4er Stromdichte $(j^\mu) = \begin{pmatrix} c \cdot \rho \\ j_x \\ j_y \\ j_z \end{pmatrix}$

$\partial_\mu j^\mu = \frac{\partial \rho}{\partial t} + \nabla \cdot j = 0$

- 4er Potential $A^\mu = \begin{pmatrix} \Phi \\ A_x \\ A_y \\ A_z \end{pmatrix}$ Lorenz-Eichung $\partial_\mu A^\mu = 0$

$\partial^\nu \partial_\nu A^\mu = \frac{4\pi}{c} j^\mu$ $(\partial^\mu) = \begin{pmatrix} \frac{1}{c}\partial_t \\ -\partial_x \\ -\partial_y \\ -\partial_z \end{pmatrix} = \eta^{\mu\nu} \partial_\nu$

$\partial_\mu x^\mu = 4$

- Feldstärketensor $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$ antisymm.

inv. gegenüber EichTrafos $A^\nu \to A^\nu + \partial^\nu \Lambda$ ← skalare Funktion

$F^{\mu\nu} \to F^{\mu\nu}$

$\partial_\mu F^{\mu\nu} = \frac{4\pi}{c} j^\nu$ $\partial^\nu F^{\mu\lambda} + \partial^\lambda F^{\nu\mu} + \partial^\mu F^{\lambda\nu} = 0$

4 inhomog. Maxwell-Gl. 4 homogene Maxwell-Gl.

$\nu \neq \mu \neq \lambda \neq \nu$

Lorentz-Kraft $\underline{F}_L = q\cdot\underline{E} + \frac{q}{c}\cdot\underline{v}\times\underline{B}$ $\qquad d\tau = dt\sqrt{1-\beta^2} = \frac{dt}{\gamma}$

$$\boxed{\frac{dp^\mu}{d\tau} = m_0\cdot\frac{du^\mu}{d\tau} = \frac{q}{c}F^{\mu\nu}\cdot u_\nu}$$

$$(u^\mu) = \begin{pmatrix}\gamma\cdot c\\ \gamma\cdot v_x\\ \gamma\cdot v_y\\ \gamma\cdot v_z\end{pmatrix}$$

$\underline{\mu=0}$: $\underbrace{m_0\gamma\frac{d}{dt}}_{\frac{d}{d\tau}}(\gamma\cdot c) = \frac{q}{c}F^{0i}\underset{\substack{\uparrow\\ i\in\{1,2,3\}}}{u_i}$

$$= \frac{q}{c}(-\underline{E})\cdot(-\gamma\cdot\underline{v})$$

$$(F^{\mu\nu}) = \begin{pmatrix}0 & -E_x & -E_y & -E_z\\ +E_x & 0 & -B_z & +B_y\\ +E_y & +B_z & 0 & -B_x\\ +E_z & -B_y & +B_x & 0\end{pmatrix}$$

$$\underbrace{\frac{d}{dt}(\gamma m_0 c^2)}_{\text{Änd. der Gesamtenergie}} = \underbrace{q\,\underline{E}\cdot\underline{v}}_{\text{mech. Leistung}}$$

F^{i0} $\qquad F^{ij}$

$\underline{\mu\in\{1,2,3\}}$: $\frac{d}{dt}(\underbrace{\gamma m_0}_{m_{eff}=\gamma\cdot m_0}\cdot\underline{v}) = q\cdot\underline{E} + \frac{q}{c}\underline{v}\times\underline{B}$

5.7.6. Transf. der Felder

$$A'^\mu = \Lambda^\mu{}_\nu A^\nu \qquad \partial'^\mu = \Lambda^\mu{}_\nu\partial^\nu$$

$$F'^{\mu\nu} = \partial'^\mu A'^\nu - \partial'^\nu A'^\mu = \Lambda^\mu{}_\sigma\Lambda^\nu{}_\kappa F^{\sigma\kappa} = \Lambda^\mu{}_\sigma F^{\sigma\kappa}\underbrace{\Lambda^\nu{}_\kappa}_{\Lambda_\kappa{}^\nu}$$

$$(F'^{\mu\nu}) = \Lambda F\Lambda$$

für $\underline{v} = v\cdot\underline{e}_x$ $\qquad E_x' = E_x \qquad E_y' = \gamma\cdot E_y - \beta\gamma B_z \qquad E_z' = \gamma\cdot E_z + \beta\gamma\cdot B_y$

$\qquad B_x' = B_x \qquad B_y' = \gamma\cdot B_y + \beta\gamma\cdot E_z \qquad B_z' = \gamma\cdot B_z - \beta\gamma\cdot E_y$

5.7.7. ~~Beispiel~~. Gleichförmig ~~bewegte~~ Ladungsverteilung

im RS der LV $\quad (j'^\mu) = \begin{pmatrix}c\cdot\varrho_0(x',y',z')\\ 0\\ 0\\ 0\end{pmatrix}$

falls $\varrho_0(\underline{r}')$ räumlich begrenzt $\rightsquigarrow$ Elektrostatik

$$\underline{\Phi}'(r') = \int\frac{\varrho_0(\bar{x}',\bar{y}',\bar{z}')}{|\underline{r}'-\bar{\underline{r}}'|}d^3\bar{r}' \rightsquigarrow E'(r') = -\nabla'\underline{\Phi}'$$

$$\underline{A}'(r') = \underline{0} \longrightarrow B'(r') = 0$$

$\rightsquigarrow \frac{1}{c}\partial_{t'}\underline{\Phi}' + \nabla'\cdot A' = 0$ $\quad$ Lorenz-Eichung

$\partial'_\mu A'^\mu = 0$ $\quad\rightsquigarrow$ gilt in allen IS transformiere die 4er Stromdichte

$$\begin{pmatrix} c\rho \\ j_x \end{pmatrix} = \begin{pmatrix} \gamma & +\gamma\beta \\ +\gamma\beta & \gamma \end{pmatrix} \begin{pmatrix} c\rho_0(x',y',z') \\ 0 \end{pmatrix} \qquad \begin{pmatrix} ct' \\ x' \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta \\ -\gamma\beta & \gamma \end{pmatrix} \begin{pmatrix} ct \\ x \end{pmatrix}$$

$$\rho(\underline{r},t) = \gamma\cdot\rho_0(x',y',z') = \gamma\cdot\rho_0(\underline{\gamma(x-v\cdot t)}, y, z)$$

• Ladungsverteilung erscheint gestaucht in Richtung der Relativ-Geschwindigkeit

$$j_x(\underline{r},t) = v\cdot\gamma\cdot\rho_0(\gamma(x-v\cdot t), y, z) = v\cdot\rho(\underline{r},t)$$

Gesamtladung ist invariant

$$\int\rho(\underline{r},t)\,d^3r = \gamma\int\rho_0(\gamma(x-vt),y,z)\,d^3r = Q = Q'$$

im LS muss gelten (wegen Lorenz-Eichung)

$$\Box\Phi = -4\pi\rho(r,t) \qquad \begin{pmatrix} \Phi \\ A_x \end{pmatrix} = \begin{pmatrix} \gamma & +\beta\gamma \\ +\beta\gamma & \gamma \end{pmatrix} \begin{pmatrix} \Phi' \\ 0 \end{pmatrix} \qquad A_y' = A_y = 0$$

$$\Box\underline{A} = -\frac{4\pi}{c}\underline{j}(r,t) \qquad A_z' = A_z = 0$$

$$\Phi(\underline{r},t) = \gamma\int\frac{\rho_0(\bar{r}')}{\sqrt{(\underbrace{\gamma x-\beta\gamma ct}_{x'}-\bar{x}')^2+(y-\bar{y}')^2+(z-\bar{z}')^2}}\,d^3\bar{r}' \qquad A_x = \beta\cdot\Phi(\underline{r},t)$$

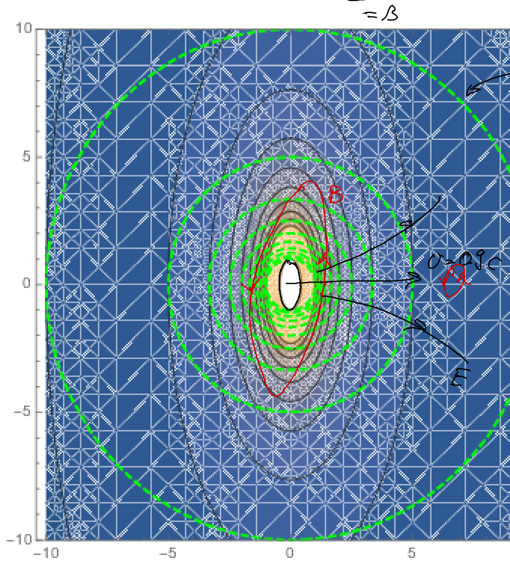
$$\Box\Phi = \gamma\int\left\{\partial_{x'}^2+\partial_{y'}^2+\partial_{z'}^2\cancel{-\frac{1}{c^2}\partial_{t'}^2}\right\}\frac{\rho_0(\bar{r}')}{\sqrt{\ldots}}\,d^3\bar{r}'$$

$$= -4\pi\gamma\rho_0(x',y',z') = -4\pi\gamma\rho_0(\gamma x-\beta\gamma ct, y, z) = -4\pi\rho(r,t)$$

$$\Delta\frac{1}{|r-\bar{r}'|} = -4\pi\delta(r-\bar{r}')$$

$$\Box A_x = -\frac{4\pi}{c}\cdot j_x$$

$$\beta\,\Box\Phi = -4\pi\underbrace{\frac{v}{c}}_{=\beta}\rho$$



Potential einer radialsymm. LV mit homog. Ladungsdichte in einer Kugel

$$\Phi'(r'>R) = \frac{Q}{r'}$$

5.7.8. Doppler-Effekt

Welcher 4er Vektor beschreibt eine ebene Welle

$$(k^\mu) = \begin{pmatrix} \omega/c \\ k_x \\ k_y \\ k_z \end{pmatrix} \qquad \omega = |\underline{k}|c \qquad \begin{pmatrix} \omega'/c \\ k_x' \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma \\ -\beta\gamma & \gamma \end{pmatrix} \begin{pmatrix} \omega/c \\ k_x \end{pmatrix}$$

$$x_\mu k^\mu = ct\,\frac{\omega}{c} - \underline{k}\cdot\underline{r} \qquad \underline{A} = \mathrm{Re}\left\{A_0\, e^{-i x_\mu k^\mu}\right\}$$ Vektorpot. einer ebenen Welle

+ betrachte Signal, welches von einer bewegten Quelle (K') ausgesandt wird
+ wird von ruhenden Beobachter gemessen (K)
+ K' bewege sich mit $\underline{v} = v\cdot\underline{e}_x$ in K

$$\omega' = \gamma\cdot\omega - \beta\cdot\gamma\, c\cdot k_x = \gamma\omega - \gamma\,\underline{v}\cdot\underline{k} = \gamma\cdot\omega - \gamma v\cdot \underbrace{k}_{\frac{\omega}{c}}\cdot\cos\vartheta$$
$$= \gamma\omega\left[1-\beta\cos\vartheta\right]$$

$$\boxed{\omega = \frac{\omega'\sqrt{1-\beta^2}}{1-\beta\cdot\cos\vartheta}}$$ Kreisfreq. in Laborsystem

$\vartheta = \sphericalangle(\underline{k},\underline{v})$ rel. Doppler-Effekt

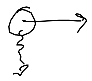
a.) longitudinaler Doppler-Effekt ($\beta > 0$)

- $\cos\vartheta = +1$ $\quad \omega > \omega'$ Blauverschiebung $\quad \omega = \frac{\omega'\sqrt{1-\beta^2}}{1-\beta}$ Quelle bewegt sich auf Empfänger zu
- $\cos\vartheta = -1$ $\quad \omega < \omega'$ Rotverschiebung $\quad \omega = \frac{\omega'\sqrt{1-\beta^2}}{1+\beta}$ Quelle bewegt sich weg

b.) transversaler Doppler-Effekt ($\beta > 0$)

$\cos\vartheta = 0$

$$\omega = \omega'\sqrt{1-\beta^2} < \omega'$$



- Akustischer Doppler-Effekt hängt von beiden Relativ-v (Sender & Empfänger) ab
  Sender-Medium
  Empfänger-Medium
- relativ. Doppler-Effekt hängt nur von v_{Rel} Sender-Empfänger ab

Mathl. Einschub: Funktionentheorie